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ANALYSIS OF FLOW ENTITIES ON THE SKIN FRICTION FOR A FLOW PAST SEMI INFINITE VERTICAL PLATE WITH VISCOUS DISSIPATION UNDER THE INFLUENCE OF MAGNETIC FILD

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ABSTRACT

The aim of the present paper is to examine the nature of skin friction due to various flow entities of on an unsteady two dimensional laminar convective boundary layer flow of a viscous in compressible, chemically reacting fluid on a semi infinite vertical plate with suction by taking into account the effects of dissipation. It is observed that as Schmidt number increases the skin friction also increases. Further, as the chemical reaction parameter increases the skin friction is observed to be proportional and for similar values of chemical reaction parameter as Schmidt number the increasing the magnetic field contributes inversely. When the Schmidt number is held constant, increase in the concentration parameter contributes to the direct increase in skin friction. In addition to the above, it is observed that increase in the applied magnetic intensity contributes to the decrease in the skin friction.

***Keywords**: Heat transfer, viscous dissipation, Radiation, Chemical Reaction, Applied Magnetic field and Porosity parameter*

Nomenclature:

u'	:	Velocity component in x'-direction
v'	:	Velocity component in y'-direction
t'	:	Time parameter
ρ	:	Fluid density
ν	:	Kinematic viscosity
c_p	:	Specific heat at constant Pressure
G	:	Acceleration due to gravity
β	:	Volumetric coefficient of thermal expansion
β^*	:	Volumetric coefficient of concentration expansion
T	:	Dimensional temperature
C	:	Dimensional concentration
α	:	Fluid thermal diffusivity
μ	:	Coefficient of viscosity
D	:	Mass diffusivity
k_r'	:	Chemical reaction parameter
U_0	:	Scale of free stream velocity
C_w	:	Wall dimensional concentration
T_w	:	Wall dimensional temperature
C_∞	:	Free stream dimensional concentration
T_∞	:	Free stream dimensional temperature
n'	:	The constant
σ_s	:	Stefan – Boltzmann constant
K_e	:	Mean absorption coefficient
A	:	Real positive constant
εA	:	A real positive constant <<1
V_0	:	Non – zero positive constant
G_c	:	Solutal Grashof number
P_r	:	Prandtl number
R	:	Radiation parameter
E_c	:	Eckert number
S_c	:	Schmidt number
k_r	:	Chemical reaction parameter
M	:	Applied Magnetic field
k_1	:	Porosity parameter

1. 1. Introduction:

Due to its wide range of applications in the areas of soil physics, geothermal energy extraction, chemical engineering, glass production, furnace design, space technology application, flight aerodynamics and plasma physics which operates at extremely high temperature the effects of the radiation are found to be highly significant. The influence of radiation in all such applications and understanding the boundary layer development and heat transfer characteristics are of primary requirement to investigate the problem in detail for the design and development of equipment for high rate of accuracy. The conservation of energy equation becomes more complicated due to the inclusion of radiation component which leads to a non-linear partial differential equation.

Analytical expressions for boundary layer thickness, local and overall heat flux was studied by Cheng and Minkowyiz [1] for the case of free convective heat transfer characteristics for vertical plate in porous medium. Later, using boundary layer approximations the problem of mixed convection on inclined surface was examined by Cheng [2]. Subsequently, studied the effects of transverse magnetic field on the heat transfer characteristics in a porous medium was investigated by Ananda Rao, *et al* [3] who brought out the effects of porous parameter on temperature and Nusselt number. The radiative free convection flow of an optically thin grey-gas past a semi infinite vertical plate was examined by Soundalgekar and Takhar [4]. Thereafter, the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction concentration of the fluid under consideration was studied by Das *et al* [5]. Later, Hussain and Thakar [6] examined the radiation effects on mixed convection along an isothermal vertical plate. Thereafter, the effects of thermal radiation on free convective flow past a moving vertical plane was studied by Raptis and Perdikis [7] while, the effects of radiation on free convection flow past a semi infinite vertical plate with mass transfer was examined by Chamkha *et al* [8]. Subsequently, Sobha and Ramakrishna [9] presented the effects of magnetic field on heat transfer characteristics in porous medium under natural convection, whereas, Muthucumaraswamy and Ganesan [10] have studied the radiation effects on flow past an impulsively started infinite vertical plate with variable temperature. Further, the problem of unsteady two-dimensional flow of a radiating and chemically reacting fluid with time dependent suction was studied by Prakash and Ogulu [11].

In all above studies, the influence of participating parameters on the flow field has been studied. However, not much of importance has been paid to the concept of skin friction. In chemical reaction chambers and high pressure boilers, the factors that affect the bounding surface is important for accurate design of the system. Therefore, an attempt has been made in this paper to study the influence of all such participating parameters on the skin friction.

The fluid under consideration is assumed to be viscous incompressible, chemically reacting fluid along a semi infinite vertical plate with suction by taking into account the effects of dissipation. The equations of continuity, linear momentum, energy and diffusion that govern the flow field are examined in detail. The characteristic performance of various parameters that effects the skin friction have been discussed qualitatively and illustrated graphically.

2. Mathematical formulation of the problem:

An unsteady two-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects is considered, by taking the effect of viscous dissipation into account. The x'-axis is taken along the vertical infinite plate in the upward direction and the y'-axis normal to the plate as shown below.

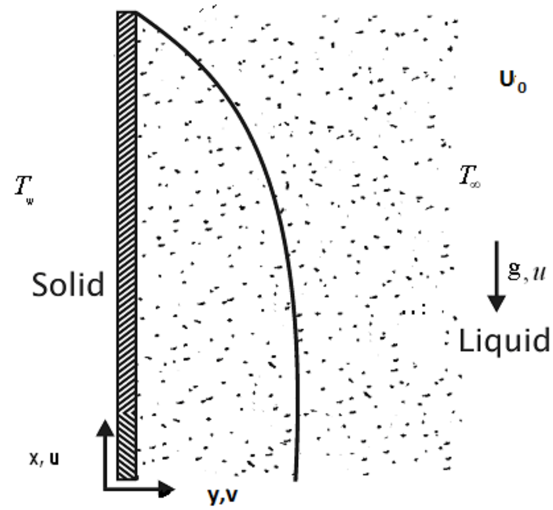

Figure – 1: Geometry of the flow field

Under the assumption that the level of concentration of foreign mass is assumed to be low, the Soret and Dufour effects can be neglected. By considering Boussinesq's approximation, the flow field is governed by the following equations: $\frac{\partial v'}{\partial y'} = 0$ (1)

$$\left.\begin{aligned}\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} &= \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty)\\ &+ g\beta^*(C - C_\infty)\end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned}\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} &= \alpha\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial qr}{\partial y'}\\ &+ \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2\end{aligned}\right\}. \quad (3)$$

$$\frac{\partial C}{\partial t'} + u'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} - k_r'^2(C - C_\infty) \quad (4)$$

The boundary conditions for the velocity, temperature, and concentration fields are:

$$\left.\begin{aligned}&u' = U_0,\ T = T_w + \varepsilon(T_w - T_\infty)e^{n't'},\\ &C = C_w + \varepsilon(C_w - C_\infty)e^{n't'} \quad \text{at } y' = 0\\ &u' \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as } y' \to \infty\end{aligned}\right\} \quad (5)$$

By using Rosseland approximation, the radiative heat flux is given by

$$q_r = \frac{-4\sigma_s \partial T^4}{3K_e \partial y'} \quad (6)$$

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It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then Eqn (6) can be linearised by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms take the form:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

In view of Eqns (6) and (7), Eqn (3) reduces to

$$\left.\begin{aligned}\frac{\partial T}{\partial t'} + v\frac{\partial T}{\partial y'} = \frac{k}{\rho\, c_p}\frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma_s}{3\rho\, c_p\, K_e} \\ -T_\infty^3\frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho\, c_p}\left(\frac{\partial u'}{\partial y'}\right)^2\end{aligned}\right\} \tag{8}$$

From the continuity Eqn (1), it is clear that the suction velocity normal to the plate which can be considered either a constant or a function of time. In the fitness of the present situation, it is assumed in the form:

$$v' = -V_0(1+\varepsilon A e^{n't'}) \tag{9}$$

the negative sign indicates that these suction is towards the plate.

In order to governing equations and boundary conditions to make dimension
less quantities, the following non dimensional scheme is introduced:

$$u = \frac{u'}{U_0},\quad y = \frac{V_0 y'}{v},\quad t = \frac{t' V_0^2}{v},\quad p_r = \frac{\rho\, c_p v}{k} = \frac{v}{\alpha},$$

$$S_c = \frac{v}{D},\ \phi = \frac{C - C_\infty}{C_w - C_\infty}$$

$$n = \frac{n' v}{V_0^2},\qquad R = \frac{16\sigma_s T_\infty^3}{3K_e k},\qquad k_r^2 = \frac{k_r'^2 v}{V_0^2},$$

$$G_c = \frac{g\,\beta^* v (C_w - C_\infty)}{U_0 V_0^2} \tag{10}$$

In view of the Eqns (9) and (10), Eqns (2), (8) and (4) are reduced to the following dimensionless form.

$$\left.\begin{aligned}\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} \\ + G_c\phi + Mu\end{aligned}\right\} \tag{11}$$

$$0 = E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

$$\frac{\partial \phi}{\partial t} - \left(1+\varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} - k_r^2\phi \tag{13}$$

Where G_r, G_c, P_r, R, E_c, S_c, M and k_r are the thermal Grashof number, Solutal Grashof number, Prandtl number, Radiation parameter, Eckert number,
Schmidt number, intensity of applied transverse magnetic field and chemical reaction parameter respectively.

The corresponding dimensionless boundary conditions are

$$\left.\begin{aligned}&u = 1,\quad \phi = 1 + \varepsilon e^{nt} \text{ at } y = 0 \\ &u \to 0,\ \phi \to o \text{ as } y \to \infty\end{aligned}\right\} \tag{14}$$

3. Solution of the problem:

Eqns (11) - (13) are coupled non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as

$$\phi(y,\, t) = \phi_0(y) + \varepsilon\, e^{nt}\phi_1(y) + \mathrm{O}(\varepsilon)^2 \tag{15}$$

$$u(y,t) = u_0(y) + \varepsilon\, e^{nt}\, u_1(y) + o(\varepsilon)^2 \tag{16}$$

Substituting the Eqn (15) and Eqn (16) in Eqns (11) - (13) and equating the harmonic and non-harmonic terms, and neglecting the higher – order terms of $\mathrm{O}(\varepsilon)^2$, we obtain

$$\phi_0'' + S_c\phi_0' - k_r^2 S_c\phi_0 = 0 \tag{17}$$

$$\phi_1'' + S_c\,\phi_1' - \left(k_r^2 + n\right)S_c\phi_1 = -AS_c\phi_0' \tag{18}$$

$$u_0'' + u_0' + M\,u_0 = -G_c\,\phi_0 \tag{19}$$

$$u_1'' + u_1' + (M - n)u_1 = -A u_0' - G_c\,\phi_1 \tag{20}$$

Where $h = \dfrac{\Pr}{1+R}$ and prime denotes ordinary differentiation with respect to y.

The corresponding boundary conditions can be re written as:

$$\left.\begin{aligned}&u_0 = 1, u_1 = 0,\ \phi_1 = 1, \text{at } y = 0 \\ &u_0 \to 0, u_1 \to 0,\ \phi_0 \to 0,\ \phi_1 \to 0, \text{as } y \to \infty\end{aligned}\right\} \tag{21}$$

The solutions of Eqns (17) – (20) in explicit form is given by:

$$\phi_0(y) = e^{-\left(\frac{S_c + \alpha}{2}\right)y} \text{ and}$$

$$\phi_1(y) = \frac{4A\,S_c\,\gamma}{\alpha^2 - S_c^2 - 4S_c\,\beta^2}\left(e^{-\gamma y} - e^{-\delta y}\right) + e^{-\delta y}$$

$$u_0(y) = \left(1 + \frac{G_c}{m_1 + n}\right) e^{-t_0 y} - \frac{G_c}{m_1 + n} e^{-\gamma y} \qquad (22)$$

$$u_1(y) = \left(\left[\frac{A}{4n}\left(1 + \frac{G_c}{m_1+n}\right) t_0 + \frac{A\gamma G_c}{(m_1+n)m_1} + \frac{4AG_cS_c\gamma}{m_2}\left(\frac{1}{m_1} - \frac{1}{m_3}\right) + \frac{G_c}{m_3} \right] e^{-t_1 y} - \frac{A}{4n}\left(1+\frac{G_c}{m_1+n}\right) t_0\, e^{-t_0 y} - \frac{A\gamma G_c}{(m_1+n)m_1} e^{-\gamma y} - \frac{4AG_cS_c\gamma}{m_2}\left(\frac{e^{-\gamma y}}{m_1} - \frac{e^{-\delta y}}{m_3}\right) - \frac{G_c}{m_3} e^{-\delta y} \right)$$

Where $\alpha = \sqrt{S_c^2 + 4S_cK_r^2}$, $\beta = \sqrt{K_r^2 + n}$

$$\gamma = \frac{S_c + \alpha}{2}, \qquad \delta = \frac{S_c + \sqrt{S_c^2 + 4S_c\beta^2}}{2},$$

$$\theta = \sqrt{1-4M}\ ,\ \xi = \sqrt{1-4M-4n}$$

$$m_1 = \gamma^2 - \gamma + M - n \Rightarrow m_1 + n = \gamma^2 + \gamma + M$$

$$m_2 = \alpha^2 - S_c^2 - 4\beta^2 S_c$$

$$m_3 = \delta^2 - \delta + M - n \Rightarrow m_3 + n = \delta^2 - \delta + M$$

$$t_0 = \frac{\theta+1}{2}\ ,\quad t_1 = \frac{\xi+1}{2}$$

In view of Eqn (16), the required solution is given by:

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y)$$

The skin friction on lower plate is:

$$\left.\frac{\partial u}{\partial y}\right|_{at\ y=0} = -t_0\left(1 + \frac{G_c}{m_1+n}\right) + \frac{\gamma G_c}{m_1+n} + \varepsilon e^{nt}\left[\left(\frac{A}{4n}\left(1+\frac{G_c}{m_1+n}\right)t_0 + \frac{A\gamma G_c}{(m_1+n)m_1} + \frac{4AG_cS_c\gamma}{m_2}\left(\frac{1}{m_1} - \frac{1}{m_3}\right) + \frac{G_c}{m_3} \right)(-t_1) + \frac{A}{4n}\left(1+\frac{G_c}{m_1+n}\right)t_0^2 + \frac{A\gamma^2 G_c}{(m_1+n)m_1} - \frac{4AG_cS_c\gamma}{m_2}\left(-\frac{\gamma}{m_1} + \frac{\delta}{m_3}\right) + \frac{\delta G_c}{m_3} \right]$$

4. Results and conclusions:

1. Fig.2, Fig.3, Fig.4, Fig.5 and Fig.6 illustrates the consolidated influence of Schmidt number and Chemical reaction parameter on skin friction. In each of the illustrations, when the Magnetic intensity is held constant, it is observed that as Schmidt number increases the skin friction also increases. Also, as the Chemical reaction parameter increases, the skin friction is observed to be proportional. Further, for similar values of chemical reaction parameter and Schmidt number, increase in magnetic field contributes to decrease in the skin friction.

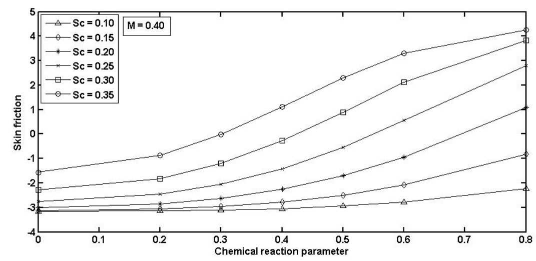

Fig.2: Effect of Schmidt number on Skin friction.

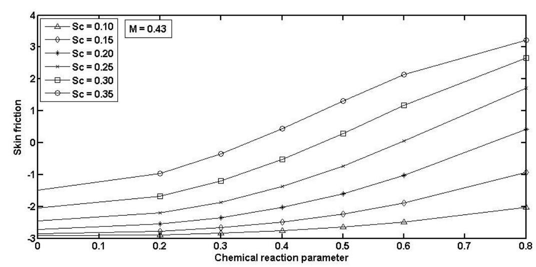

Fig.3: Influence of Schmidt number on skin friction.

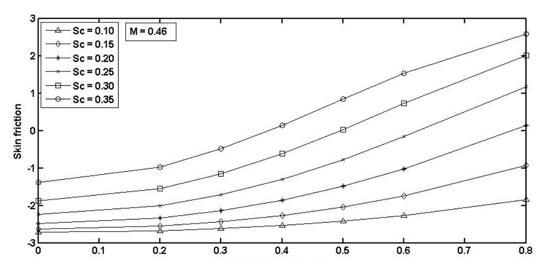

Fig.4: Contribution of Schmidt number on skin friction

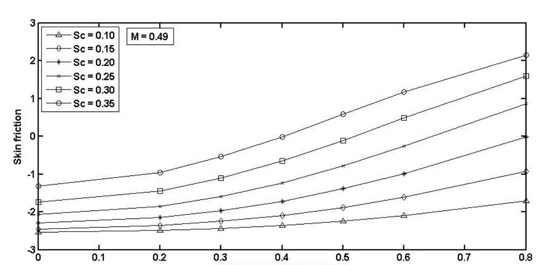

Fig.5: Effect of Schmidt number on skin friction.

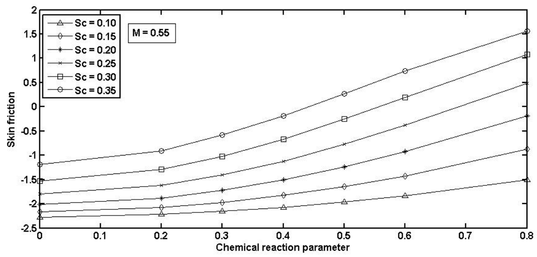

Fig.6: Influence of Schmidt number on skin friction

2. The combined effect of Schmidt number and concentration parameter with respect to the applied magnetic intensity is illustrated in Fig.7, Fig.8 Fig.9, Fig.10 and Fig.11. In each of the representations it is observed that, while all other parameters are held constant increase in Schmidt number contributes to increase in skin friction. Further, for a fixed Schmidt number increase in the concentration parameter contributes to the increase in skin friction. In addition to the above, it is observed that increase in the applied magnetic intensity contributes to the decrease in the skin friction.

From the illustrations it is noticed that the dispersion in the profiles is not that significant within the boundary layer region. However, it is found to be quiet distinct for the higher values of Schmidt number, concentration parameter and also for smaller values of applied magnetic intensity

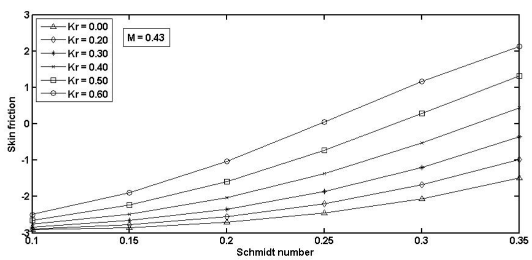

Fig.7: Contribution of Chemical reaction parameter on skin friction.

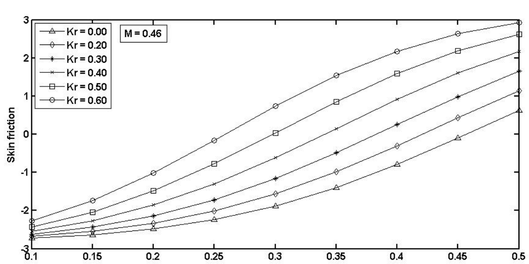

Fig.8: Effect of Chemical reaction parameter on skin friction.

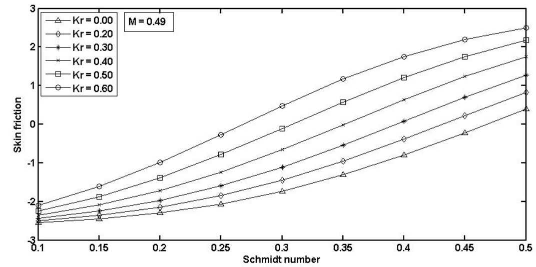

Fig.9: Influence of Chemical reaction parameter on Skin friction

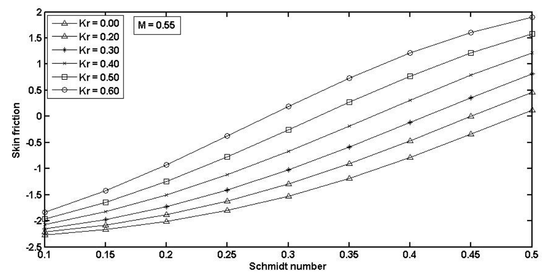

Fig.10: Contribution of Chemical Reaction parameter of skin friction.

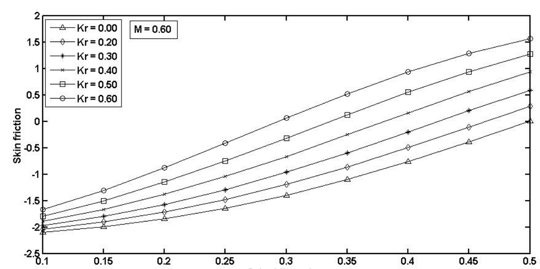

Fig.11: Effect of Chemical reaction parameter on skin friction

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