

ON TOTAL EDGE FIXED GEODOMINATING SETS AND POLYNOMIALS OF GRAPHS

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ABSTRACT

We introduce total edge fixed geodomination sets and polynomial of a graph G . The total edge fixed geodomination polynomial of G is defined as $G_t(G, x) = \sum_{e_k \in E(G)} G_{e_k}(G, x)$ where $G_{e_k}(G, x) = \sum_{i=g_{e_k}(G)}^{n-2} g_{e_k}(G, i)x^i$, $g_{e_k}(G, i)$ is the number of edge fixed geodominating sets of graph G with cardinality i , and e_k is a fixed edge of G and $g_{e_k}(G)$ is the e_k -geodomination number of G . we obtain some properties of $G_t(G, x)$ and its coefficients. Also, we compute the polynomial for some specific graphs.

Keywords: Edge fixed geodominating set, Total edge fixed geodomination polynomial, unimodal.

1. INTRODUCTION

Let $G = (u, v)$ be a simple graph of order n . An edge of a graph is said to be pendant if one of its vertices is a pendant vertex. A vertex of a graph is said to be pendant if its neighbourhood contains exactly one vertex. The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of the shortest $u - v$ path in G . A $u - v$ path of length $d(u, v)$ is called $u - v$ geodesic. The closed interval $I[u, v]$ consists of all vertices lying on some $u - v$ geodesic of G , while for $S \subseteq V$, $I[S] = \bigcup_{u, v \in S} I[u, v]$. A set S of vertices is a geodetic set if $I[S] = V$, and the minimum cardinality of a geodetic set is the geodetic number, $g(G)$. The concept of edge fixed domination was introduced in [8]. Let $e = xy$ be any edge of a connected graph G of order atleast 3. A set S of vertices of G is an e -geodominating set if every vertex of G is lies on either an x - u geodesic or a y - u geodesic in G for some element u in S . The minimum cardinality of an e -geodominating set of G is defined as the e -geodomination number of G and is denoted by $g_e(G)$ or $g_{xy}(G)$.

The corona of two graphs G_1 and G_2 , as defined by Frucht and Harary in [6] is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 , where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 . The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G , where for each vertex $v \in V(G)$, a new vertex v' and a pendant edge vv' are added.

A finite sequence of real numbers $(a_0, a_1, a_2, \dots, a_n)$ is said to be unimodal if there is some $k \in \{0, 1, \dots, n\}$ called the mode of a sequence, such that $a_0 \leq \dots \leq a_{k-1} \leq a_k \geq a_{k+1} \geq \dots \geq a_n$; the mode is unique if $a_{k-1} < a_k > a_{k+1}$. A polynomial is called unimodal if the sequence of its coefficients is unimodal.

In the next section, we introduce the total edge fixed geodomination polynomial. In section 3 we study the coefficients of the total edge fixed geodomination polynomial. In the last section, we study the total edge fixed geodetic polynomial of the graph $G \circ K_1$, where $G \circ K_1$, is the corona of two graphs G and K_1 .

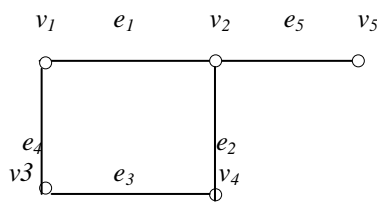
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2. TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL OF A GRAPH

Definition 2.1: The edge fixed geodomination polynomial is defined as $G_{e_k}(G, x) = \sum_{i=g_{e_k}(G)}^{n-2} g_{e_k}(G, i)x^i$, where $g_{e_k}(G, i)$ is the number of edge fixed geodominating sets of graph G with cardinality i, and e_k is a fixed edge of G. The total edge fixed geodomination polynomial of G is defined as $G_t(G, x) = \sum_{e_k \in E(G)} G_{e_k}(G, x)$ clearly, $1 \leq g_{e_k}(G) \leq n-2$.

If $g_t(G) = \min_{e_k \in E(G)} \{g_{e_k}(G)\}$, then we can write $G_t(G, x) = \sum_{i=g_t(G)}^{n-2} g_t(G, i)x^i$, where $g_t(G, i)$ is the number of total edge fixed geodominating sets of cardinality i.

Example 2.2: Consider a graph G.



Clearly $g_{e_1}(G) = 2$

$g_{e_1}(G, 2) = 2$. Here $S = \{\{v_5, v_3\}, \{v_5, v_4\}\}$ are the edge fixed geodomination set of cardinality 2.

$g_{e_1}(G, 3) = 1$. Here $S = \{v_3, v_4, v_5\}$ is the edge fixed geodominating set.

Therefore $G_{e_1}(G, x) = 2x^2 + x^3$.

Similarly $G_{e_2}(G, x) = 2x^2 + x^3$

$G_{e_3}(G, x) = 2x^2 + x^3$

$g_{e_4}(G) = 1$

$g_{e_4}(G, 1) = 1$. Here $S = \{v_5\}$

$g_{e_4}(G, 2) = 2$. Here $S = \{\{v_5, v_3\}, \{v_5, v_4\}\}$

$g_{e_4}(G, 3) = 1$. Here $S = \{v_3, v_4, v_5\}$

Therefore $G_{e_4}(G, x) = x + 2x^2 + x^3$

$g_{e_5}(G) = 1$.

$g_{e_5}(G, 1) = 1$. Here $S = \{v_3\}$

$g_{e_5}(G, 2) = 2$. Here $S = \{\{v_3, v_1\}, \{v_3, v_4\}\}$

$g_{e_5}(G, 3) = 1$. Here $S = \{(v_1, v_3, v_4)\}$

Therefore $G_{e_5}(G, x) = x + 2x^2 + x^3$

Hence $G_t(G, x) = 2x^2 + x^3 + 2x^2 + x^3 + 2x^2 + x^3 + x + 2x^2 + x^3 + x + 2x^2 + x^3$

$$= 2x + 10x^2 + 5x^3.$$

Theorem 2.3: The total edge fixed geodomination polynomial of a complete graph K_n ($n \geq 3$) is $G_t(K_n, x) = n C_2 x^{n-2}$.

Proof: Let v_1, v_2, \dots, v_n be the vertices of K_n . Let $e_1, e_2, \dots, e_{nC_2}$ be the edges of K_n . By theorem (2.8) in [8], $g_{e_k}(K_n) = n - 2$, for any edge e_k in K_n . Fix a edge e_1 in K_n . Then $g_{e_1}(K_n, n - 2) = 1$.

$$g_{e_{nC_2}}(K_n, n-2) = 1.$$

Since $1 \leq g_{e_k}(G) \leq n-2$ for any edge e_k in G we have $G_{e_1}(K_n, x) = x^{n-2}$, $G_{e_2}(K_n, x) = x^{n-2}, \dots, G_{e_{nC_2}} = x^{n-2}$.

$$\begin{aligned} \text{Therefore } G_t(K_n, x) &= (x^{n-2} + x^{n-2} + \dots + x^{n-2}) nC_2 \text{ times} \\ &= nC_2 x^{n-2}. \end{aligned}$$

Theorem 2.4: The total edge fixed geodomination polynomial of a stargraph $K_{1,n}$ ($n \geq 2$) is $G_t(K_{1,n}, x) = n x^{n-1}$.

Proof: Let u, v_1, v_2, \dots, v_n be the vertices of $K_{1,n}$.

By a theorem 2.7 in [8],

$$g_{e_k}(K_{1,n}) = n-1 \text{ for any edge } e_k \text{ in } K_{1,n}. \text{ Fix a edge } e_1 \text{ in } k_{1,n}$$

$$\text{Then } g_{e_1}(K_{1,n}, n-1) = 1$$

$$\text{Similarly, } g_{e_2}(K_{1,n}, n-1) = 1, \dots, g_{e_n}(K_{1,n}, n-1) = 1$$

$$\text{Then, we have } G_{e_1}(K_{1,n}, x) = x^{n-1}, G_{e_2}(K_{1,n}, x) = x^{n-1}, \dots, G_{e_n}(K_{1,n}, x) = x^{n-1}.$$

$$\begin{aligned} \text{Therefore, } G_t(K_{1,n}, x) &= (x^{n-1} + x^{n-1} + \dots + x^{n-1}) n \text{ times} \\ &= n x^{n-1}. \end{aligned}$$

Theorem 2.5: The total edge fixed geodomination polynomial of a complete bipartite graph $K_{m,n}$ ($m \geq n$) is

$$(i) G_t(K_{m,n}, x) = 2nx ((1+x)^{n-1}), m=2.$$

$$(ii) G_t(K_{m,n}, x) = mn \{ [(1+x)^{n-1} - 1] [(1+x)^{m-1} - 1] + x^{m-1} + x^{n-1} \}, m \geq 3$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with bipartition (X, Y) . Let $X = \{ u_1, u_2, \dots, u_m \}$ and $Y = \{ v_1, v_2, \dots, v_n \}$.

Then $|X| = m$ and $|Y| = n$

Case (i): If $m = 2$.

Then $X = \{ u_1, u_2 \}$ and $Y = \{ v_1, v_2, \dots, v_n \}$.

Let e_1, e_2, \dots, e_{2n} be the edges of $K_{m,n}$.

By a theorem (2.13) in [8], $g_{e_k}(K_{m,n}) = 1$, for any edge e_k in $K_{m,n}$. Fix e_k

$$g_{e_k}(K_{m,n}, 1) = 1$$

$$g_{e_k}(K_{m,n}, 2) = (n-1)C_1$$

$$g_{e_k}(K_{m,n}, 3) = (n-1)C_2 \dots \dots \dots$$

$$g_{e_k}(K_{m,n}, n) = 1 = (n-1)C_{n-1}$$

$$\text{Therefore, } G_{e_k}(K_{m,n}, x) = x + (n-1)C_1x^2 + (n-1)C_2x^3 + \dots + (n-1)C_{n-1}x^n.$$

$$= x(1 + (n-1)C_1x + (n-1)C_2x^2 + \dots + (n-1)C_{n-1}x^{n-1}).$$

$$= x(1+x)^{n-1}, \text{ for } k = 1, 2, \dots, 2n.$$

$$\text{Therefore, } G_t(K_{m,n}, x) = [x(1+x)^{n-1} + x(1+x)^{n-1} + \dots + x(1+x)^{n-1}] (2n \text{ times})$$

$$= 2nx [(1+x)^{n-1}]$$

Case (ii): If $m \geq 3$.

Let e_1, e_2, \dots, e_{mn} be the edges of $K_{m,n}$.

By a theorem (2.13) in [8] $g_{e_k}(K_{m,n}) = 2$ for any edge e_k in $K_{m,n}$. Fix e_k in G .

$$g_{e_k}(K_{m,n}, 2) = (m-1) C_1 x (n-1) C_1.$$

$$g_{e_k}(K_{m,n}, 3) = (m-1) C_2 x (n-1) C_1 + (m-1) C_1 x (n-1) C_2$$

$$g_{e_k}(K_{m,n}, 4) = (m-1) C_3 x (n-1) C_1 + (m-1) C_2 x (n-1) C_2 + (m-1) C_1 x (n-1) C_3$$

$$g_{e_k}(K_{m,n}, m-2) = (m-1) C_{m-3} x (n-1) C_1 + (m-1) C_{m-4} x (n-1) C_2 + (m-1) C_{m-5} x (n-1) C_3 \dots\dots + (m-1) C_1 x (n-1) C_{m-3}$$

$$g_{e_k}(K_{m,n}, m) = (m-1) C_{m-1} x (n-1) C_1 + (m-1) C_{m-2} x (n-1) C_2 + \dots\dots + (m-1) C_1 x (n-1) C_{m-1}$$

$$g_{e_k}(K_{m,n}, n-2) = (m-1) C_{m-2} x (n-1) C_{n-m} + (m-1) C_{m-3} x (n-1) C_{n-m+1} + \dots\dots + (m-1) C_1 x (n-1) C_{n-3}$$

$$g_{e_k}(K_{m,n}, n-1) = (m-1) C_{m-1} x (n-1) C_{n-m} + (m-1) C_{m-2} x (n-1) C_{n-m+1} + \dots\dots + (m-1) C_1 x (n-1) C_{n-2} + (n-1) C_{n-1}$$

$$g_{e_k}(K_{m,n}, m+n-3) = (m-1) C_{m-1} x (n-1) C_{n-2} + (n-1) C_{n-1} x (m-1) C_{m-2}$$

$$g_{e_k}(K_{m,n}, m+n-2) = (m-1) C_{m-1} x (n-1) C_{n-1}$$

Therefore,

$$\begin{aligned} G_{e_k}(K_{m,n}, x) &= [(m-1) C_1 x (n-1) C_1] x^2 + [(m-1) C_2 x (n-1) C_1 + (m-1) C_1 x (n-1) C_2] x^3 \\ &+ [(m-1) C_3 x (n-1) C_1 + (m-1) C_2 x (n-1) C_2 + (m-1) C_1 x (n-1) C_3] x^4 + \dots \\ &+ [(m-1) C_{m-1} x (n-1) C_{n-2} + (n-1) C_{n-1} x (m-1) C_{m-2}] x^{m+n-3} + (m-1) C_{m-1} x (n-1) C_{n-1} x^{m+n-2} \\ &+ (m-1) C_{m-1} x^{m-1} + (n-1) C_{n-1} x^{n-1}. \\ &= (m-1) C_1 [(n-1) C_1 x^2 + (n-1) C_2 x^3 + \dots\dots + (n-1) C_{n-1} x^n] + (m-1) C_2 [(n-1) C_1 x^3 \\ &+ (n-1) C_2 x^4 + \dots\dots + (n-1) C_{n-1} x^{n+1}] + (m-1) C_3 [(n-1) C_1 x^4 + (n-1) C_2 x^5 + \dots\dots \\ &+ (n-1) C_{n-1} x^{n+2}] + \dots\dots + (m-1) C_{m-2} [(n-1) C_1 x^{m-1} + (n-1) C_2 x^m + \dots\dots + (n-1) C_{n-1} x^{m+n-3}] \\ &+ (m-1) C_{m-1} [(n-1) C_1 x^m + (n-1) C_2 x^{m+1} + \dots\dots + (n-1) C_{n-1} x^{m+n-2}] + x^{m-1} + x^{n-1} \\ &= (m-1) C_1 x [(1+x)^{n-1} - 1] + (m-1) C_2 x^2 [(1+x)^{n-1} - 1] + (m-1) C_3 x^3 [(1+x)^{n-1} - 1] + \dots \\ &+ (m-1) C_{m-2} x^{m-2} [(1+x)^{n-1} - 1] + (m-1) C_{m-1} x^{m-1} [(1+x)^{n-1} - 1] + x^{m-1} + x^{n-1} \\ &= [(1+x)^{n-1} - 1] + [(m-1) C_1 x + (m-1) C_2 x^2 + (m-1) C_3 x^3 + \dots\dots + (m-1) C_{m-2} x^{m-2} \\ &+ (m-1) C_{m-1} x^{m-1}] + x^{m-1} + x^{n-1} \\ &= [(1+x)^{n-1} - 1] x [(1+x)^{m-1} - 1] + x^{m-1} + x^{n-1} \end{aligned}$$

Since $K_{m,n}$ has mn edges

$$G_t(K_{m,n}, x) = mn \{[(1+x)^{n-1} - 1] [(1+x)^{m-1} - 1] + x^{m-1} + x^{n-1}\}.$$

3. COEFFICIENTS OF TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL

In this section we obtain some properties of total edge fixed geodomination polynomial of G .

Theorem 3.1: Let G be a graph with $|V(G)| = n, n \geq 3$.

Then (i) If G is connected, then $g_{e_k}(G, n-2) = 1$ for any edge e_k in G .

(ii) $g_{e_k}(G, i) = 0$ if and only if $i < g_{e_k}(G)$ or $i > n-2$.

(iii) $G_t(G, x)$ has no constant term.

(iv) $G_t(G, x)$ is strictly increasing function in $[0, \infty)$

(v) Let G be a graph and H be any induced subgraph of G , then $\deg(G_t(G, x)) \geq \deg(G_t(H, x))$

(iv) zero is a root of $(G_t(G, x))$ with multiplicity $g_t(G)$.

Proof: Proof is obvious.

4. TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL OF $G \circ K_1$

Let G be any graph with vertex set $\{v_1, v_2, \dots, v_n\}$. Add a new set of vertices $\{u_1, u_2, \dots, u_n\}$ and join u_i to v_i for $1 \leq i \leq n$. By the definition of corona of two graphs. We shall denote this graph by $G \circ K_1$.

Lemma 4.1: For any graph G of order n ,

$$g_{e_k}(G \circ K_1) = \begin{cases} n-1 & \text{if } e_k \text{ is a pendant edge} \\ n & \text{if } e_k \text{ is a non-pendant edge} \end{cases}$$

Proof:

Case (i): e_k is a pendant edge. If G_1 is a edge fixed geodominating set of G then $G_1 = \{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_n\}$ or $G_1 = \{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_n\} \cup A$ where $A \subset \{v_1, v_2, \dots, v_n\}$.

Therefore $|G_1| \geq n-1$. since $G_1 = \{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_n\}$ is a edge fixed geodominating set of $G \circ K_1$, we have $g_{e_k}(G \circ K_1) = n-1$.

Case (ii): e_k is a non pendant edge. If G_1 is a edge fixed geodominating set of G , then $G_1 = \{u_1, u_2, \dots, u_n\}$ or $G_1 = \{u_1, u_2, \dots, u_n\} \cup A$ where $A \subset \{v_1, v_2, \dots, v_n\}$.

Therefore $|G_1| \geq n$. since $G_1 = \{u_1, u_2, \dots, u_n\}$ is a edge fixed geodominating set of $G \circ K_1$, we have $g_{e_k}(G \circ K_1) = n$.

By lemma 4.1 $g_{e_k}(G \circ K_1, m) = 0$ if $m < n-1$ for pendant edge e_k and if $m < n$ for non pendant edge e_k . so we shall compute $g_e(G \circ K_1, m)$ for $n-1 \leq m \leq 2(n-1)$ if e_k is a pendant edge and for $n \leq m \leq 2(n-1)$ if e_k is a non pendant edge.

Theorem 4.2: For any graph G of order n and p be the number of edges. Then

$$g_{e_k}(G \circ K_1, m) = \begin{cases} (n-1)C_{m-(n-1)} & \text{if } e_k \text{ is a pendant edge and } n-1 \leq m \leq 2(n-1) \\ (n-2)C_{m-n} & \text{if } e_k \text{ is a non-pendant edge and } n \leq m \leq 2(n-1) \end{cases}$$

Hence $G_t(G \circ K_1, x) = (1+x)^{n-2} [nx^{n-1} + (n+p)x^n]$.

Proof:

Case (i): e_k is a pendant edge.

Suppose that G_1 is a edge fixed geodominating set of size m . When $m = n-1$, the edge fixed geodminating set with cardinality $n-1$ is $\{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_n\}$. Therefore, $g_{e_k}(G \circ K_1, n-1) = (n-1) C_0$. When $m = n$, the edge fixed geodominating set with cardinality n is $\{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_n\} \cup \{v_j\}$, $1 \leq j \leq n-1$.

Therefore $g_{e_k}(G \circ K_1, n) = (n-1) C_1$. Similarly, $g_{e_k}(G \circ K_1, n+1) = (n-1) C_2, \dots, g_{e_k}(G \circ K_1, 2(n-1)) = (n-1) C_{n-1}$.

$$\begin{aligned} \text{Therefore } G_{e_k}(G \circ K_1, x) &= (n-1) C_0 x^{n-1} + (n-1) C_1 x^n + (n-1) C_2 x^{n+1} + \dots + (n-1) C_{n-2} x^{2n-3} + (n-1) C_{n-1} x^{2(n-1)} \\ &= x^{n-1} (1 + (n-1) C_1 x + (n-1) C_2 x^2 + \dots + (n-1) C_{n-1} x^{n-1}) \\ &= x^{n-1} (1+x)^{n-1} \end{aligned}$$

Case (ii): e_k is a non pendant edge. Suppose that G_1 is a edge fixed geodominating set of size m . When $m = n$, the edge fixed geodominating set with cardinality n is $\{u_1, u_2, \dots, u_n\}$. Therefore $g_{e_k}(G \circ K_1, n) = (n-2) C_0$. when $m = n+1$, the edge fixed geodominating set with cardinality $n+1$ is $\{u_1, u_2, \dots, u_n\} \cup \{v_j\}, 1 \leq j \leq n-2$.

Therefore, $g_{e_k}(G \circ K_1, n+1) = (n-2) C_1$.

Similarly $g_{e_k}(G \circ K_1, n+2) = (n-2) C_2, \dots, g_{e_k}(G \circ K_1, 2n-3) = (n-2) C_{(n-3)}$.

$$g_{e_k}(G \circ K_1, 2n-2) = (n-2) C_{(n-2)}.$$

Therefore, $G_{e_k}(G \circ K_1, x) = (n-2) C_0 x^n + (n-2) C_1 x^{n+1} + (n-2) C_2 x^{n+2} + \dots + (n-2) C_{n-3} x^{2n-3} + (n-2) C_{n-2} x^{2n-2}$

$$= x^n (1 + (n-2) C_1 x + (n-2) C_2 x^2 + \dots + (n-2) C_{n-3} x^{n-3} + (n-2) C_{n-2} x^{n-2})$$

$$= x^n (1+x)^{n-2}$$

Since $G \circ K_1$ has n pendant edges and p non pendant edges, we have,

$$G_t(G \circ K_1, x) = nx^{n-1} (1+x)^{n-1} + px^n (1+x)^{n-2}$$

$$= x^{n-1} (1+x)^{n-2} [n(1+x) + px]$$

$$= (1+x)^{n-2} [nx^{n-1} + (n+p)x^n].$$

Here, we discuss about unimodality of the geodetic polynomial of $G_n \circ K_1$, where G_n denote a graph with n vertices.

Let us denotes $G \circ K_1$ simply by G^*

Theorem 4.3: For every $n \in \mathbb{N}$,

$$\begin{cases} g_{e_k}(G_n^*, n-1) = g_{e_k}(G_n^*, 2(n-1)) = 1, \text{ if } e_k \text{ is a pendant edge} \\ g_{e_k}(G_n^*, n) = g_{e_k}(G_n^*, 2(n-1)) = 1 \text{ if } e_k \text{ is a non-pendant edge} \end{cases}$$

Proof: By lemma 4.2, If e_k is a pendant edge, then $g_{e_k}(G_n^*, n-1) = (n-1) C_0 = 1, g_{e_k}(G_n^*, 2(n-1)) = (n-1) C_{n-1} = 1$.

If e_k is a non pendant edge, then $g_{e_k}(G_n^*, n) = (n-2) C_0 = 1, g_{e_k}(G_n^*, 2(n-1)) = (n-2) C_{n-2} = 1$.

Theorem 4.4: (Unimodal theorem for $G \circ K_1$)

For every $n \in \mathbb{N}$

(a) If e_k is a pendant edge, then

$$(i) \quad 1 = g_{e_k}(G_{3n}^*, 3n-1) < g_{e_k}(G_{3n}^*, 3n) < \dots < g_{e_k}(G_{3n}^*, 4n-2) > g_{e_k}(G_{3n}^*, 4n-1) > g_{e_k}(G_{3n}^*, 4n) \dots > g_{e_k}(G_{3n}^*, 6n-3) > g_{e_k}(G_{3n}^*, 6n-1) = 1.$$

$$(ii) \quad 1 = g_{e_k}(G_{3n+1}^*, 3n) < g_{e_k}(G_{3n+1}^*, 3n+1) < \dots < g_{e_k}(G_{3n+1}^*, 4n-1) > g_{e_k}(G_{3n+1}^*, 4n) > g_{e_k}(G_{3n+1}^*, 4n+1) > \dots > g_{e_k}(G_{3n+1}^*, 6n-1) > g_{e_k}(G_{3n+1}^*, 6n) = 1.$$

$$(iii) \quad 1 = g_{e_k}(G_{3n+2}^*, 3n+1) < g_{e_k}(G_{3n+2}^*, 3n+2) < \dots < g_{e_k}(G_{3n+2}^*, 4n) > g_{e_k}(G_{3n+2}^*, 4n+1) > g_{e_k}(G_{3n+2}^*, 4n+2) > \dots > g_{e_k}(G_{3n+2}^*, 6n+1) > g_{e_k}(G_{3n+2}^*, 6n+2) = 1.$$

(b) If e_k is a non pendant edge, then

$$(i) \quad 1 = g_{e_k}(G_{3n}^*, 3n) < g_{e_k}(G_{3n}^*, 3n+1) < \dots < g_{e_k}(G_{3n}^*, 4n-1) > g_{e_k}(G_{3n}^*, 4n+1) > \dots > g_{e_k}(G_{3n}^*, 6n-3) > g_{e_k}(G_{3n}^*, 6n-2) = 1.$$

$$(ii) 1 = g_{e_k}(G_{3n+1}^*, 3n+1) < g_{e_k}(G_{3n+1}^*, 3n+2) < \dots < g_{e_k}(G_{3n+1}^*, 4n) > g_{e_k}(G_{3n+1}^*, 4n+1) > g_{e_k}(G_{3n+1}^*, 4n+2) \\ > \dots > g_{e_k}(G_{3n+1}^*, 6n-1) > g_{e_k}(G_{3n+1}^*, 6n) = 1.$$

$$(iii) 1 = g_{e_k}(G_{3n+2}^*, 3n+2) < g_{e_k}(G_{3n+2}^*, 3n+3) < \dots < g_{e_k}(G_{3n+2}^*, 4n+2) > g_{e_k}(G_{3n+2}^*, 4n+3) \square > g_{e_k}(G_{3n+2}^*, 4n+4) \\ > \dots > g_{e_k}(G_{3n+2}^*, 6n+1) > g_{e_k}(G_{3n+2}^*, 6n+2) = 1.$$

Proof: Since the proofs of all parts are similar, we only prove the part (a) (i).

(i) Clearly $g_{e_k}(G_{3n}^*, 3n-1) = 1$ and $g_{e_k}(G_{3n}^*, 6n-2) = 1$.

we shall prove that $g_{e_k}(G_{3n}^*, i) < g_{e_k}(G_{3n}^*, i+1)$ for $3n-1 \leq i \leq 4n-2$ and $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ for $4n-1 \leq i \leq 6n-2$.

Suppose that $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ by theorem 4.2, we have

$$\binom{3n-1}{i - (3n-1)} < \binom{3n-1}{i+1 - (3n-1)}$$

So we have $i \leq 4n-2$. But $i \geq 3n-1$. Together we have $3n-1 \leq i \leq 4n-2$. Similarly, we have $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ for $4n-1 \leq i \leq 6n-2$.

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