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PREDICTING THE THRESHOLD IN AN ORGANIZATION ON MAX (Y_1, Y2, Y3, Y4) THROUGH GRADE SYSTEM

P. Pandiyan, M. Thirunaukkarasu, R. Vinoth*, R. M. Palanivel and K. Kannadasan

Department of Statistics, Annamalai University, Annamalai Nagar, 608002, India

*E-mail: *maalinga@gmail.com*

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ABSTRACT

Manpower planning involves the attempt to forecast the demand and supply for people with various skills and qualifications and to bring them into balance. The rate of accumulation of damage determines the life time of the component or device. In this paper a mathematical model is developed to obtain the expected time to reach the threshold level, in the context with the assumptions that the times between decision epochs are independent and identically distributed (i.i.d) random variable, the number of exits at each period time are i.i.d. random variables and that the threshold level is a random variable following three Exponential distribution and one Erlang distribution.

INTRODUCTION

Manpower planning is an interdisciplinary activity. It requires the combined technical skills of statisticians, economists and behavioral scientists together with the practical knowledge of managers and planners. Esary, Marshall and Proschan (1973) discussed that any component or device when exposed to shocks which cause damage to the device or system is likely to fail when the total accumulated damage exceeds a level called the threshold. A detailed account of the application of stochastic processes in manpower planning models can be had from books by Bartholomew and Morris (1971), Grinold and Marshall (1977), Bartholomew and Forbes (1979), Bennison and Casson (1984).

The organization consists of four grades of marketing personnel. The organization takes policy decisions depending upon the market environment, production, etc., the mobility or transfer of manpower from one grade to the other grade where there is more depletion is allowed. Each grade has its individual random threshold and if the loss of manpower crosses the maximum of the four thresholds, recruitment becomes necessary. The time between two consecutive policy decisions forms a sequence of independent and identically distributed random variables. The process which gives rise to policy revisions and the threshold random variables are statistically independent.

NOTATIONS

X_i : a continuous random variable denoting the amount of loss of manpower (in man hours) caused to the system on the i[th] occasion of policy announcement (Shock), 1, 2….k and X_i's are i.i.d

Y_1, Y_2, Y_3, Y_4: Continuous random variable denoting the threshold levels for the four grades.

$g(.)$: The probability density function of X; $g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: the k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive policy announcement with the corresponding c.d.f. $F(.)$.

$F_k(.)$: k-fold convolution of $F(.)$; $S(.)$: Survival function.

$V_k(t)$: Probability of exactly k policy announcements;$L(t)$:$1 - S(t)$.

RESULTS

Distribution function of the four graded threshold

$$H(x) = (1 - e^{-\lambda_1 x})(1 - e^{-\lambda_2 x})(1 - e^{-\lambda_2 x})(1 - e^{-\lambda_4 x} + \lambda_4 e^{\lambda_4 x})$$

The corresponding Survival function is

Let $\bar{H}(x) = 1 - H(x)$

***Corresponding author: R. Vinoth*, *E-mail: maalinga@gmail.com**

$$\bar{H}(x) = \left[\left\{\sum_{i=1}^{4} e^{-\lambda_i x} - \sum_{i=1}^{3} e^{-(\lambda_i+\lambda_{i+1})x}\right\} + \left\{-e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_1+\lambda_3)x} - e^{-(\lambda_1+\lambda_4)x}\right\} + e^{-(\lambda_1+\lambda_2+\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_4)x} + e^{-(\lambda_1+\lambda_3+\lambda_4)x} + e^{-(\lambda_2+\lambda_3+\lambda_4)x} - e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)x} + \left\{-e^{\lambda_4 x} + e^{-(\lambda_3-\lambda_4)x} + e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_2+\lambda_3-\lambda_4)x} + e^{-(\lambda_1-\lambda_4)x} - e^{-(\lambda_1+\lambda_3-\lambda_4)x} - e^{-(\lambda_1+\lambda_2-\lambda_4)x} + e^{-(\lambda_1+\lambda_2+\lambda_3-\lambda_4)x}\right\}\lambda_4\right]$$

$P(X_1 + X_2 + \cdots + X_k < Y) =$ P [the system does not fail, after k epochs of exits].

There may be no practical way to inspect an individual item to determine its threshold Y. In this case, the threshold must be a random variable. In general, assuming that the threshold Y follows Exponential and Erlang Distribution with parameter θ, it can be shown that

$$P(X_i < Y) = \int_0^\infty g_k(x)\bar{H}(x)dx$$

$$= \int_0^\infty g_k(x)\left[\left\{\sum_{i=1}^{4} e^{-\lambda_i x} - \sum_{i=1}^{3} e^{-(\lambda_i+\lambda_{i+1})x}\right\} + \left\{-e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_1+\lambda_3)x} - e^{-(\lambda_1+\lambda_4)x}\right\} + e^{-(\lambda_1+\lambda_2+\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_4)x} + e^{-(\lambda_1+\lambda_3+\lambda_4)x} + e^{-(\lambda_2+\lambda_3+\lambda_4)x} - e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)x} + \left\{-e^{\lambda_4 x} + e^{-(\lambda_3-\lambda_4)x} + e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_2+\lambda_3-\lambda_4)x} + e^{-(\lambda_1-\lambda_4)x} - e^{-(\lambda_1+\lambda_3-\lambda_4)x} - e^{-(\lambda_1+\lambda_2-\lambda_4)x} + e^{-(\lambda_1+\lambda_2+\lambda_3-\lambda_4)x}\right\}\lambda_4\right]$$

The survival function $S(t)$ which is the probability that an individual survives for a time t

$S(t) = P(T > t) =$ Probability that the system survives beyond t

$$P(T > t) = \sum_{k=0}^{\infty} P[\text{there are exactly k instants of exit in}(0,t)] * P[\text{the system does not fail in } (0,t)]$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\, P\left[\sum_{i=1}^{k} X_i < max(y_1, y_2, y_3, y_4)\right]$$

It is also known from renewal theory that

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \qquad \text{with} \quad F_0(t) = 1$$

$$= \sum_{k=0}^{\infty} V_k(t) P(X_i < Y)$$

$$= \left[\sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_0^\infty g_k^* \left\{\sum_{i=1}^{4} e^{-\lambda_i x} - \sum_{i=1}^{3} e^{-(\lambda_i+\lambda_{i+1})x}\right\} + \left\{-e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_1+\lambda_3)x} - e^{-(\lambda_1+\lambda_4)x}\right\} + e^{-(\lambda_1+\lambda_2+\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_4)x} + e^{-(\lambda_1+\lambda_3+\lambda_4)x} + e^{-(\lambda_2+\lambda_3+\lambda_4)x} - e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)x} + \left\{-e^{\lambda_4 x} + e^{-(\lambda_3-\lambda_4)x} + e^{-(\lambda_2+\lambda_4)x} - e^{-(\lambda_2+\lambda_3-\lambda_4)x} + e^{-(\lambda_1-\lambda_4)x} - e^{-(\lambda_1+\lambda_3-\lambda_4)x} - e^{-(\lambda_1+\lambda_2-\lambda_4)x} + e^{-(\lambda_1+\lambda_2+\lambda_3-\lambda_4)x}\right\}\lambda_4\right]$$

Now $L(T) = 1 - S(t)$

Taking Laplace transform of $L(T)$, we get

$$l^*(s) = 1 - \left[\left[\left\{\frac{(\sum_{i=1}^{4} 1 - g^*(\lambda_i))f^*(S)}{[\sum_{i=1}^{4} 1 - g^*(\lambda_i)f^*(S)]} - \frac{(\sum_{i=1}^{3} 1 - g^*(\lambda_i + \lambda_{i+1}))f^*(S)}{[\sum_{i=1}^{3} 1 - g^*(\lambda_i + \lambda_{i+1})f^*(S)]} - \frac{(1 - g^*(\lambda_2 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_2 + \lambda_4)f^*(S)]} - \frac{(1 - g^*(\lambda_1 + \lambda_3))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_3)f^*(S)]} - \frac{(1 - g^*(\lambda_1 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_4)f^*(S)]} + \frac{(1 - g^*(\lambda_1 + \lambda_2 + \lambda_3))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_2 + \lambda_3)f^*(S)]} + \frac{(1 - g^*(\lambda_1 + \lambda_2 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_2 + \lambda_4)f^*(S)]} + \frac{(1 - g^*(\lambda_1 + \lambda_3 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_3 + \lambda_4)f^*(S)]} + \frac{(1 - g^*(\lambda_2 + \lambda_3 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_2 + \lambda_3 + \lambda_4)f^*(S)]} - \frac{(1 - g^*(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4))f^*(S)}{[1 - g^*(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)f^*(S)]}\right\}\right.\right.$$

$$+\left\{-\frac{\left(1-g^*(\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_4)f^*(S)]}+\frac{\left(1-g^*(\lambda_3-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_3-\lambda_4)f^*(S)]}+\frac{\left(1-g^*(\lambda_2-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_2-\lambda\lambda_4)f^*(S)]}\right.$$
$$-\frac{\left(1-g^*(\lambda_2+\lambda_3-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_2+\lambda_3-\lambda_4)f^*(S)]}+\frac{\left(1-g^*(\lambda_1-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_1-\lambda_4)f^*(S)]}-\frac{\left(1-g^*(\lambda_1+\lambda_3-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_3-\lambda_4)f^*(S)]}$$
$$\left.-\frac{\left(1-g^*(\lambda_1+\lambda_2-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_2-\lambda_4)f^*(S)]}+\frac{\left(1-g^*(\lambda_1+\lambda_2+\lambda_3-\lambda_4)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_2+\lambda_3-\lambda_4)f^*(S)]}\right\}\lambda_4\Bigg]\!\!\Bigg]$$

Where $[f^*(s)]$ is Laplace transform of $F_k(t)$ since the inter arrival times are i.i.d. The above equation can be rewritten as,

$$f^*(\cdot)\sim\exp(c);\ f^*(s)=\left(\frac{c}{c+s}\right)$$
$$=1-\Bigg[\!\!\Bigg[\frac{\sum_{i=1}^4 c\left(1-g^*(\lambda_i)\right)}{\sum_{i=1}^4(c+s-g^*(\lambda_i)c)}-\frac{\sum_{i=1}^3 c\left(1-g^*(\lambda_i+\lambda_{i+1})\right)}{\sum_{i=1}^3\left(c+s-g^*\left(\lambda_i+\lambda_{i+1_i}\right)c\right)}-\frac{c\left(1-g^*(\lambda_2+\lambda_4)\right)}{(c+s-g^*(\lambda_2+\lambda_4)c)}$$
$$-\frac{c\left(1-g^*(\lambda_1+\lambda_3)\right)}{(c+s-g^*(\lambda_1+\lambda_3)c)}-\frac{c\left(1-g^*(\lambda_1+\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_4)c)}+\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)}{(c+s-g^*(\lambda_1+\lambda_2+\lambda_3)c)}$$
$$+\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_2+\lambda_4)c)}+\frac{c\left(1-g^*(\lambda_1+\lambda_3+\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_3+\lambda_4)c)}+\frac{c\left(1-g^*(\lambda_2+\lambda_3+\lambda_4)\right)}{(c+s-g^*(\lambda_2+\lambda_3+\lambda_4)c)}$$
$$-\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_2+\lambda_3+\lambda_4)c)}+\left\{-\frac{c\left(1-g^*(\lambda_4)\right)}{(c+s-g^*(\lambda_4)c)}+\frac{c\left(1-g^*(\lambda_3-\lambda_4)\right)}{(c+s-g^*(\lambda_3-\lambda_4)c)}\right.$$
$$+\frac{c\left(1-g^*(\lambda_2-\lambda_4)\right)}{(c+s-g^*(\lambda_2-\lambda_4)c)}-\frac{c\left(1-g^*(\lambda_2+\lambda_3-\lambda_4)\right)}{(c+s-g^*(\lambda_2+\lambda_3-\lambda_4)c)}+\frac{c\left(1-g^*(\lambda_1-\lambda_4)\right)}{(c+s-g^*(\lambda_1-\lambda_4)c)}$$
$$-\frac{c\left(1-g^*(\lambda_1+\lambda_3-\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_3-\lambda_4)c)}-\frac{c\left(1-g^*(\lambda_1+\lambda_2-\lambda_4)\right)}{(c+s-g^*(\lambda_1+\lambda_2-\lambda_4)c)}$$
$$\left.+\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3-\lambda_4)\right)}{(c+s-g^*(\lambda+\lambda_2+\lambda_3-\lambda_4)c)}\right\}\lambda_4\Bigg]\!\!\Bigg]$$

$$E(T)=-\frac{d}{ds}L^*(S)\text{ given }\ s=0$$

$$E(T)=-\frac{\sum_{i=1}^4 c\left(1-g^*(\lambda_i)\right)}{\sum_{i=1}^4 c^2\left(1-g^*(\lambda_i)\right)^2}+\frac{\sum_{i=1}^3 c\left(1-g^*(\lambda_i+\lambda_{i+1})\right)}{\sum_{i=1}^3 c^2\left(1-g^*(\lambda_i+\lambda_{i+1})\right)^2}-\frac{c\left(1-g^*(\lambda_2+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_2+\lambda_4)\right)^2}+\frac{c\left(1-g^*(\lambda_1+\lambda_3)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_3)\right)^2}$$
$$+\frac{c\left(1-g^*(\lambda_1+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)^2}-\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_4)\right)^2}$$
$$-\frac{c\left(1-g^*(\lambda_1+\lambda_3+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_3+\lambda_4)\right)^2}+\frac{c\left(1-g^*(\lambda_2+\lambda_3+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_2+\lambda_3+\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\right)^2}$$
$$+\left\{\frac{c\left(1-g^*(\lambda_4)\right)}{c^2\left(1-g^*(\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_3-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_3-\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_2-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_2-\lambda_4)\right)^2}\right.$$
$$+\frac{c\left(1-g^*(\lambda_2+\lambda_3-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_2+\lambda_3-\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_1-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1-\lambda_4)\right)^2}+\frac{c\left(1-g^*(\lambda_1+\lambda_3-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_3-\lambda_4)\right)^2}$$
$$\left.+\frac{c\left(1-g^*(\lambda_1+\lambda_2-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_2-\lambda_4)\right)^2}-\frac{c\left(1-g^*(\lambda_1+\lambda_2+\lambda_3-\lambda_4)\right)}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3-\lambda_4)\right)^2}\right\}\lambda_4$$

The inter-arrival time of the threshold follows exponential distribution. The Laplace transformation of the exponential is given by $\frac{\mu}{\mu+\lambda}$

Now, $g(\cdot)\sim exp(\mu);\ \ g^*(\mu)=\frac{\mu}{\mu+\lambda}$

$$E(T)=\frac{1}{c}\left\{-\sum_{i=1}^{4}\left(\frac{\mu+\lambda_i}{\lambda_i}\right)+\sum_{i=1}^{3}\frac{\mu+\lambda_i+\lambda_{i+1}}{\lambda_i+\lambda_{i+1}}+\frac{\mu+\lambda_2+\lambda_4}{\lambda_2+\lambda_4}+\frac{\mu+\lambda_1+\lambda_3}{\lambda_1+\lambda_3}+\frac{\mu+\lambda_1+\lambda_4}{\lambda_1+\lambda_4}-\frac{\mu+\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right.$$
$$\left.-\frac{\mu+\lambda_1+\lambda_2+\lambda_4}{\lambda_1+\lambda_2+\lambda_4}-\frac{\mu+\lambda_2+\lambda_3+\lambda_4}{\lambda_2+\lambda_3+\lambda_4}-\frac{\mu+\lambda_1+\lambda_3+\lambda_4}{\lambda_1+\lambda_3+\lambda_4}-\frac{\mu+\lambda_1+\lambda_2+\lambda_3+\lambda_4}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}\right\}$$

$$+\left\{\frac{\mu+\lambda_4}{\lambda_4}-\frac{\mu+\lambda_3-\lambda_4}{\lambda_3-\lambda_4}-\frac{\mu+\lambda_2-\lambda_4}{\lambda_2-\lambda_4}+\frac{\mu+\lambda_2+\lambda_3-\lambda_4}{\lambda_2+\lambda_3-\lambda_4}-\frac{\mu+\lambda_1-\lambda_4}{\lambda_1-\lambda_4}+\frac{\mu+\lambda_1+\lambda_3-\lambda_4}{\lambda_1+\lambda_3-\lambda_4}\right.$$
$$\left.+\frac{\mu+\lambda_1+\lambda_2-\lambda_4}{\lambda_1+\lambda_2-\lambda_4}-\frac{\mu+\lambda_1+\lambda_2+\lambda_3-\lambda_4}{\lambda_1+\lambda_2+\lambda_3-\lambda_4}\right\}\lambda_4 \qquad \textit{on simplification}$$

CONCLUSION

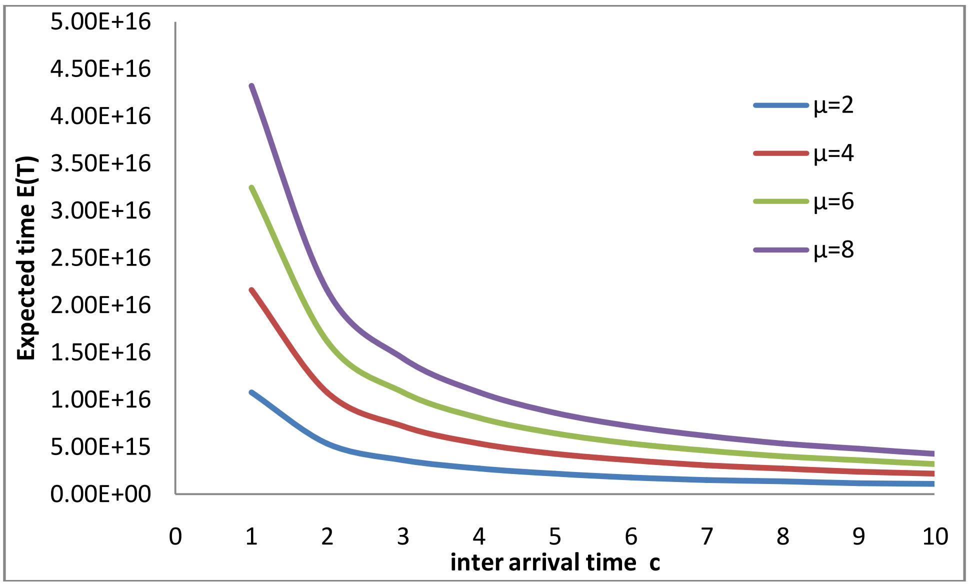
To illustrate the method described in this thesis, some limited simulation results are discussed. The theory developed was tested using stimulated data in *Mathcad* software. Numerical examples are given with one parameter varying and keeping the other parameter fixed.

When μ is kept fixed with other parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$; the inter-arrival time 'c′, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time E (T) to the system in an organization decreases, for all cases of the parameter value μ = 2,4,6,8. When the value of the parameter μ increases, the expected time is found increasing, this is observed in the Table 1 and Figure 1.

Table: 1

λ_1=0.1,λ_2=0.2,λ_3=0.4,λ_4=0.6				
c	μ=2	μ=4	μ=6	μ=8
1	1.08E+16	2.16E+16	3.24E+16	4.32E+16
2	5.40E+15	1.08E+16	1.62E+16	2.16E+16
3	3.60E+15	7.21E+15	1.08E+16	1.44E+16
4	2.70E+15	5.40E+15	8.11E+15	1.08E+16
5	2.16E+15	4.32E+15	6.49E+15	8.65E+15
6	1.80E+15	3.60E+15	5.40E+15	7.20E+15
7	1.54E+15	3.09E+15	4.63E+15	6.18E+15
8	1.35E+15	2.70E+15	4.05E+15	5.40E+15
9	1.20E+15	2.40E+15	3.60E+15	4.80E+15
10	1.08E+15	2.16E+15	3.24E+15	4.32E+15

Figure: 1



REFERENCE:

1. Esary, J.D., A.W. Marshall and F. Proschan, 1973. Shock models and wear processes. Ann. Probability, 1(4): 627-649.

2. Bartholomew, D.J. and Morries, B.R. (1971). Aspects of Manpower Planning, Elsevier Publishing Company, New York.
3. Grinold, R.C. and Marshall, K.T. (1977). Manpower Planning Models. Elsevier North-Holland, New York
4. Bartholomew, D.J., and A.F Forbes, 1979. Statistical Techniques for Manpower Planning. John Wiley and Sons.
5. Bennison, M and J. Casson (1984). The Manpower Planning Hand Book, McGraw- Hill Book Company (UK) Limted, England.

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