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TWO STAGE SPECIALLY STRUCTURED FLOW SHOP SCHEDULING WITH JOB RESTRICTIONS TO MINIMIZE THE RENTAL COST

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ABSTRACT

One of the earliest results in flow shop scheduling is an algorithm given by Johnson's (1954) for scheduling the jobs on two/three machines with minimum makespan. In most of the literature, the processing times of jobs are always considered to be random. But there are significant situations in which the processing time are not merely random but bear a well defined relationship to one another. The present paper is an attempt to develop a heuristic algorithm for two machines specially structured flow shop scheduling in which processing time of jobs are associated with their probabilities with some well defined structural relationship to one another including job block criteria. The objective of the paper is to minimize the rental cost of the machines under a specified rental policy. A computer programme followed by a numerical illustration is given to substantiate the algorithm.

Keywords: *Flow shop scheduling, Rental policy, Processing time, Utilization time, Makespan, Idle time.*

Mathematical Subject Classification: *90B30, 90B35.*

1. INTRODUCTION

Flow shop scheduling is a kind of scheduling problem where jobs are processed by a series of machines in exactly the same order. Most of literature deals with simple flow shop problems where the processing time of jobs are completely random. So far, the most frequently studied criterion for evaluation of the quality of solutions of flow shop problem is makespan, which is the total time required to finish the last job on the last machine. In 1954, Johnson proposed an algorithm called Johnson's rule, to achieve the minimum makespan for two-machines flow shop problem. The work was developed by Ignall and Scharge (1965), Bagga (1969), Gupta, J.N.D (1975), Maggu and Das (1977), Schwartz (1977), Yoshida and Hitomi (1979), Singh, T.P. (1985), Gupta Deepak and Sharma Sameer (2011) etc. Narain (2005) studied a problem to obtain a sequence which gives minimum possible rental cost while minimize total elapsed time under pre-defined rental policy. Singh, T.P. & Gupta Deepak (2006) studied n x 2 flow shop problem to minimize the rental cost of the machines under pre-defined rental policy in which the probabilities have been associated with processing time. In real situation the processing time of machines are not always random but bear a well defined relationship to one another and hence, introduces the concept of specially structured flow shop scheduling.

Gupta, Sharma and Shashi (2012) introduces the concept of specially structured flow shop scheduling to minimize the rental cost of machines, in which processing times are associated with probabilities. The present paper is an attempt to extend their study by introducing the concept of jobs restrictions. The idea of jobs restrictions has a practical significance to create a balance between the cost of providing priority in service to the customers and cost of giving service with non-priority customers thereby making the problem more wider and application in a production concern.

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2. PRACTICAL SITUATION

Many applied and experimental situations occur in our day to day working in factories and industrial concern where we have to restrict the processing of some jobs. The practical situation may be taken in a production industry; manufacturing industry etc, where some jobs has to give priority over other. It is due to some urgency or demand of one particular type of job over other. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up-gradation to new technology.

3. NOTATIONS

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ...
M_j : Machine j, j= 1,2
M : Minimum makespan
a_{ij} : Processing time of i^{th} job on machine M_j
p_{ij} : Probability associated to the processing time a_{ij}
A_{ij} : Expected processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Renal cost of i^{th} machine.
$CT(S_i)$: Total completion time of the jobs for sequence S_i
β : Equivalent job block

4. DEFINITION

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + a_{ij} \times p_{ij}$ for $j \geq 2$.
$= max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + A_{i,,j}$, where $A_{i,,j}$= Expected processing time of i^{th} job on j^{th} machine.

5. RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

6. PROBLEM FORMULATION

Let some job i (i = 1, 2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij}. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine such that either $A_{i1} \geq A_{i2}$ or $A_{i1} \leq A_{i2}$ for all values of i. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M1		Machine M2	
I	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	a_{11}	p_{11}	a_{i2}	p_{i2}
2	a_{21}	p_{21}	a_{12}	p_{12}
3	a_{31}	p_{31}	a_{22}	p_{22}
-	-	-	-	-
n	a_{n1}	p_{n1}	a_{n2}	p_{n2}

Table: 1

Mathematically, the problem is stated as: Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

7. THEOREM

If $A_{i1} \le A_{i2}$ *for all i, j, i ≠ j,* then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Solution: Let $A_{i1} \le A_{j2}$ for all $i, j, i \ne j$

i.e., max $A_{i1} \le$ min A_{j2} for all $i, j, i \ne j$

Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$

Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ ($\because A_{21} \le A_{12}$)

$\therefore k_1 \le k_2$

Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$

$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ ($\because A_{31} \le A_{22}$)

Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.

Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge \ldots\ldots \ge k_n$, a monotonically decreasing sequence.

Corollary 1: The total rental cost of machines is same for all the sequences.

Proof: The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_1 = \sum_{i=1}^{n} A_{i2} + A_{11}$.

It implies that under rental policy P the total elapsed time on machine M_2 is same for all the sequences thereby the rental cost of machines is same for all the sequences.

8. THEOREM

If $A_{i1} \ge A_{j2}$ for all $i, j, i \ne j$, then $K_1, K_2 \ldots\ldots K_n$ is a monotonically increasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \ge A_{j2}$ for all $i, j, i \ne j$ i.e., min $A_{i1} \ge$ max A_{j2} for all $i, j, i \ne j$

Here $k_1 = A_{11}$

$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \ge k_1$ ($\because A_{21} \ge A_{j2}$)

Therefore, $k_2 \ge k_1$.

Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \ge k_2$ ($\because A_{31} \ge A_{22}$)

Hence, $k_3 \ge k_2 \ge k_1$.

Continuing in this way, we can have $k_1 \le k_2 \le k_3 \ldots\ldots \le k_n$, a monotonically increasing sequence.

Corollary 2: The total elapsed time of machines is same for all the possible sequences.

Proof: The total elapsed time

$$T(S)=\sum_{i=1}^{n}A_{i2}+k_n=\sum_{i=1}^{n}A_{i2}+\left(\sum_{i=1}^{n}A_{i1}-\sum_{i=1}^{n-1}A_{i2}\right)=\sum_{i=1}^{n}A_{i1}+\left(\sum_{i=1}^{n}A_{i2}-\sum_{i=1}^{n-1}A_{i2}\right)=\sum_{i=1}^{n}A_{i1}+A_{n2}$$

Therefore total elapsed time of machines is same for all the sequences.

9. ALGORITHM

Step 1: Calculate the expected processing times, $A_{ij}=a_{ij}\times p_{ij}\forall i,\ j.$

Step 2: Take equivalent job $\beta(k,m)$ and calculate the processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ on the guide lines of Maggu and Das[7] as follows

$$A'_{\beta 1}=A'_{k1}+A'_{m1}-\min\left(A'_{m1},A'_{k2}\right),\ A'_{\beta 2}=A'_{k2}+A'_{m2}-\min\left(A'_{m1},A'_{k2}\right).$$

Step 3: Define a new reduces problem with the processing times A'_{i1} and A'_{i2} as defined in step 2 and jobs (k, m) are replaced by single equivalent job β with processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ as defined in step 2.

Step 4: Obtain the job J_1 (say) having maximum processing time on 1st machine.

Step 5: Obtain the job J_n (say) having minimum processing time on 2nd machine.

Step 6: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 7, Otherwise go to step 7.

Step 7: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 8: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.

Step 9: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences S_1, S_2 … S_r.

Step 10: Compute the total completion time $CT(S_k)$ k=1, 2…r.

Step 11: Calculate utilization time U_2 of 2nd machine $U_2 = CT(S_k) - A_{11}(S_k)$; k=1,2,…. r.

Step 12: Find rental cost $R(S_i) = \sum_{i=1}^{n} A_{i1}(S_k)\times C_1 + U_2\times C_2$, where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

10. NUMERICAL ILLUSTRATION

Consider 6 jobs, 2 machines flow shop problem with processing times are associated with probabilities. Jobs 2 and 6 are to be processed as a job block β = (2,6). The two machines M_1 and M_2 are taken on rent under rental policy P. The rental cost per unit time for machines M_1 and M_2 are 10 units and 8 units respectively. The objective is to find the optimal sequence of jobs with minimum possible cost.

Jobs	Machine M_1		Machine M_2	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}
1	24	0.1	9	0.1
2	16	0.1	8	0.3
3	25	0.2	7	0.2
4	20	0.2	7	0.1
5	24	0.3	10	0.2
6	28	0.1	12	0.1

Table: 2

Solution: As per step 1: The expected processing time for machines M_1 and M_2 are

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	2.4	0.9
2	1.6	2.4
3	5.0	1.4
4	4.0	0.7
5	7.2	2.0
6	2.8	1.2

Table: 3

As per step 2: Here β= (2, 6)

$A'_{\beta 1}$ = 1.6 + 2.8 – 2.4 = 2.0, $A'_{\beta 2}$ = 2.4 + 1.2 - 2.4 = 1.2.

As per step 3: The new reduced problem is

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	2.4	0.9
3	5.0	1.4
4	4.0	0.7
5	7.2	2.0
β	2.0	1.2

Table: 4

Here, we observe that $A_{i1} \geq A_{j2}$ for all values of i , j; $i \neq j$.

Max A_{i1}=7.2, which is for the 5th job, .i.e. J_1=5.

Min A_{i2}=0.7, which is for the 4th job, .i.e. J_n=4.

Also $J_1 \neq J_n$. On placing J_1 on first position and J_n on last position, the optimal sequences are S_1= 5 - β - 1 – 3 – 4 = 5 – 2 – 6 – 1 – 3 – 4, S_2= 5 - β - 3 – 1 – 4,

S_3= 5 - β - 1 – 3 – 4, ----------, So on. There are six possible optimal sequences. The In-Out table for any of these six optimal sequences say S_1= 5 - β - 1 – 3 – 4 = 5 – 2 – 6 – 1 – 3 – 4 is

Jobs	Machine M_1	Machine M_2
i	In-Out	In-Out
5	0.0 – 7.2	7.2 – 9.2
2	7.2 – 8.8	9.2 – 11.6
6	8.8 – 11.6	11.6 – 12.8
1	11.6 – 14.0	14.0 – 14.9
3	14.0 – 19.0	19.0 – 20.4
4	19.0 – 23.0	23.0 – 23.7

Table: 5

Here, total time elapsed $CT(S_1)$ = 23.7 units and

Utilization time of machine M_2 = $U_2(S_1)$ =16.5 units. Also $\sum_{i=1}^{n} A_{i1}$ =23 units.

Therefore the total rental cost for each of the sequence (S_k), k = 1, 2, --- 6 is

$R(S_k)$ = 23 × 10 + 16.5 × 8= 362 units.

11. REMARKS

If we solve the above problem by Johnson's (1954) methods we get the optimal sequence as S= 3 – 2 – 6 – 1 – 5 – 4. The in-out flow table is

Jobs	**Machine M_1**	**Machine M_2**
i	In - Out	In - Out
3	0.0 – 5.0	5.0 – 6.4
2	5.0 – 6.6	6.6 – 9.0
6	6.6 – 9.4	9.4 – 10.6
1	9.4 – 11.8	11.8 – 12.7
5	11.8 – 19.0	19.0 – 19.8
4	19.0 – 23.0	23.0 – 23.7

Table: 6

Therefore, the total elapsed time =CT(S) =23.7 units and

Utilization time for M_2 = $U_2(S)$ = 18.7 units. Also $\sum_{i=1}^{n} A_{i1}$ =23.0 units.

Therefore Rental Cost is R(S) = 379.6 units.

12. CONCLUSION

The algorithm proposed here for specially structured two stage flow shop scheduling problem with job restrictions is more efficient and less time consuming as compared to the algorithm proposed by Johnson's(1954) to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost. The study may further be extended by introducing the concept of independent set up time, Transportation time (Cost), Weighted jobs etc.

Appendix

Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>

int n;
float a_1[16],b_1[16],a11[16],b11[16];
float macha[16],machb[16],maxv,u2;
int j[16],j1[16],j2[16],j11[16],j12[16],j3[16];
float costa,costb,cost;
int group[16];//variables to store two job blocks
float minv,gbeta,hbeta;
int f=1;
int main()
  {
        clrscr();
        int a[16],b[16];
        float p[16],q[16],g1,g2;
        cout<<"How many Jobs (<=15) : ";
        cin>>n;
        if(n<1 || n>15)
        {
                cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";
                getch();
                exit(0);
        }
        for(int i=1;i<=n;i++)
                {
```

```
            cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine A : ";
            cin>>a[i]>>p[i];
            cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine B : ";
            cin>>b[i]>>q[i];
            //Calculate the expected processing times of the jobs for the machines:
            a_1[i] = a[i]*p[i];
            b_1[i] = b[i]*q[i];
            j[i]=i;
            }
    cout<<"\n Enter the rental cost for Machine M1 & Machine M2 :";
    cin>>costa>>costb;
    cout<<endl<<"Expected processing time of machine A and B: \n";
    for(i=1;i<=n;i++)
    {
    cout<<"\n"<<j[i]<<"\t"<<a_1[i]<<"\t"<<b_1[i]<<"\t";
    cout<<endl;
    }
    cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):";
    cin>>group[0]>>group[1];
//calculate G_Beta and H_Beta
if(a_1[group[1]]<b_1[group[0]])
    {
    minv=a_1[group[1]];
    }
else
    {
    minv=b_1[group[0]];
    }
gbeta=a_1[group[0]]+a_1[group[1]]-minv;
hbeta=b_1[group[0]]+b_1[group[1]]-minv;
cout<<endl<<endl<<"G_Beta="<<gbeta;
cout<<endl<<"H_Beta="<<hbeta;
int j1[16];
float a1[16],b1[16];
for(i=1;i<=n;i++)
    {
    if(j[i]==group[0]||j[i]==group[1])
    {
    f--;
    }
else
    {
    j1[f]=j[i];
    }
    f++;
    }
    j1[n-1]=17;

for(i=1;i<=n-2;i++)
    {
    a1[i]=a_1[j1[i]];
    b1[i]=b_1[j1[i]];
    }
    a1[n-1]=gbeta;
    b1[n-1]=hbeta;
    cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
    {
    cout<<j1[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<endl;
    }
    for(i=1;i<=n-1;i++)
    {
```



```
          if((a1[i]>=b1[i])^(a1[i]<=b1[i]))
            {
                    a1[i]=a1[i],b1[i]=b1[i];
            }
          else
          {
          cout<<"\n The data is not in standard form";
          getch();
          exit(0);
          }
          }
   int j11[16], j12[16],j2[16],j3[16];
   void sort(float [],int []);// function declaration
   for(i=1;i<=n-1;i++)
   {
   a11[i]=a1[i];
   j3[i]=j1[i];
   }
   sort(a11,j3);//fuction call
   cout<<"\nSorted processing times in ascending order of Machine A :\n";

   for(i=1;i<=n-1;i++)
   {
   j11[i]=j3[i];
   cout<<"\n"<<j11[i]<<"\t"<<a11[i];
   }
   for(i=1;i<=n-1;i++)
   {
   b11[i]=b1[i];
   j2[i]=j1[i];
   }
   sort(b11,j2);// function call
   cout <<"\nSorted processing times in ascending order of Machine B :\n";
   for(i=1;i<=n-1;i++)
   {
   j12[i]=j2[i];
   cout<<"\n"<<j12[i]<<"\t"<<b11[i];
   }

   if(j11[n-1]!=j12[1])
   {
   j3[1]=j11[n-1];j3[n-1]=j12[1];
   for(int k=2;k<=n-2;k++)
   {
   if(j11[k-1]!=j12[1])
   {
   j3[k]=j11[k-1];
   }
   else
   {
   if(j11[n-2]!=j12[1])
   {
   j3[k]=j11[n-2];
   }
   }
   }
   }
   else
   {
   g1=a11[j11[n-1]]-a11[j11[n-2]];
   g2=b11[j2[12]]-b11[j12[1]];
   if(g1<=g2)
```

```
{
j3[1]=j11[n-2];j3[n-1]=j12[1];
for(int g=2;g<=n-2;g++)
{
j3[g]=j11[g-1];
}
}
else
{
j3[1]=j11[n-1];j3[n-1]=j12[2];
for(int f=2;f<=n-2;f++)
{
j3[f]=j2[f+1];
}
}
}
int arr[16],m=1;
for(i=1;i<=n;i++)
        {
        if(j3[i]==17)
        {
         arr[m]=group[0];
        arr[m+1]=group[1];
        m=m+2;
        continue;
        }
else
{
        arr[m]=j3[i];
        m++;
        }
        }

macha[1]=a_1[arr[1]];machb[1]=macha[1]+b_1[arr[1]];

// displaying solution
cout<<"\n\n\t**********************************************************";
cout<<"\n\t"<<"optimal sequence is";
for(i=1;i<=n;i++)
{
cout<<"\t"<<arr[i];
}
float time =0.0;
cout<<endl<<endl<<"In-Out Table is"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"Machine M2"<<endl;
cout<<arr[1]<<"\t"<<time<<"--"<<macha[1]<<"\t"<<"\t"<<macha[1]<<"--"<<machb[1]<<"\t"<<endl;
for(i=2;i<=n;i++)
{
macha[i]=macha[i-1]+a_1[arr[i]];
if(machb[i-1]>macha[i])
{
maxv= machb[i-1];
}
else
{
maxv=macha[i];
}
machb[i]=maxv+b_1[arr[i]];
cout<<arr[i]<<"\t"<<macha[i-1]<<"--"<<macha[i]<<"\t"<<"\t"<<maxv<<"--"<<machb[i]<<"\t"<<endl;
}
u2=machb[n]-macha[1];
cost=macha[n]*costa+u2*costb;
```

```
cout<<"\n\nThe total rental cost of machines is:"<<cost;
cout<<"\n\n\t**************************************************************";
getch();
return 0;
}
void sort(float x[],int y[])// function decleration
{
float temp; int temp1;
//outer for loop to control no of passea
for(int k=1;k<n;k++)
{
//inner for loop for making comparison per pass
for(int m=1;m<n-k;m++)
{
	if(x[m]>x[m+1])
	{
	temp=x[m];temp1=y[m];
	x[m]=x[m+1];y[m]=y[m+1];
	x[m+1]=temp;y[m+1]=temp1;
	}
}
}
}
```

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