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ON DECOMPOSITIONS OF M-CONTINUITY

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ABSTRACT

We introduce a new type of sets called m-A set, m-t set, m-B set, m-h set and m-C set and a new class of mappings called M-A continuous, M-B continuous and M-C continuous. We obtain several characterizations of this class and study its minimal properties and investigate the relationships with other mappings like M-α-continuous.

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1. INTRODUCTION

Njastad [7] initiated the concept of nearly open sets in topological spaces. Following it many research papers were introduced by Tong [11, 12], Hatir [3, 4, 5], Dontchev [1] and Ganster [2] in the name of "Decompositions of Continuity" in topological spaces. It is an effort based on them to bring out a paper in the name of "Decompositions of M-continuity" in minimal spaces using the new sets like m-A set, m-B set and m-C set and the new mappings like M-A continuous, M-B continuous and M-C continuous. In this paper, we obtain some important results in minimal spaces. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

2. PRELIMINARIES

Definition 2.1: [6, 8] Let X be a nonempty set and $\wp(X)$ the power set of X. A subfamily m_x of $\wp(X)$ is called a minimal structure (briefly, m-structure) on X if $\phi \in m_x$ and $X \in m_x$.

A set X with an m-structure m_x is called an m-space and is denoted by (X, m_x). Each member of m_x is said to be m_x-open and the complement of an m_x-open set is said to be m_x-closed. Throughout this paper, (X, m_x) (or X) denotes minimal space.

Throughout this paper, (X, mx) (or X) denotes minimal space.

Definition 2.2: [6, 8] Let X be a nonempty set and m_x an m-structure on X. For a subset A of X, the m_x-closure of A and the m_x-Interior of A are defined as follows:
(1) $m_x\text{-Cl}(A) = \cap\{ F : A \subset F,\ X - F \in m_x \}$,
(2) $m_x\text{-Int}(A) = \cup\{ U : U \subset A,\ U \in m_x \}$.

Lemma 2.3: [6, 8] Let X be a nonempty set and m_x a minimal structure on X. For subsets A and B of X, the following properties hold :
(1) $m_x\text{-Cl}(X - A) = X - m_x\text{-Int}(A)$ and $m_x\text{-Int}(X - A) = X - m_x\text{-Cl}(A)$,
(2) If $(X - A) \in m_x$, then $m_x\text{-Cl}(A) = A$ and if $A \in m_x$, then $m_x\text{-Int}(A) = A$,

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(3) m_x-Cl(ϕ) = ϕ, m_x-Cl(X) = X, m_x-Int(ϕ) = ϕ and m_x-Int(X) = X,
(4) If $A \subset B$, then m_x-Cl(A) $\subset$ m_x-Cl(B) and m_x-Int(A) $\subset$ m_x-Int(B),
(5) $A \subset m_x$-Cl(A) and m_x-Int(A) $\subset$ A,
(6) m_x-Cl(m_x-Cl(A)) = m_x-Cl(A) and m_x-Int(m_x-Int(A)) = m_x-Int(A).

Definition 2.4: [8] A minimal space m_x on a nonempty set X is said to have property $\mathfrak{B}$ if the union of any family of subsets belonging to m_x belongs to m_x.

Lemma 2.5: [8] The following are equivalent for the minimal space (X, m_x).
(1) m_x have property $\mathfrak{B}$;
(2) If m_x-Int(E) = E, then E $\in m_{x;}$
(3) If m_x-Cl(F) = F, then $F^c \in m_x$.

Definition 2.6: [9] Let S be a subset of X. Then S is said to be

(i) m_x-α-open if S $\subseteq$ m_x-Int(m_x-Cl(m_x-Int(S)));
(ii) m_x-semi-open if S $\subseteq$ m_x-Cl(m_x-Int(S));
(iii) m_x-preopen if S $\subseteq$ m_x-Int(m_x-Cl(S)).

The family of all m_x-α-open [resp. m_x-semi-open, m_x-preopen] sets of X is denoted by m_x-αO(X) [resp. m_x-SO(X), m_x-PO(X)].

Remark 2.7: [9]
(i) Every m_x-open set is m_x-α-open but not conversely.
(ii) A m_x-semi-open [m_x-preopen] set need not be m_x-α-open.

Example 2.8: Let Y = {p, q, r} and m_y = {ϕ, Y, {p}, {q}, {p, q}}. We have
m_y-αO(Y) = {ϕ, Y, {p}, {q}, {p, q}};m_y-SO(Y) = {ϕ, Y, {p}, {q}, {p, q}, {p, r}, {q, r}} and
m_y-PO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

Definition 2.9: A minimal space (X, m_x) has the property I if the any finite intersection of m-open sets is m-open.

Remark 2.10: For subsets A and B of a minimal space (X, m_x) satisfying property I, the following holds:

m-Int (A $\cap$ B) = m-Int(A) $\cap$ m-Int(B).

Example 2.11.: For subsets A and B of a minimal space (X, m_x) satisfying property $\mathfrak{B}$, the following does not hold:

m-Int (A $\cap$ B) = m-Int(A) $\cap$ m-Int(B).

Let X= { a, b, c, d}, m_x = {ϕ, X, {a}, {a, b}, {a, c}, {b, c}, {a, b, c}}. Let A ={a, b} and B = {b, c}. Then A $\cap$ B = {b}. We have m-Int(A) = {a, b}; m-Int(B) = {b, c} and m-Int(A) $\cap$ m-Int(B) = {b}. But m-Int (A $\cap$ B) = ϕ. Hence m-Int (A $\cap$ B) $\neq$ m-Int(A) $\cap$ m-Int(B).

3. m-C SETS:

We introduce a new type of sets as follows:

Definition 3.1: A subset S of X is said to be

(i) regular m-open [10] if S = m-Int(m-Cl(S)),
(ii) regular m-closed if S = m-Cl(m-Int(S)).

The family of all regular m-closed sets of X is denoted m-RC(X).

Definition 3.2: A subset S of X is said to be

(i) a m-A set if S = M $\cap$ N where M is m-open and N $\in$ m-RC(X),
(ii) a m-t set if m-Int(m-Cl(S)) = m-Int(S),

(iii) a m-B set if $S = M \cap N$ where M is m-open and N is a m-t set,
(iv) a m-h set if m-Int(m-Cl(m-Int(S))) = m-Int(S),
(v) a m-C set if $S = M \cap N$ where M is m-open and N is a m-h set.

Example 3.3: Let X = {a, b, c} and m_x = {ϕ, X, {a}, {b}, {a, b}}. Then the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called m_x-closed.

Example 3.4: Let X = {a, b, c} and m_x ={ϕ, X, {a}, {c}, {a, b}, {a, c}}. Then the sets in {ϕ, X, {b, c}, {a, b}, {c}, {b}} are called m_x-closed.

Example 3.5: Let X = {a, b, c} and m_x = {ϕ, X, {a, b}, {b, c}}. Then the sets in {ϕ, X, {c}, {a}} are called m_x-closed.

Remark 3.6: It is evident that any m-open set of X is an m-α-open and each m-α-open set of X is both m-semi-open and m-preopen. But the separate converses are not true.

Theorem 3.7: If A and B are two m-t sets of a space X satisfying property I, then $A \cap B$ is a m-t set in X.

Proof: Since $A \subset$ m-Cl(A), m-Int($A\cap B$) $\subset$ m-Int(m-Cl($A\cap B$)) $\subset$ m-Int(m-Cl(A) $\cap$ m-Cl(B)) = m-Int(m-Cl(A)) $\cap$ m-Int(m-Cl(B)) = m-Int(A) $\cap$m-Int(B) (since A, B are m-t sets) = m-Int($A \cap B$). Thus m-Int(m-Cl($A \cap B$)) = m-Int($A \cap B$) and $A \cap B$ is m-t set.

Theorem 3.8: If A is a m-t set of X and $B \subseteq X$ with $A \subseteq B \subseteq$ m-Cl(A) then B is a m-t set.

Proof: We note that m-Cl(B) $\subseteq$ m-Cl(A). So we have m-Int(B) $\subseteq$ m-Int(m-Cl(B)) $\subseteq$ m-Int(m-Cl(A)) = m-Int(A) $\subseteq$ m-Int(B). Thus m-Int(B) = m-Int(m-Cl(B)) and therefore B is a m-t set.

Remark 3.9: The union of two m-h sets need not be a m-h set. Refer Example 3.3, {a} and {b} are m-h sets but {a, b} is not m-h set.

Remark 3.10: Let (X, m_x) have property I.Then the intersection of any two m-h sets is a m-h set.

4. COMPARISON

Theorem 4.1: Any m-open set is an m-A set.

Proof: $S = X \cap S$ where $X \in$ m-RC(X) and S is m-open. The proof is completed.

Remark 4.2: The converse of Theorem 4.1 is not true. Refer Example 3.3, {b, c} is m-A set but not m-open.

Theorem 4.3: Any m-closed set is a m-t set.

Proof: Since A = m-Cl(A), m-Int(A) = m-Int(m-Cl(A)). The proof is completed.

Remark 4.4: The converse of Theorem 4.3 is not true. Refer Example 3.3, {a} is m-t set but not m-Closed.

Theorem 4.5: A regular m-open set is a m-t set.

Proof: Since S = m-Int(m-Cl(S)), m-Int(S) = m-Int(m-Cl(S)). The proof is completed.

Remark 4.6: The converse of Theorem 4.5 is not true. Refer Example 3.3. {c} is a m-t set but not regular m-open.

Theorem 4.7: Let (X, m_x) have property $\mathfrak{B}$. Then every regular m-open set is m-open.

Proof: Suppose S = m-Int(m-Cl(S)). Then m-Int(S) = m-Int(m-Cl(S)) and we have S = m-Int(S). Thus, S is m-open.

Remark 4.8: The converse of Theorem 4.7 is not true. Refer Example 3.3, {a, b} is m-open but not regular m-open.

Theorem 4.9: Any m-t set is m-B set.

Proof: $S = X \cap S$ where X is m-open and S is m-t set. The proof is completed.

Remark 4.10: The converse of Theorem 4.9 is not true. Refer Example 3.4, {a} is a m-B set but not m-t set.

Theorem 4.11: Any m-open set is a m-B set.

Proof: Since S = X ∩ S where S is m-open and X is regular m-open, by Theorem 4.5, X is m-t set. The proof is completed.

Remark 4.12: The converse of Theorem 4.11 is not true. Refer Example 3.3, {c} is m-B set but not m-open.

Theorem 4.13: A m-closed set is a m-B set.

Proof: It follows from Theorem 4.3 and Theorem 4.9.

Theorem 4.14: Let (X, m_x) have property $\mathfrak{B}$. Then every m-A set is a m-B set.

Proof: S = X ∩ S where X is m-open and S is regular m-closed. Since S is m-closed, by Theorem 4.3, S is m-t set. The proof is completed.

Remark 4.15: The converse of Theorem 4.14 is not true. Refer Example 3.3, {c} is m-B set but not m-A set.

Theorem 4.16: Any m-t set is m-h set.

Proof: Since m-Int(S) = m-Int(m-Cl(S)), m-Cl(m-Int(S)) = m-Cl(m-Int(m-Cl(S))) implies m-Int(m-Cl(m-Int(S))) = m-Int(m-Cl(S)) = m-Int(S). The proof is completed.

Remark 4.17: The converse of Theorem 4.16 is not true. Refer Example 3.5, {b} is m-h set but not m-t set.

Theorem 4.18: Any m-h set is m-C set.

Proof: S = X ∩ S where X is m-open and S is m-h set. The proof is completed.

Remark 4.19: The converse of Theorem 4.18 is not true. Refer Example 3.4, {a} is m-C set but not m-h set.

Theorem 4.20: Any m-open set is m-C set.

Proof: S = X ∩ S where X is m-h set and S is m-open. The proof is completed.

Remark 4.21: The converse of Theorem 4.20 is not true. Refer Example 3.3, {c} is m-C set but not m-open.

Theorem 4.22: m-B set is m-C set.

Proof: S = X ∩ S where X is m-open and S is m-t set. By Theorem 4.16, S is m-h set. The proof is completed.

Remark 4.23: The converse of Theorem 4.22 is not true. Refer Example 3.5, {b} is m-C set but not m-B set.

Remark 4.24: A m-A set need not be m-semi-open as shown in the following example.

Let X = {a, b, c} and m_x = {ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, c}}. Then the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called m-closed. We have {c} is m-A set but not m-semi-open.

Remark 4.25: A m-semi-open set need not be m-A set as shown in the following example.’

Let X = {a, b, c} and m_x = {ϕ, X, {a}}. Then the sets in {ϕ, X, {b, c}} are called m-closed. We have {a, b} is m-semi-open but not m-A set.

Remark 4.26: By the previous Theorems, Examples and Remarks, we obtain the following diagram:

$$\begin{tikzcd}
 & & (5) \arrow[dl] & (6) \\
(1) & (2) \arrow[l] & (3) \arrow[l] \arrow[r] \arrow[u] & (4) \arrow[u] \arrow[d] \\
(7) \arrow[u] & (8) \arrow[l] \arrow[u] & (9) \arrow[l] \arrow[r] \arrow[u] & (10)
\end{tikzcd}$$

Here (1) = m-C set, (2) = m-B set, (3) = m-open set,
(4) = m-α-open set, (5) = m-A set, (6) = m-semi-open set,
(7) = m-h set, (8) = m-t set, (9) =regular m-open set,
(10) = m-preopen set.

5. DECOMPOSITIONS OF M-CONTINUITY

Definition 5.1:
(i) Let f: X $\rightarrow$ Y be a mapping where X has property $\mathfrak{B}$. Then f is said to be M-continuous [8] if $f^{-1}(V)$ is m_x-open in X for every m_y-open set V in Y.

(ii) Let f: X $\rightarrow$ Y be a mapping. Then f is said to be M-α-continuous [9] if $f^{-1}(V)$ is m-α-open in X for every m_y-open set V in Y.

We introduce a new class of mappings as follows.

Definition 5.2: Let f : X $\rightarrow$ Y be a mapping. Then f is said to be

(i) M-A continuous if $f^{-1}(V)$ is M-A set in X for every m_y-open set V in Y.
(ii) M-B continuous if $f^{-1}(V)$ is M-B set in X for every m_y-open set V in Y.
(iii) M-C continuous if $f^{-1}(V)$ is m-C set in X for every m_y-open set V in Y.

Theorem 5.3: Let (X, m_x) have property $\mathfrak{B}$. Then a subset S of X is regular m-open if and only if it is both m-preopen and m-t set.

Proof: Let S be a regular m-open. By Theorem 4.5, S is m-t set. Also by Theorem 4.7, S is m-open. Thus, S is m-preopen .

Conversely, let S be both m-preopen and m-t set. Since m-Int(S) $\subseteq$ S $\subseteq$ m-Int(m-Cl(S)) = m-Int(S), S = m-Int(m-Cl(S)). Hence, S is regular m-open.

Theorem 5.4: Let (X, m_x) have property $\mathfrak{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-A set.

Proof: Let S be an m-open. Then S is m-α-open and by Theorem 4.1, S is m-A set.

Conversely, let S be both m-α-open and m-A set. Since S is m-A set, S = M $\cap$N where M is m-open and N $\in$ m-RC(X). Since S is m-α-open.

M $\cap$ N $\subset$ m-Int(m-Cl(m-Int(M $\cap$ N))) $\subset$ m-Int(m-Cl(m-Int(M) $\cap$m-Int(N)))
= m-Int(m-Cl(M $\cap$ m-Int(N))) as M is m-open $\subset$ m-Int(m-Cl(M) $\cap$ m-Cl(m-Int(N)))
= m-Int(m-Cl(M) $\cap$ N) as N$\in$m-RC(X) $\subset$ m-Int(m-Cl(M)) $\cap$ m-Int(N)…(1)

Now since M $\subset$ m-Int(m-Cl(M)), by (1)
S= M $\cap$ N= (M $\cap$ N)$\bigcap$ M $\subset$ (m-Int(m-Cl(M)) $\cap$ m-Int(N)) $\cap$ M
= M $\cap$ m-Int(N)
= m-Int(M $\cap$ N) by property I
= m-Int(S)

Therefore, S $\subset$ m-Int(S). But m-Int(S) $\subset$ S. Hence, S is m-open.

Theorem 5.5: Let (X, m_x) have property $\mathfrak{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-B set.

Proof: Let S be an m-open. Then S is m-α-open. Also, by Theorem 4.11, S is m-B set.

Conversely, let S be both m-α-open and m-B set. Since S is m-B set, S = X $\cap$ S where X is m-open and S is m-t set.

Then S = X ∩ S ⊂ X ∩ m-Int(m-Cl(S)) (as S is m-preopen) = X ∩ m-Int(S) (as S is m-t set) . We have S ⊂ X ∩ m-Int(S). Hence S ⊂ m-Int(X ∩ S) by property I and S ⊂ m-Int(S). But always m-Int(S) ⊂ S. Thus S = m-Int(S) and by property $\mathfrak{B}$, S is m-open.

Theorem 5.6: Let (X, m_x) have property $\mathfrak{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-C set.

Proof: Let S be an m-open in X. Then S is m-α-open and by Theorem 4.20, S is m-C set.

Conversely, let S be both m-α-open and m-C set. Since S is m-C set, S = M ∩ N where M is m-open and N is m-h set. Since S is m-α-open set, S ⊂ m-Int(m-Cl(m-Int(S))) = m-Int(m-Cl(m-Int(M))) ∩ m-Int(m-Cl(m-Int(N))) = m-Int(m-Cl(M)) ∩ m-Int(N) (as M is m-open and N is m-h set). Now S = M∩ N = M ∩ (M ∩ N) = M ∩ S ⊂ M ∩ (m-Int(m-Cl(M)) ∩ m-Int(N)) = M ∩ m-Int(N) (as M ⊂ m-Int(m-Cl(M))) = m-Int(M ∩ N) (by property I) = m-Int(S). Thus, S ⊂ m-Int(S) and m-Int(S) ⊂S. Hence, by property $\mathfrak{B}$, S is m-open.

Theorem 5.7: Let (X, m_x) have property $\mathfrak{B}$ and property I and f : X → Y be a mapping. Then f is M-continuous if and only if

(i) it is M-α-continuous and M-A continuous.
(ii) it is M-α-continuous and M-B continuous.
(iii) it is M-α-continuous and M-C continuous.

Proof: It is the decompositions of M-continuity from Theorems 5.4, 5.5 and 5.6.

Remark 5. 8: In the above four theorems, both properties are used and so the above four theorems are nothing but topological results.

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