

## A NEW TYPE OF GENERALIZED CLOSED SETS

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### ABSTRACT

In this paper, we introduce a new classes of sets called D-closed sets, D-open sets in topological spaces and study some basic properties of D-closed sets.

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### 1. INTRODUCTION

The study of generalized closed sets in a topological space was initiated by Levine [6] and the concept of  $T_{1/2}$  space was introduced. In 1996, H. Maki, J. Umehara and T. Noiri [8] introduced the class of pregeneralized closed sets and used them to obtain properties of pre- $T_{1/2}$  spaces. In 2008, S.Jafari, T.Noiri, N.Rajesh and M. L. Thivagar [4] introduced the concept of  $\tilde{g}$  closed sets and their properties. In this paper we introduce new classes of sets called D-closed sets and D-open sets in topological spaces.

### 2. PRELIMINARIES

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  will always denoted topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When  $A$  is a subset of  $(X, \tau)$ ,  $\text{cl}(A)$ ,  $\text{Int}(A)$  and  $D(A)$  denote the closure, the interior and the derived set of  $A$  respectively.

We recall some known definitions are needed in this paper.

**Definition 2.1:** Let  $(X, \tau)$  be a topological space. A subset  $A$  of the space  $X$  is said to be

1. pre-open [7] if  $A \subseteq \text{Int}(\text{cl}(A))$  and pre-closed if  $\text{cl}(\text{Int}(A)) \subseteq A$
2. semi-open [5] if  $A \subseteq \text{cl}(\text{Int}(A))$  and semi-closed if  $\text{Int}(\text{cl}(A)) \subseteq A$
3.  $\alpha$ -open[9] if  $A \subseteq \text{Int}(\text{cl}(\text{Int}(A)))$  and  $\alpha$ -closed if  $\text{cl}(\text{Int}(\text{cl}(A))) \subseteq A$ .
4. semi-preopen[1] if  $A \subseteq \text{cl}(\text{Int}(\text{cl}(A)))$  and semi-preclosed if  $\text{Int}(\text{cl}(\text{Int}(A))) \subseteq A$ .
5. regular open if  $A = \text{Int}(\text{cl}(A))$  and regular closed if  $A = \text{cl}(\text{Int}(A))$

**Definition 2.2[8]:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$

1. pre-interior of  $A$  denoted by  $\text{pInt}(A)$  is the union of all pre-open subsets of  $A$ .
2. pre-closure of  $A$  denoted by  $\text{pcl}(A)$  is the intersection of all pre-closed sets containing  $A$ .

**Lemma 2.3[3]:** If  $A$  is regular open and  $\text{gpr}$ -closed then  $A$  is pre-closed

**Lemma 2.4[4]:** Every  $\tilde{g}$ -closed set is  $\omega$ -closed

**Lemma 2.5[1]:** For any subset  $A$  of  $X$ , the following relations hold.

1.  $\text{scl}(A) = A \cup \text{Int}(\text{cl}(A))$
2.  $\alpha\text{-cl}(A) = A \cup \text{cl}(\text{Int}(\text{cl}(A)))$

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3.  $pcl(A) = A \cup cl(Int(A))$
4.  $spcl(A) = A \cup Int(cl(Int(A)))$

**Definition 2.6:** Let  $(X, \tau)$  be a topological space. A subset  $A \subseteq X$  is said to be

1. generalized closed (g-closed)[6] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. The complement of a g-closed set is said to be g-open.
2. generalized pre-closed(gp-closed)[8] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open
3. generalized preregular-closed(gpr-closed)[3] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open
4. pregeneralized closed (pg-closed)[8] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is pre-open
5.  $g^*$ -preclosed ( $g^*p$ -closed) [14] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is g-open
6.  $g\alpha^*$ -closed[14] if  $\alpha-cl(A) \subseteq Int(U)$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open. The complement of  $g\alpha^*$ -closed is said to be  $g\alpha^*$ -open
7.  $\mu$ -preclosed( $\mu p$ -closed)[14] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g\alpha^*$ -open
8. generalized semi-preclosed (gsp-closed) [2] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open
9. pre semi-closed [14] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is g-open
10.  $\omega$ -closed [12] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open. The complement of a  $\omega$ -closed set is  $\omega$ -open.
11.  $\eta^*$ -closed[10] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\omega$ -open
12.  $^*g$ -closed[15] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\omega$ -open
13.  $\#g$ -semiclosed( $\#gs$ -closed)[13] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $^*g$ -open. The complement of  $\#gs$ -closed set is said to be  $\#gs$ -open.
14.  $\tilde{g}$ -closed set [4] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\#gs$ -open. The complement of  $\tilde{g}$ -closed set is said to be  $\tilde{g}$ -open
15.  $\rho$ -closed [11] if  $pcl(A) \subseteq Int(U)$  whenever  $A \subseteq U$  and  $U$  is  $\tilde{g}$ -open

### 3. BASIC PROPERTIES OF D-CLOSED SETS

We introduce the following definition

**Definition 3.1:** A subset  $A$  of a space  $(X, \tau)$  is said to be D-closed in  $(X, \tau)$  if  $pcl(A) \subseteq Int(U)$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$  – open.

**Theorem 3.2:** Every open and pre-closed subset of  $(X, \tau)$  is D-closed.

**Proof:** Let  $A$  be open and preclosed subset of  $(X, \tau)$ . Let  $A \subseteq U$  and  $U$  is  $\omega$  – open in  $X$ .

Then  $pcl(A) = A = Int(A) \subseteq Int(U)$ . Hence  $A$  is D-closed.

**Remark 3.3:** The converse of the above theorem is not true.

**Example 3.4:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ .

Then the set  $A = \{a, b, c, e\}$  is D-closed but neither open nor pre-closed.

**Theorem 3.5:** Every D-closed set is gp- closed

**Proof:** Let  $A$  be any D-closed set in  $X$ . Let  $A \subseteq U$  and  $U$  is open in  $X$ . Since every open set is  $\omega$ -open,

we get  $pcl(A) \subseteq Int(U) = U$ . Hence  $A$  is gp-closed

**Remark 3.6:** The Converse of the above theorem is not true.

**Example 3.7:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{b, c, d\}, \{a, b, c, d\}, \{b, c, d, e\}, X\}$ . Then the set  $A = \{c, d\}$  is gp-closed but not D-closed in  $X$ .

**Theorem 3.8:** Every D-closed set is gpr-closed.

**Proof:** Let  $A$  be any D-closed set. Let  $A \subseteq U$  and  $U$  is regular open. Since every regular open is open and every open is  $\omega$ -open, we get  $pcl(A) \subseteq Int(U) = U$ . Hence  $A$  is gpr-closed.

**Remark 3.9:** The converse of the above theorem is not true

**Example 3.10:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{e\}, \{a, b\}, \{c, e\}, \{a, b, e\}, \{a, b, c, e\}, X\}$ . Then the set  $A = \{a, b, c, e\}$  is gpr-closed but not D-closed in  $X$

**Theorem 3.11:** Every D-closed set is gsp-closed.

**Proof:** Let  $A$  be any D-closed set. Let  $A \subseteq U$  and  $U$  be open. Since every open set is  $\omega$ -open, we get  $\text{pcl}(A) \subseteq \text{Int}(U) = U$ . Since  $\text{spcl}(A) \subseteq \text{pcl}(A) \subseteq U$ , We get  $A$  is gsp closed.

**Remark 3.12:** The converse of the above theorem is not true.

**Example 3.13:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ . Then the set  $A = \{a, b\}$  is gsp-closed but not D-closed in  $X$ .

**Remark 3.14:** D-closedness and pre-closedness are independent. It is shown by the following examples

**Example 3.15:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Then the set  $A = \{a, b, c, e\}$  is D-closed but not pre-closed.

**Example 3.16:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}, X\}$ . Then the set  $A = \{a\}$  is pre-closed but not D-closed.

**Remark 3.17:** D-closedness and  $\alpha$ -closedness are independent. It is shown by the following examples.

**Example 3.18:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ . Then the set  $A = \{d\}$  is  $\alpha$ -closed but not D-closed.

**Example 3.19:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ . Then the set  $A = \{a, c\}$  is D-closed but not  $\alpha$ -closed.

**Remark 3.20:** D-closed sets are independent of semi-closed sets and semi-pre-closed sets. It is shown by the following example.

**Example 3.21:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ . Then the set  $A = \{a, b, c\}$  is D-closed but neither semi-closed nor semi-pre-closed and the set  $B = \{c, d\}$  is both semi-closed and semi-pre-closed but not D-closed.

**Remark 3.22:** D-closedness and presemi-closedness are independent. It is shown by the following examples.

**Example 3.23:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ . Then the set  $A = \{a\}$  is presemi-closed but not D-closed

**Example 3.24:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, X\}$ . Then the set  $A = \{b, c\}$  is D-closed but not presemi-closed

**Remark 3.25:** D-closedness and g-closedness are independent. It is shown by the following example

**Example 3.26:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Then the set  $A = \{a, c, d\}$  is D-closed but not g-closed and the set  $B = \{e\}$  is g-closed but not D-closed.

**Remark 3.27:** D-closedness and pg-closedness are independent. It is shown by the following examples.

**Example 3.28:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, X\}$ . Then the set  $A = \{a, b\}$  is D-closed but not pg-closed.

**Example 3.29:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then the set  $A = \{a\}$  is pg-closed but not D-closed.

**Remark 3.30:** D-closedness and  $g^*p$ -closedness are independent. It is shown by the following examples.

**Example 3.31:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{b, c\}, X\}$ . Then the set  $A = \{b\}$  is  $g^*p$ -closed but not D-closed

**Example 3.32:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, X\}$ . Then the set  $A = \{a, b\}$  is D-closed but not  $g^*p$ -closed

**Remark 3.33:** D-closedness and  $\mu p$ -closedness are independent. It is shown by the following example.

**Example 3.34:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{b\}, \{b, c\}, X\}$ . Then the set  $A = \{a, b\}$  is D-closed but not  $\mu p$ -closed and the set  $B = \{b, c\}$  is  $\mu p$ -closed but not D-closed

**Remark 3.35:** D-closedness and  $\eta^*$ -closedness are independent. It is shown by the following examples.

**Example 3.36:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Then the set  $A = \{a, b, d\}$  is D-closed but not  $\eta^*$ -closed.

**Example 3.37:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ . Then the set  $A = \{a, b\}$  is  $\eta^*$ -closed but not D-closed.

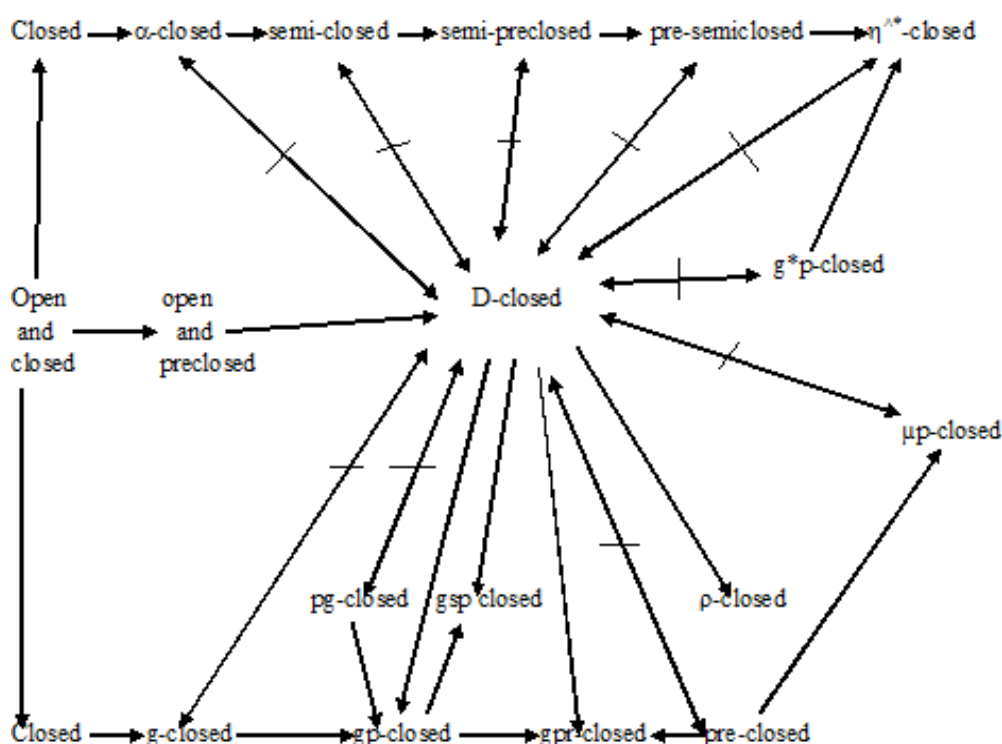
**Theorem 3.38:** Every D-closed set is  $\rho$ -closed.

**Proof:** Let  $A$  be any D-closed set. Let  $A \subseteq U$  and  $U$  be  $\tilde{g}$ -open in  $X$ . By Lemma 2.4, we get  $pcl(A) \subseteq Int(U)$ . Hence  $A$  is  $\rho$ -closed.

**Remark 3.39:** The converse of the above theorem is not true. It is shown by the following example.

**Example 3.40:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Then the set  $A = \{a, b, c\}$  is  $\rho$ -closed but not D-closed.

**Remark 3.41:** From the above discussion and known results we have the following implication  $A \rightarrow B (A \not\leftarrow B)$  represents  $A$  implies  $B$  but not conversely ( $A$  and  $B$  are independent of each other).



#### 4. PROPERTIES OF D-CLOSED SETS

**Definition 4.1:** The intersection of all  $\omega$ -open subsets of  $(X, \tau)$  containing  $A$  is called  $\omega$ -kernel of  $A$  and is denoted by  $\omega\text{-Ker}(A)$ .

**Theorem 4.2:** If a subset  $A$  of  $(X, \tau)$  is D-closed then  $pcl(A) \subseteq \omega\text{-ker}(A)$ .

**Proof:** Suppose that  $A$  is D-closed. Then  $pcl(A) \subseteq Int(U)$  whenever  $A \subseteq U$  and  $U$  is  $\omega$ -open. Let  $x \in pcl(A)$ . Suppose  $x \notin \omega\text{-ker}(A)$ . Then there is a  $\omega$ -open set  $U$  containing  $A$  such that  $x \notin U$ . Since  $U$  is a  $\omega$ -open set containing  $A$ ,  $x \in pcl(A)$ . Which is a contradiction.

**Remark 4.3:** The converse of the above theorem need not be true as seen from the following example.

**Example 4.4:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{e\}, \{a, b\}, \{c, e\}, \{a, b, e\}, \{a, b, c, e\}, X\}$ . Then the set  $A = \{a\}$  satisfies  $pcl(A) \subseteq \omega\text{-ker}(A)$ . But  $A$  is not D-closed.

**Remark 4.5:** The union of two D-closed sets need not be D-closed.

**Example 4.6:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Let  $A = \{a, d\}$  and  $B = \{b, d\}$ . Here  $A$  and  $B$  are  $D$ -closed sets. But  $A \cup B = \{a, b, d\}$  is not  $D$ -closed.

**Remark 4.7:** The intersection of two  $D$ -closed sets need not be  $D$ -closed.

**Example 4.8:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ . Let  $A = \{a, c\}$  and  $B = \{c, d\}$ . Here  $A$  and  $B$  are  $D$ -closed sets. But  $A \cap B = \{c\}$  is not  $D$ -closed.

**Theorem 4.9:** A set  $A$  is  $D$ -closed then  $\text{pcl}(A) - A$  contains no non-empty closed set.

**Proof:** Suppose  $F \subseteq \text{pcl}(A) - A$  be a non-empty closed set. Then  $F \subseteq \text{pcl}(A)$  and  $A \subseteq X$ . Since  $X - F$  is  $\omega$ -open, we get  $\text{pcl}(A) \subseteq \text{Int}(X - F) = X - \text{cl}(F)$ . Hence  $\text{cl}(F) \subseteq X - \text{pcl}(A)$ . Thus  $F \subseteq X - \text{pcl}(A)$ . Hence  $F \subseteq \text{pcl}(A) \cap (X - \text{pcl}(A)) = \emptyset$ . Which is a contradiction.

**Remark 4.10:** The converse of the above theorem need not be true as seen from the following example.

**Example 4.11:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ . Let  $A = \{a, c\}$ . Then  $\text{pcl}(A) - A = \{d\}$  contains no non-empty closed set. But  $A$  is not  $D$ -closed.

**Theorem 4.12:** A set  $A$  is  $D$ -closed then  $\text{pcl}(A) - A$  contains no non empty  $\omega$ -closed set

**Proof:** It follows from theorem 4.9.

**Theorem 4.13:** If  $A$  is  $D$ -closed and  $A \subseteq B \subseteq \text{pcl}(A)$  then  $B$  is  $D$ -closed.

**Proof:** Let  $U$  be  $\omega$ -open set of  $X$  and  $B \subseteq U$ . Then  $A \subseteq U$ .

Since  $A$  is  $D$ -closed, we get  $\text{pcl}(A) \subseteq \text{Int}(U)$ .

Now  $\text{pcl}(B) \subseteq \text{pcl}(\text{pcl}(A)) = \text{pcl}(A) \subseteq \text{Int}(U)$ . Hence  $B$  is  $D$ -closed

**Theorem 4.14:** If a subset  $A$  of  $(X, \tau)$  is  $\omega$ -open and  $D$ -closed then  $A$  is pre-closed in  $(X, \tau)$

**Proof:** Since a subset  $A$  of  $(X, \tau)$  is  $\omega$ -open, we get,  $\text{pcl}(A) \subseteq \text{Int}(A) \subseteq A$ . But  $A \subseteq \text{pcl}(A)$ . Hence  $A$  is pre-closed in  $(X, \tau)$ .

**Theorem 4.15:** A regular open set of  $(X, \tau)$  is  $\text{gpr}$ -closed iff  $A$  is  $D$ -closed in  $(X, \tau)$

**Proof:** Let  $A \subseteq U$  and  $U$  be  $\omega$ -open in  $(X, \tau)$ . Since  $A$  is regular open and  $\text{gpr}$ -closed, by lemma 2.3  $A$  is pre-closed. Since every regular open is open, we get  $A$  is open and preclosed. Hence by theorem 3.2,  $A$  is  $D$ -closed.

Conversely, let  $A \subseteq U$  and  $U$  be regular open in  $(X, \tau)$ . Since every regular open is  $\omega$ -open,

we get  $\text{pcl}(A) \subseteq \text{Int}(U) \subseteq U$ . Hence  $A$  is  $\text{gpr}$ -closed.

**Theorem 4.16:** Let  $A$  be  $D$ -closed in  $(X, \tau)$ . Then  $A$  is pre-closed iff  $\text{pcl}(A) - A$  is  $\omega$ -closed.

**Proof:** Let  $A$  be pre-closed. Then  $\text{pcl}(A) = A$ . Hence  $\text{pcl}(A) - A = \emptyset$ , which is  $\omega$ -closed.

Conversely, suppose  $\text{pcl}(A) - A$  is  $\omega$ -closed. Since  $A$  is  $D$ -closed and by theorem 4.12,  $\text{pcl}(A) - A = \emptyset$ . Then  $\text{pcl}(A) = A$ .

Hence  $A$  is preclosed.

**Theorem 4.17:** An open set  $A$  of  $(X, \tau)$  is  $\text{gp}$ -closed iff  $A$  is  $D$ -closed.

**Proof:** Let  $A$  be open and  $\text{gp}$ -closed set. Let  $A \subseteq U$  and  $U$  be  $\omega$ -open in  $X$ . Since  $A$  is open, we get  $A = \text{Int}(A) \subseteq \text{Int}(U)$ . Since  $\text{Int}(U)$  is open, we get  $\text{pcl}(A) \subseteq \text{Int}(U)$ . Hence  $A$  is  $D$ -closed. Converse is true by theorem 3.5

**Theorem 4.18:** If a subset  $A$  of  $(X, \tau)$  is open and regular closed then  $A$  is  $D$ -closed.

**Proof:** Let  $A$  be open and regular closed set. Since every regular closed set is pre-closed, we get  $A$  is open and preclosed. By theorem 3.2,  $A$  is  $D$ -closed

**Theorem 4.19:** In a topological space  $X$ , for each  $x \in X$ ,  $\{x\}$  is  $\omega$ -closed or its complement  $X - \{x\}$  is  $D$ -closed in  $(X, \tau)$

**Proof:** Suppose that  $\{x\}$  is not  $\omega$ -closed in  $(X, \tau)$ . Then  $X - \{x\}$  is not  $\omega$ -open. Hence the only  $\omega$ -open set containing  $X - \{x\}$  is  $X$ . Thus  $\text{pcl}(X - \{x\}) \subseteq X$ . Hence  $X - \{x\}$  is  $D$ -closed in  $(X, \tau)$ .

**Definition 4.20:** Let  $A$  be a subset of a topological space  $X$ . The  $D$ -closure of  $A$  is defined as the intersection of all  $D$ -closed sets that are containing  $A$  and is denoted by  $D\text{-cl}(A)$ .

**Lemma 4.21:** If a subset  $A$  of  $(X, \tau)$  is  $D$ -closed then  $A = D\text{-cl}(A)$

**Remark 4.22:** The converse of the above lemma need not be true as seen from the following example

**Example 4.23:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Let  $A = \{c\}$ . Then  $A = D\text{-cl}(A)$ . But  $A$  is not  $D$ -closed.

**Definition 4.24:** Let  $(X, \tau)$  be a topological space,  $A \subseteq X$  and  $x \in X$ . Then  $x$  is said to be a pre-limit point of  $A$  if every pre-open set containing  $x$  contains a point of  $A$  different from  $x$ .

**Definition 4.25:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . The set of all pre-limit points of  $A$  is said to be pre-derived set of  $A$  and is denoted by  $D_p[A]$ .

**Theorem 4.26:** If  $D[A] \subseteq D_p[A]$  for each subset  $A$  of a space  $(X, \tau)$  then the union of two  $D$ -closed sets is  $D$ -closed

**Proof:** Let  $A$  and  $B$  be  $D$ -closed sets of  $X$  and  $U$  be a  $\omega$ -open set such that  $A \cup B \subseteq U$ . Then  $\text{pcl}(A) \subseteq \text{Int}(U)$  and  $\text{pcl}(B) \subseteq \text{Int}(U)$ . Since for each subset  $A$  of  $X$ ,  $D_p[A] \subseteq D[A]$ .

Then  $\text{cl}(A) = \text{pcl}(A)$  and  $\text{cl}(B) = \text{pcl}(B)$ . Hence  $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) = \text{pcl}(A) \cup \text{pcl}(B) \subseteq \text{Int}(U)$

But  $\text{pcl}(A \cup B) \subseteq \text{cl}(A \cup B)$ . Hence  $\text{pcl}(A \cup B) \subseteq \text{Int}(U)$ . Hence  $A \cup B$  is  $D$ -closed

**Theorem 4.27:** A subset  $A$  of  $(X, \tau)$  is regular open iff  $A$  is open and  $D$ -closed

**Proof:** Suppose  $A$  is open and  $D$ -closed. Then  $A$  is  $\omega$ -open and  $D$ -closed. By theorem 4.14,  $A$  is pre-closed. Hence  $\text{cl}(\text{Int}(A)) \subseteq A$ . Since  $A$  is open and  $A = \text{Int}(A)$ , we get  $\text{cl}(A) = A$ .

Hence  $\text{Int}(A) = \text{Int}(\text{cl}(A))$ . Since  $A$  is open, we get  $A = \text{Int}(\text{cl}(A))$ . Hence  $A$  is regular open.

Conversely, let  $A$  be regular open. Then  $A$  is open. Let  $U$  be a  $\omega$ -open and  $A \subseteq U$ .

Suppose  $\text{pcl}(A) \not\subseteq \text{Int}(U)$ . Then  $A \cup \text{cl}(\text{Int}(A)) \not\subseteq \text{Int}(U)$ ,  $\text{cl}(\text{Int}(A)) \not\subseteq \text{Int}(U)$ . Since  $A$  is open and  $A = \text{Int}(A)$ , we get  $\text{cl}(A) \not\subseteq \text{Int}(A)$ . Which is a contradiction.

## 5. D-OPENSETS

**Definition 5.1:** A subset  $A$  of  $(X, \tau)$  is said to be  $D$ -open if its complement  $X - A$  is  $D$ -closed in  $(X, \tau)$ .

**Example 5.2:** Let  $X = \{a, b, c, d, e\}$  and  $\tau = \{\emptyset, \{e\}, \{a, b\}, \{c, e\}, \{a, b, e\}, \{a, b, c, e\}, X\}$ . Then the set  $A = \{a, b, c, e\}$  is  $D$ -open.

**Theorem 5.3:** Every clopen subset of  $(X, \tau)$  is  $D$ -open

**Proof:** Let  $A$  be any clopen subset of  $(X, \tau)$ . Let  $X - A \subseteq U$  and  $U$  is  $\omega$ -open set in  $X$ .

Since every closed set is pre-closed, we get  $\text{pcl}(X - A) = X - A = \text{Int}(X - A) \subseteq \text{Int}(U)$ . Hence  $X - A$  is  $D$ -closed and hence  $A$  is  $D$ -open

**Theorem 5.4:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . Then  $A$  is  $D$ -open if and only if  $\text{cl}(S) \subseteq \text{pInt}(A)$  whenever  $S \subseteq A$  and  $S$  is  $\omega$ -closed.

**Proof:** Let  $A$  be the  $D$ -open set in  $(X, \tau)$ . Let  $S \subseteq A$  and  $S$  be  $\omega$ -closed.

Then  $X - A$  is  $D$ -closed and it is contained in the  $\omega$ -open set  $X - S$ .

Hence  $pcl(X-A) \subseteq Int(X-S)$ ,  $X-pint(A) \subseteq X-cl(S)$ .

Hence  $cl(S) \subseteq pint(A)$ .

Conversely, If  $S$  is  $\omega$ -closed set such that  $cl(S) \subseteq pInt(A)$  whenever  $S \subseteq A$ , it follows that  $X-A \subseteq X-S$  and  $X-pInt(A) \subseteq X-cl(S)$ . Thus  $pcl(X-A) \subseteq Int(X-S)$ . Hence  $X-A$  is D-closed and  $A$  is D-open set.

**Theorem 5.5:** If  $pInt(A) \subseteq B \subseteq A$  and  $A$  is D-open then  $B$  is D-open.

**Proof:** If  $pInt(A) \subseteq B \subseteq A$  then  $X-A \subseteq X-B \subseteq X-pInt(A)$ . That is  $X-A \subseteq X-B \subseteq pcl(X-A)$ .

Since  $X-A$  is D-closed and by theorem 4.13,  $X-B$  is D-closed and hence  $B$  is D-open.

**Theorem 5.6:** If  $A \subseteq S$  is D-closed then  $pcl(A) - A$  is D-open.

**Proof:** Let  $A$  be D-closed. Then by theorem 4.12,  $pcl(A) - A$  contains no nonempty  $\omega$ -closed set.

Hence  $\phi = S \subseteq pcl(A) - A$  and  $\phi = S$  be  $\omega$ -closed. Clearly  $cl(S) \subseteq pInt(pcl(A) - A)$ .

Hence by theorem 5.4,  $pcl(A) - A$  is D-open.

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