International Journal of Mathematical Archive-3(4), 2012, Page: 1423-1428
Available online through www.ijma.info ISSN 2229 – 5046

THE UPPER EDGE-TO-VERTEX GEODETIC NUMBER OF A GRAPH

J. John

Department of Mathematics, Government College of Engineering, Tirunelveli - 627007, India
E-mail: johnramesh1971@yahoo.co.in

A. Vijayan[1] & S. Sujitha[2*]

Department of Mathematics, N. M. Christian College, Marthandam- 629165, India
E-mail: vijayan2020@yahoo.co.in, sujivenki@rediffmail.com

(Received on: 20-03-12; Accepted on: 05-04-12)

ABSTRACT

Let G be a non-trivial connected graph with at least three vertices. For subsets A and B of V(G), the distance d(A, B) is defined as d(A, B) = min{d(x, y) : x ∈A, y ∈B}. A u – v path of length d(A, B) is called an A – B geodesic joining the sets A, B ⊆ V(G), where u ∈ A and v ∈ B. A vertex x is said to lie on an A – B geodesic if x is a vertex of an A – B geodesic. A set S ⊆ E is called an edge-to-vertex geodetic set of G if every vertex of G is either incident with an edge of S or lies on a geodesic joining a pair of edges of S. The minimum cardinality of an edge-to-vertex geodetic set of G is $g_{ev}(G)$. *Any edge-to-vertex geodetic set of cardinality* $g_{ev}(G)$ *is called an edge-to-vertex geodetic basis of G. An edge-to-vertex geodetic set S in a connected graph G is called a minimal edge-to-vertex geodetic set if no proper subset of S is an edge-to-vertex geodetic set of G. The upper edge-to-vertex geodetic number* $g_{ev}^+(G)$ *of G is the maximum cardinality of a minimal edge-to-vertex geodetic set of G. Some general properties satisfied by this concept are studied. For a connected graph G of size q with upper edge-to-vertex geodetic number q or q – 1 are characterized. It is shown that for every two positive integers a and b, where* $2 \le a \le b$, *there exists a connected graph G with* $g_{ev}(G) = a$ *and* $g_{ev}^+(G) = b$.

Keywords: *distance, geodesic, edge-to-vertex geodetic basis, edge-to-vertex geodetic number, upper edge-to-vertex geodetic number.*

AMS Subject Classification: *05C12.*

1. INTRODUCTION

By a *graph* $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. We consider connected graphs with at least three vertices. For basic definitions and terminologies we refer to [1, 5]. For vertices u and v in a connected graph G, the *distance* $d(u, v)$ is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ *geodesic*. For subsets A and B of $V(G)$, the *distance* $d(A, B)$ is defined as $d(A, B) = min\{d(x, y) : x \in A, y \in B\}$. An $u - v$ path of length $d(A, B)$ is called an $A - B$ *geodesic* joining the sets A, B, where $u \in A$ and $v \in B$. A vertex x is said to *lie* on an $A - B$ geodesic if x is a vertex of an $A - B$ geodesic. For $A = \{u, v\}$ and $B = \{z, w\}$ with uv and zw edges, we write an $A - B$ geodesic as $uv - zw$ geodesic and $d(A, B)$ as $d(uv, zw)$. A set $S \subseteq E$ is called an *edge-to-vertex geodetic set* if every vertex of G is either incident with an edge of S or lies on a geodesic joining a pair of edges of S. The *edge-to-vertex geodetic number* $g_{ev}(G)$ of G is the minimum cardinality of its edge-to-vertex geodetic sets and any edge-to-vertex geodetic set of cardinality $g_{ev}(G)$ is called an *edge-to-vertex geodetic basis* of G or a $g_{ev}(G)$-set of G. The geodetic number of a graph was studied in [1, 2, 3]. The edge-to-vertex geodetic number of a graph was introduced and studied by Santhakumaran and John in [6] and further studied in [7]. The upper geodetic number of a graph was introduced and studied in [4]. For a nonempty set X of edges, the subgraph <X> *induced by* X has edge set X and consists of all vertices that are incident with at least one edge in X. This subgraph is called an *edge-induced subgraph* of G. For a cut vertex v in a connected graph G and a component H of G-v, the subgraph H and the vertex v together with all edges joining v and $V(H)$ is called a *branch* of G at v. A double star is a tree with diameter three. A vertex v is an *extreme vertex* of a graph G if the subgraph induced by its neighbors is complete. An edge of a connected graph G is called an *extreme edge* of G if one of its ends is an extreme vertex of G.

***Corresponding author: S. Sujitha[2*]**, ***E-mail:** sujivenki@rediffmail.com*

Consider the graph G given in Figure 1.1 with $A = \{v_4, v_5\}$ and $B = \{v_1, v_2, v_7\}$, the paths $P : v_5, v_6, v_7$ and $Q : v_4, v_3, v_2$ are the only two $A – B$ geodesics so that $d(A, B) = 2$. For the graph G given in Figure 1.2, the three $v_1v_6 – v_3v_4$ geodesics are $P : v_1,v_2,v_3$; $Q : v_1,v_2,v_4$; and $R : v_6,v_5,v_4$ with each of length 2 so that $d(v_1v_6, v_3v_4) = 2$. Since the vertices v_2 and v_5 lie on the $v_1v_6 – v_3v_4$ geodesics P and R respectively, $S = \{v_1v_6,v_3v_4\}$ is an edge-to-vertex geodetic basis of G so that $g_{ev}(G) = 2$.

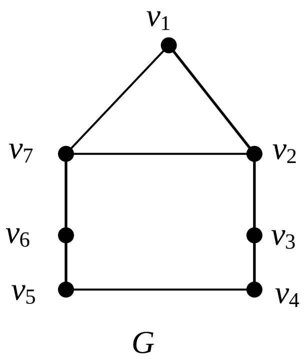

Figure: 1.1

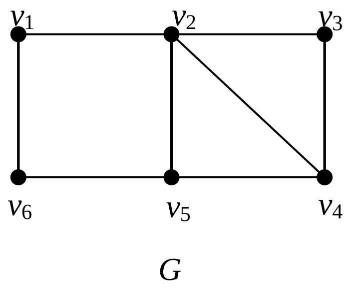

Figure: 1.2

In section 2 we give some general properties and obtain the upper edge-to-vertex geodetic number of some family of graphs. In section 3 we give some general results and sharp bounds for the upper edge-to-vertex geodetic number. In section 4 we present realization result on the edge-to-vertex geodetic number and upper edge-to-vertex geodetic number of a graph. The following theorems are used in sequel.

Theorem 1.1: [6] If v is an extreme vertex of a connected graph G, then every edge-to-vertex geodetic set contains at least one extreme edge that is incident with v.

Theorem 1.2: [6] Let G be a connected graph and S be a g_{ev}-set of G. Then no cut edge of G which is not an end-edge of G belongs to S.

Theorem 1.3: [7] For any connected graph G of size $q \geq 2$, $g_{ev}(G) = q$ if and only if G is a star.

Theorem 1.4: [7] For any connected graph G of size $q \geq 4$, $g_{ev}(G) = q – 1$ if and only if G is a double star.

Throughout the following G denotes a connected graph with at least three vertices.

2. THE UPPER EDGE-TO-VERTEX GEODETIC NUMBER OF A GRAPH

In this section we look closely at the concept of the upper edge-to-vertex geodetic number of a graph and obtain the upper edge-to-vertex geodetic number of some family of graphs.

Definition 2.1: An edge-to-vertex geodetic set S in a connected graph G is called a *minimal edge-to-vertex geodetic set* if no proper subset of S is an edge-to-vertex geodetic set of G. The *upper edge-to-vertex geodetic number* $g_{ev}^{+}(G)$ of G is the maximum cardinality of a minimal edge-to-vertex geodetic set of G.

Example 2.2: For the graph G given in Figure 2.1, $S = \{v_1v_2, v_4v_5\}$ is an edge-to-vertex geodetic basis of G so that $g_{ev}(G) = 2$. The set $S_1 = \{v_1v_2, v_3v_4, v_5v_6\}$ is an edge-to-vertex geodetic set of G and it is clear that no proper subset of S_1 is an edge-to-vertex geodetic set of G and so S_1 is a minimal edge-to-vertex geodetic set of G. Also it is easily verified that no four element or five element subset of edge set is a minimal edge-to-vertex geodetic set of G, it follows that $g_{ev}^{+}(G) = 3$.

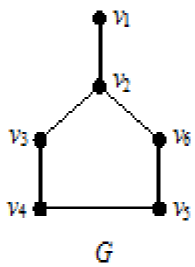

Figure: 2.1

Remark 2.3: Every minimum edge-to-vertex geodetic set of G is a minimal edge-to-vertex geodetic set of G and the converse is not true. For the graph G given in Figure 2.1, $S_1 = \{v_1v_2, v_3v_4, v_5v_6\}$ is a minimal edge-to-vertex geodetic set but not a minimum edge-to-vertex geodetic set of G.

Theorem 2.4: Let G be a connected graph with cut-vertices and S an edge-to-vertex geodetic set of G. Then every branch of G contains an element of S.

Proof: Assume that there is a branch B of G at a cut-vertex v such that B contains no element of S. Then by Theorem 1.1, B does not contain any end-edge of G. Hence it follows that no vertex of B is an end vertex of G. Let u be any vertex of B such that $u \neq v$ (such a vertex exists since $|V(B)| \geq 2$). Then u is not incident with any edge of S and so u lies on a $e - f$ geodesic P: $u_1, u_2, \ldots, u, \ldots, u_t$, where u_1 is an end of e and u_t is an end of f with $e, f \in S$. Since v is a cut-vertex of G, the $u_1 - u$ and $u - u_t$ subpaths of P both contain v and so P is not a path, which is a contradiction. Hence every branch of G contains an element of S.

Corollary 2.5: Let G be a connected graph with cut-edges and S an edge-to-vertex geodetic set of G. Then for any cut-edge e of G, which is not an end-edge, each of the two components of $G - e$ contains an element of S.

Proof: Let $e = uv$. Let G_1 and G_2 be the two components of $G - e$ such that $u \in V(G_1)$ and $v \in V(G_2)$. Since u and v are cut-vertices of G, it follows that G_1 contains at least one branch at u and G_2 contains at least one branch at v. Hence it follows from Theorem 2.4 that each of G_1 and G_2 contains an element of S.

Theorem 2.6: Let G be a connected graph and S be a minimal edge - to -vertex geodetic set of G. Then no cut edge of G which is not an end-edge of G belongs to S.

Proof: Let S be any minimal edge-to-vertex geodetic set of G. Suppose that $e = uv$ be a cut edge of G which is not an end-edge of G such that $e \in S$. Let G_1 and G_2 be the two components of $G - e$. Let $S' = S - \{uv\}$. We claim that S' is an edge-to-vertex geodetic set of G. By Corollary 2.5, G_1 contains an edge xy and G_2 contains an edge $x'y'$, where $xy, x'y' \in S$. Let z be any vertex of G. Assume without loss of generality that z belongs to G_1. Since uv is a cut edge of G, every path (in particular every geodesic) joining a vertex of G_1 with a vertex of G_2 contains the edge uv. Suppose that z is incident with uv or the edge xy of S or that lies on a geodesic joining xy and uv. If z is incident with uv, then $z = u$. Let $P : y, y_1, y_2, \ldots, z = u$ be a $xy - u$ geodesic. Let $Q : v, v_1, v_2, \ldots, y'$ be a $v - x'y'$ geodesic. Then, it is clear that $P \cup \{uv\} \cup Q$ is a $xy - x'y'$ geodesic. Thus z lies on the $xy - x'y'$ geodesic. If z is incident with xy, then there is nothing to prove. If z lies on a $xy - uv$ geodesic say $y, y_1, y_2, \ldots, z, \ldots, u$, then let $v, v_1, v_2, \ldots, y'$ be $v - x'y'$ geodesic. Then clearly $y, y_1, y_2, \ldots, z, \ldots, u, v, v_1, v_2, \ldots, y'$ is a $xy - x'y'$ geodesic. Thus z lies on a geodesic joining a pair of edges of S'. Thus we have proved that a vertex that is incident with uv or an edge of S or that lies on a geodesic joining xy and uv of S also is incident with an edge of S' or lies on a geodesic joining a pair of edges of S'. Hence it follows that S' is an edge-to-vertex geodetic set such that $S' \underset{\neq}{\subset} S$, which is a contradiction to S is a minimal edge-to-vertex geodetic set of G. Hence the theorem follows.

In the following we determine the upper edge-to-vertex geodetic number of some standard graphs.

Theorem 2.7: For any non-trivial tree T with k end-edges, $g_{ev}^+(T) = k$.

Proof: By Theorem 1.1, any edge-to-vertex geodetic set contains all the end-edges of T. By Theorem 2.6, no cut-edge of T belongs to any minimal edge-to-vertex geodetic set of G. Hence it follows that the set of all end-edges of T is the unique minimal edge-to-vertex geodetic set of T so that $g_{ev}^+(T) = k$. Thus the proof is complete.

Theorem 2.8: For a complete graph $G = K_p (p \geq 4)$, $g_{ev}^+(G) = p - 1$.

Proof: Let S be any set of $p-1$ adjacent edges of K_p incident at a vertex,say v. Since each vertex of K_p is incident with an edge of S, it follows that S is an edge-to-vertex geodetic set of G. If S is not a minimal edge-to-vertex geodetic set of G, then there exists a proper subset S' of S such that S' an edge-to-vertex geodetic set of G. Therefore there exists at least one vertex, say u of K_p such that u is not incident with any edge of S'. Hence u is neither incident with any edge of S' nor lies on a geodesic joining a pair of edges of S' and so S' is not an edge-to-vertex geodetic set of G, which is a contradiction. Hence S is a minimal edge-to-vertex geodetic set of G. Therefore $g_{ev}^+(G) \geq p - 1$. Suppose that there exists a minimal edge-to-vertex geodetic set M such that $|M| \geq p$. Since M contains at least p edges, $\langle M \rangle$ contains at least one cycle. Let $M' = M - \{e\}$, where e is an edge of a cycle which lies in $\langle M \rangle$. It is clear that M' is an edge-to-vertex geodetic set with $M' \underset{\neq}{\subset} M$, which is a contradiction. Therefore, $g_{ev}^+(G) = p - 1$.

Theorem 2.9: For the complete bipartite graph $G = K_{m,n} (2 \le m \le n)$, $g_{ev}^+(G) = n + m - 2$.

Proof: Let $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$ be a bipartition of G. Let $S_i = \{x_iy_1, x_iy_2, ..., x_iy_{n-1}, x_1y_n, x_2y_n, \ldots, x_{i-1}y_n, x_{i+1}y_n, \ldots, x_my_n\}$ $(1 \le i \le m)$, $M_j = \{x_1y_j, x_2y_j, ..., x_{m-1}y_j, x_my_1, x_my_2, \ldots, x_my_{j-1}, x_my_{j+1}, \ldots, x_my_n\}$ $(1 \le j \le n)$ and $N_k = \{x_1y_1, x_2y_2, ..., x_{m-1}y_{m-1}, x_my_m, x_my_{m+1}, \ldots, x_my_n\}$ with $|S_i| = |M_j| = n + m - 2$ and $|N_k| = n$. It is easily verified that any minimal edge-to-vertex geodetic set of G is of the form either S_i or M_j *or* N_k. Since no proper subset of S_i $(1 \le i \le m)$, $M_j (1 \le j \le n)$ and N_k is an edge-to-vertex geodetic set of G, it follows that, $g_{ev}^+(G) = n + m - 2$.

3. THE EDGE-TO-VERTEX GEODETIC NUMBER AND UPPER EDGE-TO-VERTEX GEODETIC NUMBER OF A GRAPH

In this section, connected graphs G of size q with upper edge-to-vertex geodetic number q or q-1 are characterized.

Theorem 3.1: For a connected graph G, $2 \le g_{ev}(G) \le g_{ev}^+(G) \le q$.

Proof: Any edge-to-vertex geodetic set needs at least two edges and so $g_{ev}(G) \ge 2$. Since every minimal edge-to-vertex geodetic set is an edge-to-vertex geodetic set, $g_{ev}(G) \le g_{ev}^+(G)$. Also, since $E(G)$ is an edge-to-vertex geodetic set of G, it is clear that $g_{ev}^+(G) \le q$. Thus $2 \le g_{ev}(G) \le g_{ev}^+(G) \le q$.

Remark 3.2: The bounds in Theorem 3.1 are sharp. For any non-trivial path P, $g_{ev}(P) = 2$. For any tree T, $g_{ev}(T) = g_{ev}^+(T)$ and $g_{ev}^+(K_{1,q}) = q$ for $q \ge 2$. Also, all the inequalities in the theorem are strict. For the complete graph $G = K_5$, $g_{ev}(G) = 3$, $g_{ev}^+(G) = 4$ and $q = 10$ so that $2 < g_{ev}(G) < g_{ev}^+(G) < q$.

Theorem 3.3: For a connected graph G, $g_{ev}(G) = q$ if and only if $g_{ev}^+(G) = q$.

Proof: Let $g_{ev}^+(G) = q$. Then $S = E(G)$ is the unique minimal edge-to-vertex geodetic set of G. Since no proper subset of S is an edge-to-vertex geodetic set, it is clear that S is the unique minimum edge-to-vertex geodetic set of G and so $g_{ev}(G) = q$. The converse follows from Theorem 3.1.

As a consequence of this result, we have the following corollary.

Corollary 3.4: For a connected graph G of size q, the following are equivalent:
(i) $g_{ev}(G) = q$
(ii) $g_{ev}^+(G) = q$
(iii) $G = K_{1,q}$

Proof: This follows from Theorems 1.3 and 3.3.

Theorem 3.5: Let G be a connected graph of size $q \ge 4$ which is not a star and has no cut edge. Then $g_{ev}^+(G) \le q - 2$.

Proof: Suppose that $g_{ev}^+(G) \ge q$-1. Then by Corollary 3.4, $g_{ev}^+(G) = q - 1$. Let e be an edge of G which is not an end edge of G and let $M = E(G) - \{e\}$ be a minimal edge-to-vertex geodetic set of G. Since e is not a cut edge of G, $\langle E(G) - e\rangle$ is connected. Let f be an edge of $\langle E(G) - e\rangle$ which is independent of e and also which is not an end edge of G. Then $M_1 = M - \{f\}$ is an edge-to-vertex geodetic set of G. Since $M_1 \subsetneq M$, M is not a minimal edge-to-vertex geodetic set of G, which is a contradiction. Therefore $g_{ev}^+(G) \le q - 2$.

Remark 3.6: The bound in Theorem 3.5 is sharp. For the graph G given in Figure 3.1, $S_1 = \{v_1v_2, v_4v_5\}$, $S_2 = \{v_1v_2, v_3v_4, v_3v_5\}$, $S_3 = \{v_1v_3, v_2v_3, v_4v_5\}$ and $S_4 = \{v_1v_3, v_2v_3, v_3v_4, v_3v_5\}$ are the only four minimal edge-to-vertex geodetic set of G so that $g_{ev}^+(G) = 4 = q - 2$.

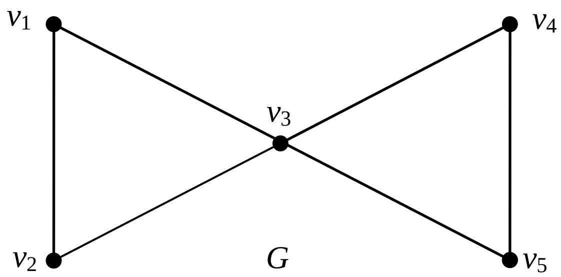

Figure: 3.1

Theorem 3.7: For a connected graph G of size $q \geq 4$, $g_{ev}(G) = q - 1$ if and only if $g_{ev}^+(G) = q - 1$.

Proof: Let $g_{ev}(G) = q - 1$. Then it follows from Theorem 3.1 that $g_{ev}^+(G) = q$ or $q - 1$. If $g_{ev}^+(G) = q$, then by Theorem 3.3, $g_{ev}(G) = q$, which is a contradiction. Hence $g_{ev}^+(G) = q - 1$. Conversely, let $g_{ev}^+(G) = q - 1$, then it follows from Corollary 3.4 that G is not a star. Hence by Theorem 3.5, G contains a cut edge, say e. Since $g_{ev}^+(G) = q - 1$, hence it follows from Theorem 2.4 that $M = E(G) - \{e\}$ is the unique minimal edge-to-vertex geodetic set of G. We claim that $g_{ev}(G) = q - 1$. Suppose that $g_{ev}(G) < q - 1$.Then there exists a minimum edge-to-vertex geodetic set M_1 such that $|M_1| < q - 1$. By Theorem 1.2, $e \notin M_1$. Then it follows that $M_1 \underset{\neq}{\subset} M$, which is a contradiction. Therefore $g_{ev}(G) = q - 1$.

Corollary 3.8: For a connected graph G of size $q \geq 4$, the following are equivalent:
(i) $g_{ev}(G) = q - 1$
(ii) $g_{ev}^+(G) = q - 1$
(iii) G is a double star.

Proof: This follows from Theorems 1.4 and 3.7.

4. REALIZATION RESULT

In view of Theorem 3.1, we have the following realization result.

Theorem 4.1: For every two positive integers a and b, where $2 \leq a \leq b$, there exists a connected graph G with $g_{ev}(G) = a$ and $g_{ev}^+(G) = b$.

Proof: If $a = b$, let $G = K_{1,a}$. Then by Corollary 3.4, $g_{ev}(G) = g_{ev}^+(G) = a$. So, let $2 \leq a < b$. Let $P : x, y$ be a path on two vertices. Let G be the graph in Figure 4.1 obtained from P by adding new vertices $z, v_1, v_2, \ldots, v_{b-a+1}, u_1, u_2, \ldots, u_{a-1}$ and joining each vertex $u_i (1 \leq i \leq a - 1)$ and each vertex v_i $(1 \leq i \leq b - a + 1)$ with z, each vertex v_i $(2 \leq i \leq b - a + 1)$ with x and v_1 with y . Let $S = \{zu_1, zu_2, \ldots, zu_{a-1}\}$ be the set of end edges of G. By Theorem 1.1, S is contained in every edge-to-vertex geodetic set of G. It is clear that S is not an edge-to-vertex geodetic set of G and so $g_{ev}(G) \geq a$. However $S' = S \cup \{xy\}$ is an edge-to-vertex geodetic set of G so that $g_{ev}(G) = a$.

Now, $T = S \cup \{yv_1, xv_2, \ldots, xv_{b-a+1}\}$ is an edge-to-vertex geodetic set of G. We show that T is a minimal edge-to-vertex geodetic set of G. Let W be any proper subset of T. Then there exists at least one edge say $e \in T$ such that $e \notin W$. First assume that $e = zu_i$ for some $i(1 \leq i \leq a - 1)$. Then the vertex u_i is neither incident with an edge of W nor lies on any geodesic joining a pair of edges of W and so W is not an edge-to-vertex geodetic set of G. Now, assume that $e = xv_j$ for some $j(2 \leq j \leq b - a + 1)$. Then the vertex v_j is neither incident with an edge of W nor lies on a geodesic joining any pair of edges of W and so W is not an edge-to-vertex geodetic set of G. Next, assume that $e = yv_1$. Then the vertex v_1 is neither incident with an edge of W nor lies on a geodesic joining any pair of edges of W and so W is not an edge-to-vertex geodetic set of G. Hence T is a minimal edge-to-vertex geodetic set of G so that $g_{ev}^+(G) \geq b$. Now, we show that there is no minimal edge-to-vertex geodetic set X of G with $|X| \geq b + 1$. Suppose that there exists a minimal edge-to-vertex geodetic set X of G such that $|X| \geq b + 1$. Then by Theorem 1.1, $S \subseteq X$. Since S' is an edge-to-vertex geodetic set of G , it follows that $xy \notin X$. Let $M_1 = \{yv_1, xv_2, xv_3 \ldots, xv_{b-a+1}\}$ and $M_2 = \{zv_1, zv_2, zv_3 \ldots, zv_{b-a+1}\}$. Let $X = S \cup S_1 \cup S_2$, where $S_1 \subseteq M_1$ and $S_2 \subseteq M_2$ .First we show that $S_1 \underset{\neq}{\subset} M_1$ and $S_2 \underset{\neq}{\subset} M_2$. Suppose that $S_1 = M_1$. Then $T \subseteq X$ and so X is not a minimal edge-to-vertex geodetic set of G, which is a contradiction. Suppose that $S_2 = M_2$. If $yv_1 \notin X$, then y is neither incident with an edge of X nor lies on a geodesic joining any pair of edges of X and so X is not an edge-to-vertex geodetic set of G, which is a contradiction. If $yv_1 \in X$ and if xv_i do not belong to S_1 for all i$(2 \leq i \leq b - a + 1)$, then x is neither incident with an edge of X nor lies on a geodesic joining any pair of edges of X and so X is not an edge-to-vertex geodetic set of G, which is a contradiction. Therefore xv_i belong to S_1 for some $i(2 \leq i \leq b - a + 1)$. Without loss of generality let us assume that $xv_2 \in S_1$. Then $X' = X - \{zv_2\}$ is an edge-to-vertex geodetic set of G with $X' \underset{\neq}{\subset} X$, which is a contradiction. Therefore, $S_1 \underset{\neq}{\subset} M_1$ and $S_2 \underset{\neq}{\subset} M_2$. Next we show that $V(<S_1>) \cap V(<S_2>)$ contains no $v_i(1 \leq i \leq b - a + 1)$. Suppose that $V(<S_1) \cap V(<S_2)$ contains v_i for some $i(1 \leq i \leq b - a + 1)$. Without loss of generality let us assume that $v_2 \in V(<S_1>) \cap V(<S_2>)$. Then $X'' = X - \{zv_2\}$ is an edge-to-vertex geodetic set of G with $X'' \underset{\neq}{\subset} X$, which is a contradiction. Therefore $|S_1 \cup S_2| = b - a + 1$. Hence it follows that $|X| = a - 1 + b - a + 1 = b$, which is a contradiction to $|X| \geq b + 1$.

Therefore $g_{ev}^+(G) = b$.

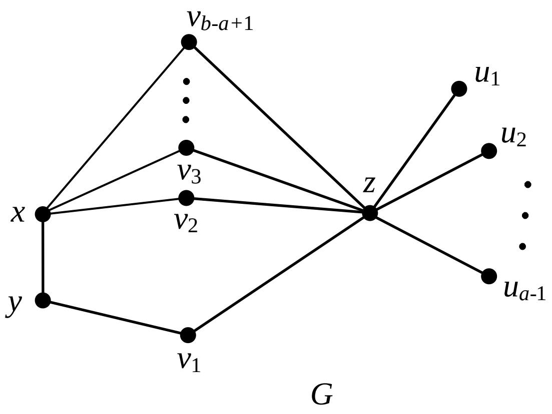

Figure: 4.1

REFERENCES

[1] F. Buckley and F. Harary, *Distance in Graphs*, Addition-Wesley, Redwood City, CA, 1990.

[2] F. Buckley, F. Harary and L.V. Quintas, *Extremal results on the Geodetic Number of a graph*, Scientia A2 (1988) 17–22.

[3] G. Chartrand, F. Harary and P. Zhang, *Geodetic sets in Graphs*, Discussiones Mathematicae Graph Theory 20(2000)129 – 138.

[4] G. Chartrand, F. Harary and P. Zhang, *On the Geodetic Number of a Graph*, Networks Vol. 39(1), (2002) 1 – 6.

[5] F. Harary, *Graph Theory*, Addision-Wesley (1969).

[6] A.P. Santhakumaran and J. John, *The Edge-to-Vertex Geodetic Number of a Graph*, Serdica Journal of Mathematics (In press).

[7] A.P. Santhakumaran and J. John, *On the Edge-to-Vertex Geodetic Number of a Graph*, Miskolc Mathematical Notes (In press).

****

