International Journal of Mathematical Archive-3(4), 2012, Page: 1463-1466
Available online through www.ijma.info ISSN 2229 – 5046

ON SUPER-RESOLUTION BY MINIMIZATION OF FULL-WIDTH AT HALF-MAXIMUM WITH A LINEAR APODISATION FILTERS

[1]Alety. Srisailam* & [2]V. Dharmaiah

[1,2]Department of Mathematics, Osmania University, Hyderabad-500007, India
E-mail: asmou22@yahoo.com

(Received on: 02-04-12; Accepted on: 19-04-12)

ABSTRACT

The "Full-Width at Half-Maximum" (FWHM) is a very useful image-quality assessment parameter when the optical system is not diffraction-limited. In the present paper, we have studied this important parameter for optical systems apodised with Bartlett window functions.

Mathematics Subject Classification: *78A45.*

Key-words: *Apodisation, Resolution, FWHM, etc.*

---

1. INTRODUCTION

The **"Full-width at Half–Maximum"** (FWHM) is a direct corollary of the PSF of the system. It is also known as the **"Half-Power Diameter"** (HPD), as it is defined as the diameter of the PSF at 50% of its peak value. The FWHM, as such, is not a very useful point-image quality-assessment parameter for comparing diffraction–limited and near diffraction–limited optical systems. The shape of the PSF and the value of the central maximum vary little with increasing wave-front error in all normal situations. A central obscuration, viz., imaging with an annular aperture, in fact, reduces the half-power diameter.

Further the half-power diameter is most useful when the image quality is limited by external factors such as image motion or atmospheric turbulence, where it correlates directly with the degrading factors. It is also useful where large amounts of aberrations are present [1]. In such cases the first minimum in the diffraction pattern is non-zero and, this makes the application of Rayleigh's criterion of resolution, invalid.

2. MATHEMATICAL FORMULA OF FWHM

The intensity Point Spread Function (PSF) for an optical system, in general, is given by the Fourier Transform of the exit pupil of the aperture. The analytical expression for the same can be written as [2],

$$I(y,Z) = \left| 2\int_0^1 f(r) e^{-iyr^2/2} J_0(Zr) r dr \right|^2 \tag{1}$$

Where , f(r)=Pupil function of the system considered.

Y=defocus error of the wave front incident on the system; Z is usual dimension less-diffraction variable from the centre of the diffraction pattern; $J_0(Zr)$ is the Bessel function of the first kind and order zero, r is the radial co-ordinate of an arbitrary point on the exit pupil.

For the optical system for our present study we have considered a set of linear apodisation filters whose pupil function f(r) can be mathematically expressed as

$$f(r) = (1 - \beta r) \tag{2}$$

Where β is the apodisation parameter which controls the degree of pupil transmission f(r) $[0 \le f(r) \le 1]$. Substituting (2) in (1), we get

Corresponding author: [1]Alety. Srisailam, *E-mail: asmou22@yahoo.com

$$I(y,Z)=\left|2\int_0^1 f(1-\beta r)e^{-iyr^2/2}\,J_0(Zr)rdr\right|^2 \tag{3}$$

Further, we have restricted our present study in the focused plan of observation (y=0). Eqn .(3),

Therefore, reduces to

$$I(0,Z)=[2\int_0^1(1-\beta r)J_0(Zr)rdr]^2 \tag{4}$$

3. RESULTS AND DISCUSSIONS

For results we have used Mathematica4.1 software and we given program as input got the results. We have used the expression (4) to evaluate the intensity PSF for various values of the apodisation parameterβ. The computed values of the PSF have been shown in table-1. Fig.1 shows the intensity distribution curves for various values of β. It is found from the graphs that as the values of β is increased, the central maximum steadily decreases becoming minimum at $\beta=0$.

As for the effects of β on the FWHM, we have plotted the variation of FWHM with β in fig.2. The values of FWHM have also been tabulated in table-2. We find from both the figure and table, that as value of β is increased from 0.0 to 0.4, the value of FWHM decreases gradually. Thereafter for $0.4\leq\beta<1.0$, the value of FWHM starts to increase, though at a slow rate.

Before concluding, we wish to point out that the Fourier Transform of a function with circular symmetry is a Bessel function of the first kind. Thus the diffraction limited PSF of a two dimensional optical system with radial symmetry, will be a Bessel function described intensity distribution in the Airy pattern. This diffraction-limited point spread function will be considerably modified when one includes the effects of various amplitude and phase filters, aberrations, image motion, atmospheric turbulence and other factors external to the optical system [3]

Table-1

Computed values of FWHM for various values of β

β	τ
0.0	1.0000
0.1	0.87111
0.2	0.075111
0.3	0.64
0.4	0.53777
0.5	0.44444
0.6	0.36
0.7	0.28444
0.8	0.217778
0.9	0.16
1.0	0.1111

Fig.1: FWHM for various values of beta

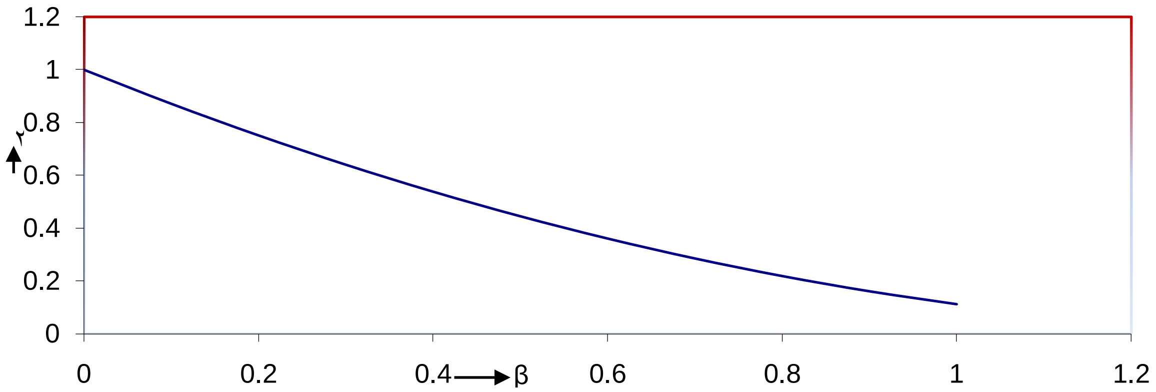
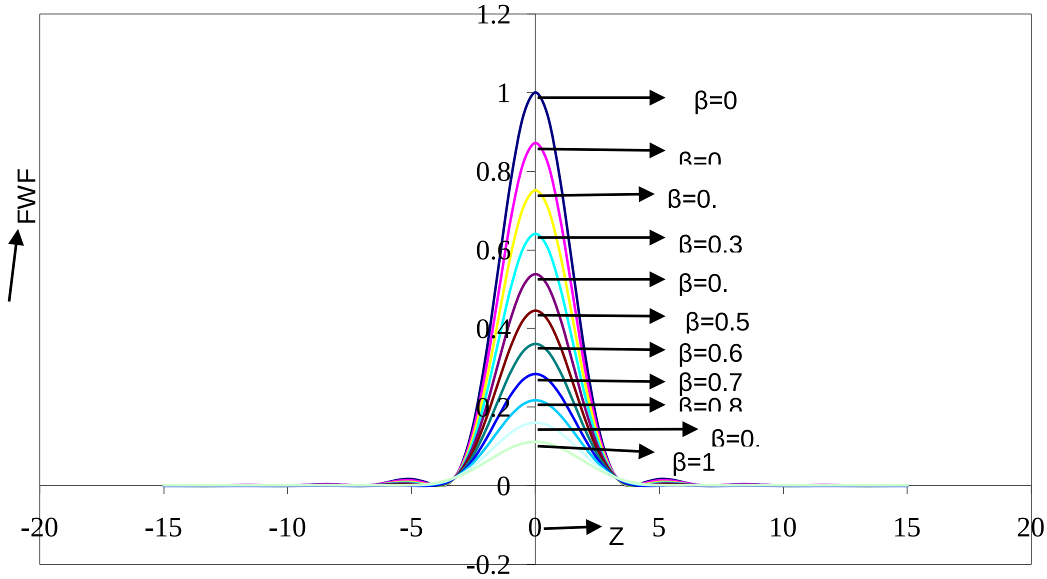


Table-2

FULL-WIDTH AT HALF-MAXIMUM

Z values	□=0	□=0.1	□=0.2	□=0.3	□=0.4	□=0.5	□=0.6	□=0.7	□=0.8	□=0.9	□=1
-15	0.00075	0.00061	0.00049	0.00038	0.00028	0.0002	0.00013	7.7E-05	3.8E-05	1.2E-05	7.1E-07
-14.5	0.00071	0.00058	0.00046	0.00035	0.00026	0.00018	0.00011	6.3E-05	2.8E-05	6.9E-06	3E-09
-14	0.00036	0.00029	0.00023	0.00017	0.00012	8.2E-05	5E-05	2.6E-05	9.4E-06	1.1E-06	8.6E-07
-13.5	3.2E-05	2.4E-05	1.8E-05	1.2E-05	7.7E-06	4.2E-06	1.8E-06	3.9E-07	9.1E-09	6.6E-07	2.3E-06
-13	0.00012	9.8E-05	8.1E-05	6.5E-05	5.1E-05	3.9E-05	2.8E-05	1.9E-05	1.2E-05	6.4E-06	2.6E-06
-12.5	0.0007	0.00057	0.00046	0.00036	0.00027	0.00019	0.00013	7.5E-05	3.8E-05	1.3E-05	1.1E-06
-12	0.00139	0.00112	0.00088	0.00068	0.0005	0.00034	0.00022	0.00012	5.3E-05	1.2E-05	5.5E-08
-11.5	0.00158	0.00126	0.00098	0.00074	0.00053	0.00035	0.00021	0.00011	4E-05	4.5E-06	4.2E-06
-11	0.00103	0.00081	0.00062	0.00045	0.00031	0.0002	0.00011	4.6E-05	1E-05	2.2E-07	1.7E-05
-10.5	0.00023	0.00017	0.00012	7.6E-05	4.4E-05	2.1E-05	6E-06	1.3E-07	3E-06	1.5E-05	3.5E-05
-10	7.6E-05	7.3E-05	7E-05	6.7E-05	6.5E-05	6.2E-05	5.9E-05	5.7E-05	5.4E-05	5.2E-05	5E-05
-9.5	0.00115	0.00098	0.00082	0.00067	0.00054	0.00042	0.00032	0.00023	0.00015	9.5E-05	5E-05
-9	0.00297	0.00246	0.002	0.00159	0.00122	0.00091	0.00064	0.00041	0.00024	0.00011	3.2E-05
-8.5	0.00413	0.00338	0.0027	0.0021	0.00158	0.00113	0.00076	0.00046	0.00023	8.3E-05	8.8E-06
-8	0.00344	0.00278	0.00219	0.00167	0.00122	0.00084	0.00053	0.00029	0.00013	2.8E-05	4E-07
-7.5	0.0013	0.00103	0.00078	0.00057	0.0004	0.00025	0.00014	6.1E-05	1.4E-05	5.1E-08	1.8E-05
-7	1.8E-06	3.5E-06	5.8E-06	8.6E-06	1.2E-05	1.6E-05	2.1E-05	2.6E-05	3.2E-05	3.8E-05	4.5E-05
-6.5	0.00224	0.00187	0.00153	0.00123	0.00096	0.00072	0.00052	0.00035	0.00021	0.00011	4.1E-05
-6	0.00851	0.00692	0.0055	0.00424	0.00314	0.00221	0.00144	0.00084	0.0004	0.00012	3.3E-06
-5.5	0.01542	0.0123	0.00953	0.00712	0.00505	0.00334	0.00199	0.00098	0.00033	2.3E-05	7.2E-05
-5	0.01717	0.01331	0.00994	0.00706	0.00467	0.00278	0.00137	0.00046	3.2E-05	1E-04	0.00066
-4.5	0.01055	0.00764	0.0052	0.00323	0.00173	0.00069	0.00012	1.8E-05	0.00038	0.00121	0.00251
-4	0.00109	0.00046	0.0001	1.9E-06	0.00017	0.00059	0.00128	0.00223	0.00345	0.00493	0.00667
-3.5	0.00616	0.00682	0.0075	0.00822	0.00898	0.00976	0.01058	0.01144	0.01232	0.01324	0.01419
-3	0.05109	0.04818	0.04535	0.0426	0.03994	0.03737	0.03488	0.03248	0.03016	0.02793	0.02578
-2.5	0.15815	0.14307	0.12875	0.11518	0.10237	0.09032	0.07902	0.06847	0.05868	0.04965	0.04137
-2	0.33261	0.29539	0.26039	0.22758	0.19699	0.1686	0.14242	0.11845	0.09669	0.07713	0.05978
-1.5	0.55341	0.48664	0.42416	0.36597	0.31207	0.26247	0.21715	0.17613	0.1394	0.10695	0.0788
-1	0.77458	0.67733	0.58659	0.50238	0.4247	0.35353	0.28888	0.23076	0.17916	0.13408	0.09552
-0.5	0.9391	0.81881	0.70675	0.60294	0.50737	0.42004	0.34095	0.2701	0.2075	0.15314	0.10701
0	**1**	**0.87111**	**0.75111**	**0.64**	**0.53778**	**0.44444**	**0.36**	**0.28444**	**0.21778**	**0.16**	**0.11111**
0.5	0.9391	0.81881	0.70675	0.60294	0.50737	0.42004	0.34095	0.2701	0.2075	0.15314	0.10701
1	0.77458	0.67733	0.58659	0.50238	0.4247	0.35353	0.28888	0.23076	0.17916	0.13408	0.09552
1.5	0.55341	0.48664	0.42416	0.36597	0.31207	0.26247	0.21715	0.17613	0.1394	0.10695	0.0788
2	0.33261	0.29539	0.26039	0.22758	0.19699	0.1686	0.14242	0.11845	0.09669	0.07713	0.05978
2.5	0.15815	0.14307	0.12875	0.11518	0.10237	0.09032	0.07902	0.06847	0.05868	0.04965	0.04137
3	0.05109	0.04818	0.04535	0.0426	0.03994	0.03737	0.03488	0.03248	0.03016	0.02793	0.02578
3.5	0.00616	0.00682	0.0075	0.00822	0.00898	0.00976	0.01058	0.01144	0.01232	0.01324	0.01419
4	0.00109	0.00046	0.0001	1.9E-06	0.00017	0.00059	0.00128	0.00223	0.00345	0.00493	0.00667
4.5	0.01055	0.00764	0.0052	0.00323	0.00173	0.00069	0.00012	1.8E-05	0.00038	0.00121	0.00251
5	0.01717	0.01331	0.00994	0.00706	0.00467	0.00278	0.00137	0.00046	3.2E-05	1E-04	0.00066
5.5	0.01542	0.0123	0.00953	0.00712	0.00505	0.00334	0.00199	0.00098	0.00033	2.3E-05	7.2E-05
6	0.00851	0.00692	0.0055	0.00424	0.00314	0.00221	0.00144	0.00084	0.0004	0.00012	3.3E-06
6.5	0.00224	0.00187	0.00153	0.00123	0.00096	0.00072	0.00052	0.00035	0.00021	0.00011	4.1E-05
7	1.8E-06	3.5E-06	5.8E-06	8.6E-06	1.2E-05	1.6E-05	2.1E-05	2.6E-05	3.2E-05	3.8E-05	4.5E-05
7.5	0.0013	0.00103	0.00078	0.00057	0.0004	0.00025	0.00014	6.1E-05	1.4E-05	5.1E-08	1.8E-05
8	0.00344	0.00278	0.00219	0.00167	0.00122	0.00084	0.00053	0.00029	0.00013	2.8E-05	4E-07
8.5	0.00413	0.00338	0.0027	0.0021	0.00158	0.00113	0.00076	0.00046	0.00023	8.3E-05	8.8E-06
9	0.00297	0.00246	0.002	0.00159	0.00122	0.00091	0.00064	0.00041	0.00024	0.00011	3.2E-05
9.5	0.00115	0.00098	0.00082	0.00067	0.00054	0.00042	0.00032	0.00023	0.00015	9.5E-05	5E-05
10	7.6E-05	7.3E-05	7E-05	6.7E-05	6.5E-05	6.2E-05	5.9E-05	5.7E-05	5.4E-05	5.2E-05	5E-05
10.5	0.00023	0.00017	0.00012	7.6E-05	4.4E-05	2.1E-05	6E-06	1.3E-07	3E-06	1.5E-05	3.5E-05
11	0.00103	0.00081	0.00062	0.00045	0.00031	0.0002	0.00011	4.6E-05	1E-05	2.2E-07	1.7E-05
11.5	0.00158	0.00126	0.00098	0.00074	0.00053	0.00035	0.00021	0.00011	4E-05	4.5E-06	4.9E-06
12	0.00139	0.00112	0.00088	0.00068	0.0005	0.00034	0.00022	0.00012	5.3E-05	1.2E-05	5.5E-08
12.5	0.0007	0.00057	0.00046	0.00036	0.00027	0.00019	0.00013	7.5E-05	3.8E-05	1.3E-05	1.1E-06
13	0.00012	9.8E-05	8.1E-05	6.5E-05	5.1E-05	3.9E-05	2.8E-05	1.9E-05	1.2E-05	6.4E-06	2.6E-06
13.5	3.2E-05	2.4E-05	1.8E-05	1.2E-05	7.7E-06	4.2E-06	1.8E-06	3.9E-07	9.1E-09	6.6E-07	2.3E-06
14	0.00036	0.00029	0.00023	0.00017	0.00012	8.2E-05	5E-05	2.6E-05	9.4E-06	1.1E-06	8.6E-07
14.5	0.00071	0.00058	0.00046	0.00035	0.00026	0.00018	0.00011	6.3E-05	2.8E-05	6.9E-06	3E-09
15	0.00075	0.00061	0.00049	0.00038	0.00028	0.0002	0.00013	7.7E-05	3.8E-05	1.2E-05	7.1E-07

Fig.2:



ACKNOWLEDGEMENTS

Authors express their gratitude to University Grants Commission, New Delhi, India for sanctioning DRS-III/SAP-I to the Department of Mathematics, Osmania University, Hyderabad,A.P., India wherein the facilities are used for under taking this work. They are also indicted to Prof.M.V.Ramanamurthy, Department of Mathematics, Osmania University, Hyderabad for his guidance.

REFERENCES

[1] V.N. MAHAJAN, "Diffraction Theory of Optical Images and Aberrations", Part-II, SPIE Press, Massachusetts, U.S.A. , (2004).

[2] M.BORN and E. WOLF, "Principles of Optics" 7th Edition, Pergamon Press, N.Y. 2009

[3] P.V.V.S. MURTY, (1992), "performance of Optical Systems Apodisation with Co-sinusoidal Filters" Ph.D. Theses Osmania University, Hyderabad, (1991).

****

