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A NEW TYPE OF GENERALIZED CLOSED SETS

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ABSTRACT

In this paper,we introduce a new classes of sets called D-closed sets, D-open sets in topological spaces and study some basic properties of D-closed sets.

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1. INTRODUCTION

The study of generalized closed sets in a topological space was initiated by Levine [6] and the concept of $T_{1/2}$ space was introduced. In 1996, H. Maki, J. Umehara and T. Noiri [8] introduced the class of pregeneralized closed sets and used them to obtain properties of pre-$T_{1/2}$ spaces. In 2008, S.Jafari, T.Noiri, N.Rajesh and M. L. Thivagar [4] introduced the concept of $\tilde{g}$ closed sets and their properties . In this paper we introduce new classes of sets called D-closed sets and D-open sets in topological spaces**.**

2. PRELIMINARIES

Throughout this paper (X,τ), (Y, σ) and (Z, η) will always denoted topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X,τ), cl(A), Int(A) and D(A) denote the closure, the interior and the derived set of A respectively.

We recall some known definitions are needed in this paper.

Definition 2.1: Let (X,τ) be a topological space. A subset A of the space X is said to be

1. pre-open [7] if $A\subseteq Int(cl(A))$ and pre-closed if $cl(Int(A))\subseteq A$
2. semi-open [5] if $A\subseteq cl(Int(A))$ and semi-closed if $Int(cl(A))\subseteq A$
3. α–open[9] if $A\subseteq Int(cl(Int(A)))$ and α–closed if $cl(Int(cl(A)))\subseteq A$.
4. semi-preopen[1] if $A \subseteq cl(Int(cl(A)))$ and semi-preclosed if $Int(cl(Int(A)))\subseteq A$.
5. regular open if $A = Int(cl(A))$ and regular closed if $A = cl(Int(A))$

Definition 2.2[8]: Let (X,τ) be a topological space and $A\subseteq X$

1. pre-interior of A denoted by pInt(A) is the union of all pre-open subsets of A.
2. pre-closure of A denoted by pcl(A) is the intersection of all pre-closed sets containing A.

Lemma 2.3[3]: If A is regular open and gpr-closed then A is pre-closed

Lemma 2.4[4]: Every $\tilde{g}$ -closed set is ω-closed

Lemma 2.5[1]: For any subset A of X, the following relations hold.

1. $scl(A) = A\cup Int(cl(A))$
2. $\alpha\text{-}cl(A) = A\cup cl(Int(cl(A)))$

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3. pcl(A) = A∪cl(Int(A))
4. spcl(A) = A∪Int(cl(Int(A)))

Definition 2.6: Let (X,τ) be a topological space. A subset A⊆ X is said to be

1. generalized closed (g-closed)[6] if cl(A) ⊆ U whenever A⊆ U and U is open. The complement of a g-closed set is said to be g-open.
2. generalized pre-closed(gp-closed)[8] if pcl(A) ⊆ U whenever A ⊆ U and U is open
3. generalized preregular-closed(gpr-closed)[3] if pcl(A)⊆U whenever A⊆ U and U is regular open
4. pregeneralized closed (pg-closed)[8] if pcl(A) ⊆ U whenever A⊆ U and U is pre-open
5. g*-preclosed (g*p-closed) [14] if pcl(A) ⊆ U whenever A ⊆ U and U is g-open
6. gα*-closed[14] if α-cl(A)⊆Int(U) whenever A ⊆ U and U is α-open. The complement of gα*-closed is said to be gα*-open
7. μ-preclosed(μp-closed)[14] if pcl(A) ⊆ U whenever A ⊆ U and U is gα*-open
8. generalized semi-preclosed (gsp-closed) [2] if spcl(A) ⊆ U whenever A⊆ U and U is open
9. pre semi-closed [14] if spcl(A) ⊆ U whenever A⊆ U and U is g-open
10. ω-closed [12] if cl(A) ⊆ Uwhenever A⊆ U and U is semi-open. The complement of a ω-closed set is ω-open.
11. $\eta^{\wedge *}$-closed[10] if spcl(A) ⊆ U whenever A ⊆ U and U is ω-open
12. *g-closed[15] if cl(A) ⊆ U whenever A ⊆ U and U is ω-open
13. #g-semiclosed(#gs-closed)[13] if scl(A) ⊆ U whenever A ⊆ U and U is *g-open.The complement of #gs-closed set is said to be #gs-open.
14. $\tilde{g}$-closed set [4] if cl(A) ⊆ U whenever A ⊆ U and U is #gs-open. The complement of $\tilde{g}$-closed set is said to be $\tilde{g}$-open
15. ρ-closed [11] if pcl(A) ⊆ Int(U) whenever A ⊆ U and U is $\tilde{g}$-open

3. BASIC PROPERTIES OF D-CLOSED SETS

We introduce the following definition

Definition 3.1: A subset A of a space (X, τ) is said to be D-closed in (X,τ) if pcl(A) ⊆ Int(U), whenever A⊆ U and U is ω – open.

Theorem 3.2: Every open and pre-closed subset of (X, τ) is D-closed.

Proof: Let A be open and preclosed subset of (X, τ). Let A⊆ U and U is ω – open in X.

Then pcl(A)=A=Int(A)⊆ Int (U). Hence A is D-closed.

Remark 3.3: The converse of the above theorem is not true.

Example 3.4: Let X={a, b, c, d, e} and τ ={φ, {a, b},{a, b, d},{a, b, c, d}, {a, b, d, e}, X}.

Then the set A= {a, b, c, e} is D-closed but neither open nor pre-closed.

Theorem 3.5: Every D-closed set is gp- closed

Proof: Let A be any D-closed set in X. Let A ⊆ U and U is open in X. Since every open set is ω-open,

we get pcl(A) ⊆ Int(U) = U. Hence A is gp-closed

Remark 3.6: The Converse of the above theorem is not true.

Example 3.7: Let X = {a, b, c, d, e} and τ = {ϕ,{b, c, d},{a, b, c, d},{b, c, d, e},X}. Then the set A = {c, d} is gp-closed but not D-closed in X.

Theorem 3.8: Every D-closed set is gpr-closed.

Proof: Let A be any D-closed set. Let A ⊆ U and U is regular open. Since every regular open is open and every open is ω-open, we get pcl(A) ⊆ Int(U) =U. Hence A is gpr-closed.

Remark 3.9: The converse of the above theorem is not true

Example 3.10: Let X = {a, b, c, d, e} and τ = {ϕ,{e},{a, b},{c, e},{a, b, e},{a, b, c, e},X}. Then the set A = {a, b, c, e} is gpr-closed but not D-closed in X

Theorem 3.11: Every D-closed set is gsp-closed.

Proof: Let A be any D-closed set. Let A⊆U and U be open. Since every open set is ω-open, we get pcl(A) ⊆Int(U)=U. Since spcl(A) ⊆pcl(A) ⊆U, We get A is gsp closed.

Remark 3.12: The converse of the above theorem is not true.

Example 3.13: Let X={a, b, c, d, e} and τ ={φ, {a, b},{c, d},{a, b, c, d}, X}. Then the set A = {a, b} is gsp-closed but not D-closed in X.

Remark 3.14: D-closedness and pre-closedness are independent. It is shown by the following examples

Example 3.15: Let X = {a, b, c, d, e} and τ = {ϕ,{a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}. Then the set A = {a, b, c,e} is D-closed but not pre-closed.

Example 3.16: Let X = {a, b, c, d, e} and τ = {ϕ,{a, b},{a, b, c},{a, b, d},{a, b, c, d},X}. Then the set A = {a} is pre-closed but not D-closed.

Remark 3.17: D-closedness and α-closedness are independent. It is shown by the following examples.

Example 3.18: Let X= {a, b, c, d} and τ ={φ, {b},{c},{b, c},{b, c, d}, X}.Then the set A={d}is α- closed but not D-closed.

Example 3.19: Let X = {a, b, c} and τ = {ϕ , {c}, {b, c}, X}. Then the set A={a, c} is D-closed but not α-closed.

Remark 3.20: D-closed sets are independent of semi-closed sets and semi-preclosed sets. It is shown by the following example.

Example 3.21: Let X={a, b, c, d} and τ={φ, {b},{c},{b, c},{b, c, d}, X}.Then the set A={a, b, c} is D-closed but neither semi-closed nor semi-preclosed and the set B={c, d} is both semi-closed and semi-preclosed but not D-closed.

Remark 3.22: D-closedness and presemi-closedness are independent.It is shown by the following examples.

Example 3.23: Let X = {a, b, c} and τ = {ϕ,{a},{c}{a, c},X}. Then the set A = {a} is presemi-closed but not D-closed

Example 3.24: Let X = {a, b, c} and τ = {ϕ,{a},X}. Then the set A = {b, c}is D-closed but not presemi-closed

Remark 3.25: D-closedness and g-closedness are independent.It is shown by the following example

Example 3.26: Let X = {a, b, c, d, e} and τ = {ϕ,{a, b}{a, b, d},{a, b, c, d} {a, b, d, e},X}. Then the set A = {a, c, d} is D-closed but not g-closed and the set B = {e} is g-closed but not D-closed.

Remark 3.27: D-closedness and pg-closedness are independent. It is shown by the following examples.

Example 3.28: Let X= {a, b, c} and τ={φ, {a}, X}. Then the set A={a, b} is D- closed but not pg-closed.

Example 3.29: Let X={a, b, c} and τ={φ, {a, b}, X}. Then the set A= {a} is pg- closed but not D-closed.

Remark 3.30: D-closedness and g*p-closedness are independent. It is shown by the following examples.

Example 3.31: Let X = {a, b, c} and τ = {ϕ,{b, c},X}. Then the set A = {b} is g*p-closed but not D-closed

Example 3.32: Let X = {a, b, c} and τ = {ϕ,{a},X}. Then the set A = {a, b} is D-closed but not g*p-closed

Remark 3.33: D-closedness and μp-closedness are independent. It is shown by the following example.

Example 3.34: Let X = {a, b, c} and τ = {ϕ,{b},{b, c},X}. Then the set A = {a, b} is D-closed but not μp-closed and the set B = {b, c} is μp-closed but not D-closed

Remark 3.35: D-closedness and $\eta^{\wedge *}$- closedness are independent. It is shown by the following examples.

Example 3.36: Let X = {a, b, c, d, e} and τ = {ϕ,{a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}. Then the set A = {a, b, d} is D-closed but not $\eta^{\wedge *}$- closed.

Example 3.37: Let X = {a, b, c, d} and τ = {ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c},X}. Then the set A = {a, b} is $\eta^{\wedge *}$-closed but not D-closed.

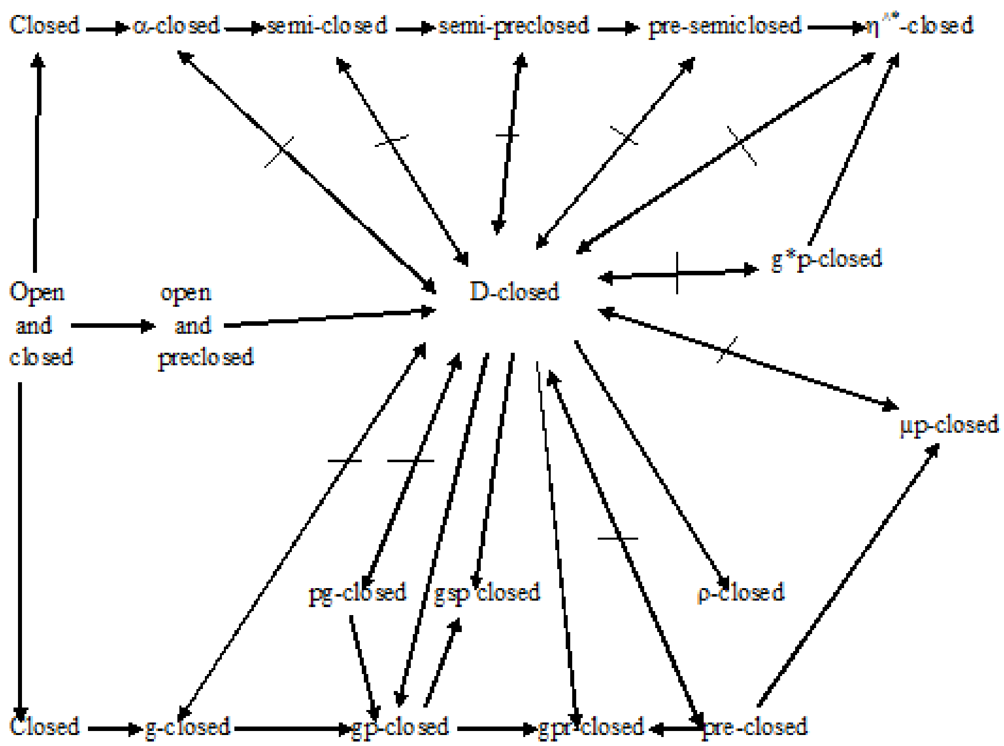
Theorem 3.38: Every D-closed set is ρ-closed.

Proof: Let A be any D-closed set. Let A ⊆ U and U be $\tilde{g}$ -open in X. By Lemma 2.4, we get pcl(A) ⊆ Int(U). Hence A is ρ-closed.

Remark 3.39: The converse of the above theorem is not true. It is shown by the following example.

Example 3.40: Let X = {a, b, c, d, e} and τ = {ϕ,{a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}. Then the set A = {a, b, c} is ρ-closed but not D-closed.

Remark 3.41: From the above discussion and known results we have the following implication A ⟶ B(A ⟷̸ B) represents A implies B but not conversely (A and B are independent of each other).



4. PROPERTIES OF D-CLOSED SETS

Definition 4.1: The intersection of all ω-open subsets of (X,τ) containing A is called ω-kernel of A and is denoted by ω-Ker(A).

Theorem 4.2: If a subset A of (X,τ) is D-closed then pcl(A) ⊆ ω-ker(A).

Proof: Suppose that A is D-closed.Then pcl(A)⊆Int(U) whenever A⊆U and U is ω-open. Let x∈pcl(A). Suppose x∉ω-ker(A). Then there is a ω-open set U containing A such that x∉ U.Since U is a ω-open set containing A, x∉ pcl(A). Which is a contradiction.

Remark 4.3: The converse of the above theorem need not be true as seen from the following example.

Example 4.4: Let X={a, b, c, d, e} and τ={ϕ,{e},{a, b},{c, e},{a, b, e},{a, b, c, e},X}. Then the set A= {a} satisfies pcl(A) ⊆ ω-ker(A). But A is not D-closed.

Remark 4.5: The union of two D-closed sets need not be D-closed.

Example 4.6: Let X={a, b, c, d, e} and τ={ϕ,{a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}.Let A={a, d} and B={b, d}. Here A and B are D-closed sets. But A∪B= {a, b, d} is not D-closed.

Remark 4.7: The intersection of two D-closed sets need not be D-closed.

Example 4.8: Let X = {a, b, c, d} and τ={ϕ, {c},{b, c},X}. Let A= {a, c} and B={c, d}. Here A and B are D-closed sets. But A∩ B= {c} is not D-closed.

Theorem 4.9: A set A is D-closed then pcl(A) - A contains no non-empty closed set.

Proof: Suppose F ⊆ pcl(A)-A be a non-empty closed set. Then F ⊆ pcl(A) and A ⊆ X. Since X-F is ω-open, we get pcl(A)⊆ Int(X-F)=X-cl(F). Hence cl(F) ⊆ X-pcl(A). Thus F ⊆ X-pcl(A). Hence F⊆ pcl(A)∩ (X-pcl(A))=ϕ. Which is a contradiction.

Remark 4.10: The converse of the above theorem need not be true as seen from the following example.

Example 4.11: Let X={a, b, c, d, e} and τ={ϕ,{a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}. Let A = {a, c}. Then pcl(A)-A={d} contains no non-empty closed set. But A is not D-closed.

Theorem 4.12: A set A is D-closed then pcl(A)-A contains no non empty ω-closed set

Proof: It follows from theorem 4.9.

Theorem 4.13: If A is D-closed and A ⊆ B ⊆ pcl(A) then B is D-closed.

Proof: Let U be ω-open set of X and B ⊆ U. Then A ⊆ U.

Since A is D-closed, we get pcl(A) ⊆ Int(U).

Now pcl(B) ⊆ pcl(pcl(A))=pcl(A)⊆Int(U). Hence B is D-closed

Theorem 4.14: If a subset A of (X, τ) is ω-open and D-closed then A is pre-closed in (X, τ)

Proof: Since a subset A of (X, τ) is ω-open, we get, pcl(A) ⊆Int(A)⊆A. But A ⊆ pcl(A).Hence A is pre-closed in (X,τ).

Theorem 4.15: A regular open set of (X, τ) is gpr-closed iff A is D-closed in (X, τ)

Proof: Let A ⊆ U and U be ω-open in (X, τ). Since A is regular open and gpr-closed, by lemma 2.3 A is pre-closed. Since every regular open is open, we get A is open and preclosed. Hence by theorem 3.2, A is D-closed.

Conversely, let A ⊆ U and U be regular open in (X, τ). Since every regular open is ω-open,

we get pcl(A) ⊆ Int(U) ⊆ U. Hence A is gpr-closed.

Theorem 4.16: Let A be D-closed in (X, τ).Then A is pre-closed iff pcl(A)-A is ω-closed.

Proof: Let A be pre-closed. Then pcl(A)=A. Hence pcl(A)-A =ϕ, which is ω-closed.

Conversely, suppose pcl(A)-A is ω-closed. Since A is D-closed and by theorem 4.12, pcl(A)-A=ϕ. Then pcl(A)=A.

Hence A is preclosed.

Theorem 4.17: An open set A of (X, τ) is gp-closed iff A is D-closed.

Proof: Let A be open and gp-closed set.Let A⊆U and U be ω-open in X. Since A is open, we get A=Int(A) ⊆ Int(U). Since Int(U) is open, we get pcl(A)⊆Int(U). Hence A is D-closed. Converse is true by theorem 3.5

Theorem 4.18: If a subset A of (X, τ) is open and regular closed then A is D-closed.

Proof: Let A be open and regular closed set. Since every regular closed set is pre-closed, we get A is open and preclosed. By theorem 3.2, A is D-closed

Theorem 4.19: In a topological space X, for each x∈X, {x} is ω-closed or its complement X-{x} is D-closed in (X, τ)

Proof: Suppose that {x} is not ω-closed in (X, τ). Then X-{x} is not ω-open. Hence the only ω-open set containing X-{x} is X. Thus pcl(X-{x}) ⊆ X. Hence X-{x} is D-closed in (X, τ).

Definition 4.20: Let A be a subset of a topological space X. The D-closure of A is defined as the intersection of all D-closed sets that are containing A and is denoted by D-cl(A).

Lemma 4.21: If a subset A of (X, τ) is D-closed then A=D-cl(A)

Remark 4.22: The converse of the above lemma need not be true as seen from the following example

Example 4.23: Let X={a, b, c, d} and τ={ϕ,{c},X}. Let A= {c}.Then A=D-cl(A). But A is not D-closed.

Definition 4.24: Let (X, τ) be a topological space, A ⊆ X and x∈X. Then x is said to be a pre-limit point of A if every pre-open set containing x contains a point of A different from x.

Definition 4.25: Let (X, τ) be a topological space and A ⊆ X. The set of all pre-limit points of A is said to be pre-derived set of A and is denoted by D_p[A].

Theorem 4.26: If D[A] ⊆ D_p[A] for each subset A of a space (X,τ) then the union of two D-closed sets is D-closed

Proof: Let A and B be D-closed sets of X and U be a ω-open set such that A∪B ⊆ U. Then pcl(A) ⊆Int(U) and pcl(B) ⊆ Int(U). Since for each subset A of X, D_p[A] ⊆ D[A].

Then cl(A) = pcl(A) and cl(B)= pcl(B). Hence cl(A∪B) =cl(A)∪cl(B)=pcl(A)∪pcl(B) ⊆ Int(U)

But pcl(A∪B) ⊆ cl(A∪B). Hence pcl(A∪B) ⊆ Int(U). Hence A∪B is D-closed

Theorem 4.27: A subset A of (X,τ) is regular open iff A is open and D-closed

Proof: Suppose A is open and D-closed.Then A is ω-open and D-closed. By theorem 4.14, A is pre-closed. Hence cl(Int(A))⊆A.Since A is open and A=Int(A),we get cl(A)=A.

Hence Int(A)= Int(cl(A)). Since A is open, we get A=Int(cl(A)). Hence A is regular open.

Conversely, let A be regular open. Then A is open. Let U be a ω-open and A ⊆ U.

Suppose pcl(A) ⊄ Int(U). Then A∪cl(Int(A)) ⊄Int(U),cl(Int(A)) ⊄Int(U). Since A is open and A=Int(A),we get cl(A)⊄Int(A). Which is a contradiction.

5. D-OPENSETS

Definition 5.1: A subset A of (X, τ) is said to be D-open if its complement X-A is D-closed in (X, τ).

Example 5.2: Let X = {a, b, c, d, e}and τ = {ϕ,{e},{a, b}{c, e},{a, b, e},{a, b, c, e},X}.Then the set A={a, b, c, e} is D-open.

Theorem 5.3: Every clopen subset of (X, τ) is D-open

Proof: Let A be any clopen subset of (X, τ).Let X-A ⊆ U and U is ω-open set in X.

Since every closed set is pre-closed, we get pcl(X-A) =X-A=Int(X-A)⊆Int(U). Hence X-A is D-closed and hence A is D-open

Theorem 5.4: Let (X,τ) be a topological space and A⊆ X. Then A is D-open if and only if cl(S)⊆ pInt(A) whenever S⊆ A and S is ω-closed.

Proof: Let A be the D-open set in (X, τ). Let S⊆ A and S be ω-closed.

Then X-A is D-closed and it is cantained in the ω-open set X-S

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Hence pcl(X-A) ⊆ Int(X-S), X-pint(A)⊆ X-cl(S).

Hence cl(S) ⊆ pint(A).

Conversely, If S is ω-closed set such that cl(S)⊆ pInt(A) whenever S⊆ A, it follows that X-A ⊆ X-S and X-pInt(A)⊆ X-cl(S).Thus pcl(X-A)⊆ Int(X-S). Hence X-A is D-closed and A is D-open set.

Theorem 5.5: If pInt (A) ⊆ B ⊆ A and A is D-open then B is D-open.

Proof: If pInt(A) ⊆ B ⊆ A then X – A ⊆ X –B ⊆ X - pInt (A).That is X-A ⊆ X - B ⊆ pcl(X - A).

Since X-A is D-closed and by theorem 4.13, X - B is D-closed and hence B is D-open.

Theorem 5.6: If A ⊆ S is D-closed then pcl(A) – A is D-open.

Proof: Let A be D-closed.Then by theorem 4.12, pcl (A) – A contains no nonempty ω-closed set.

Hence φ = S ⊆ pcl(A) - A and φ = S be ω-closed. Clearly cl(S) ⊆ pInt (pcl (A) - A).

Hence by theorem 5.4, pcl(A) - A is D-open.

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