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# BIANCHI TYPE III DARK ENERGY COSMOLOGICAL MODEL IN SCALAR TENSOR THEORY OF GRAVITATION

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### *ABSTRACT*

*Bianchi Type-III Dark Energy model with variable equation of state (EoS) parameter is resented in the scalar tensor theory of Gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 1986). To get a determinate solution of the field equation we take the help of special law of variation for Hubble parameter presented by Barman which yields a cosmological model with negative constant deceleration parameter. Some physical and kinematical properties of model are also discussed.*

***Keywords**: Scalar-Tensor theory. Dark energy, cosmological model.*

## 1. INTRODUCTION

The recent discovery of the accelerating expansion of the Universe [1, 2] has prompted many theoretical speculations about the underlying mechanism. The most likely mechanism is a cosmological constant, which is the simplest model and is in good agreement with observational data [3]. More complicated models involve new dynamical sources of gravity that act as dark energy, and/or medications to general relativity on large scales. A plethora of models have been postulated and explored in recent years, including Quintessence, K-essence [4, 5], Ghost Condensates [6], DGP gravity [7], and f(R) gravity, to name but a few. See Refs. [8, 9, 10, 11, 12, 13, 14] for detailed reviews of these and other models. In recent years there has been a considerable interest in scalar – tensor theories of gravitation proposed by Brans and Dicke[15], Nordtvedt [16], Lyra [17], Sen and Dunn [18] and Saez and Ballester [19]. Brans-Dicke theory includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez-Ballester scalar – tensor theory the metric is coupled with a dimensionless scalar field in a simple manner.

The field equation of Saez-Ballester Theory for the combine scalar and tensor field is

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij}, \quad (1)$$

and the Scalar field $\phi$ satisfy the equation

$$2\phi^n\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0, \quad (2)$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein Tensor, *R* the scalar curvature *w* and *n* are constant, $T_{ij}$ is the stress tensor of matter and comma and semicolon denotes partial and covariant differentiation respectively.

Also, we have energy conservation equation,

$$T^{ij}_{;i} = 0, \quad (3)$$

which is the consequence of the field equation (1) and (2).

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The study of cosmological models in the frame work of scalar - tensor theories has been the active area of research for the last few decades. Rahaman and Bera [20, 21], Rahaman and Chakraborty [22], Soleng [23], Reddy and Venkateswarlu [24], Rahaman et al. [25] and Rahaman et al. [26] are some of the authors who have investigated several aspects of cosmology with in the frame work of Lyra [17] geometry. Singh and Agrawal [27], Shri Ram and Singh [28], Shri Ram and Tiwari [29], Reddy and Naidu [30], Rao et al. [31, 32] have studied Saez-Ballester scalar – tensor theory with special reference to cosmological models.

In recent years various form of time dependent $\omega$ have been used for variable models by Mukhopadhya et al. [33]. Setare [34, 35, 36] and Setare and Saridakis [37] have also studied the DE models in different contexts. Recently, dark energy models with variable EoS parameter have been studied by Ray et al. [38], Akarsu and Kilinc [39,40], Yadav et al. [41], Yadav and Yadav [42], Pradhan and Amirhashchi [43], Pradhan et al. [44].Recently Katore et.al [45] have investigated Bianchi type-V dark energy model with variable EOS parameter in Saez–Ballester scalar–tensor theory of gravitation.

It is well known that spatially homogeneous and a anisotropic cosmological models in the presence of scalar fields play a vital role in the discussion of large scale structure of the universe. With this motivation we investigate, in this paper, Bianchi type-III dark energy model with variable EOS parameter in Saez–Ballester scalar–tensor theory of gravitation.

## 2. METRIC AND FIELD EQUATIONS

We consider the Bianchi type-III metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 dz^2, \tag{4}$$

where *A*, *B*, *C*, are the function of *t* only .

The Energy momentum tensor of fluid is taken as

$$T_j^i = diag\left[T_0^0, T_1^1, T_2^2, T_3^3\right], \tag{5}$$

One can parameterize this as follows,

$$\begin{aligned} T_j^i &= diag\left[\rho, -p_x, -p_y, -p_z\right], \\ &= diag\left[1, -w_x, -w_y, -w_z\right]\rho, \\ &= diag\left[1, -w_x, -(w+\delta)_y, -(w+\delta)_z\right]\rho, \end{aligned} \tag{6}$$

where $\rho$ is the energy density of the fluid $p_x, p_y, p_z$ are the pressure and $w_x, w_y, w_z$ are the directional EoS parameter along the *x,y,z* axis respectively.

$w(t) = \dfrac{p}{\rho}$ is the deviation free EoS parameter of the fluid we have parameterized the deviation from isotropy by setting $w_x = w$ and then introducing skew ness parameter $\delta$ which is the deviation from *w* along both *y* and *z* axes.

In Co-moving co-ordinate system Saez-Ballester field equations (1)-(3) for the metric (4) with the help of (5) and (6) take the form

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{w}{2}\phi^n \phi_4^2 = -w\rho, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{w}{2}\phi^n \phi_4^2 = -(w+\delta)\rho, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{1}{A^2} - \frac{w}{2}\phi^n \phi_4^2 = -(w+\delta)\rho, \tag{9}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{1}{A^2} + \frac{w}{2}\phi^n \phi_4^2 = \rho, \tag{10}$$

 

$$\frac{B_4}{B}-\frac{A_4}{A}=0 \ , \tag{11}$$

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \, , \tag{12}$$

$$\rho_4+(w+\delta)\rho\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \, . \tag{13}$$

From equation (11), without loss of generality, we get

$$A=B \, . \tag{14}$$

Using equations (7),(8) and (14) , we have

$$\delta=0 \, . \tag{15}$$

Hence the above set of equations reduces to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4}{B}\frac{C_4}{C}-\frac{w}{2}\phi^n\phi_4^2=-w\rho \ , \tag{16}$$

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}-\frac{1}{B^2}-\frac{w}{2}\phi^n\phi_4^2=-w\rho \, , \tag{17}$$

$$\frac{B_4^2}{B^2}+\frac{2B_4}{B}\frac{C_4}{C}-\frac{1}{B^2}+\frac{w}{2}\phi^n\phi_4^2=\rho \, , \tag{18}$$

$$\phi, \ +\phi_4\left(\frac{2B_4}{B}+\frac{C_4}{C}\right)+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \, , \tag{19}$$

$$\rho_4+w\rho\left(\frac{2B_4}{B}+\frac{C_4}{C}\right)=0 \, . \tag{20}$$

Here and in what follows a suffix 4 after an unknown function indicates ordinary differentiation with respect to *t*.

## 3. SOLUTION OF FIELD EQUATIONS

Equations (16)-(19) are four independent equations in five unknowns $B, C, \phi, w, \rho$ . To get a determinate solution, one extra condition is needed. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition however these procedure have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may leads to a non-physical situation.

We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's Parameter proposed by Berman[46] which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter model define by

$$q=\frac{-RR_{44}}{R_4^2}=cons\tan t \, . \tag{21}$$

where $R=\left(B^2Ce^x\right)^{1/3}$ is the overall scale factor. Here the constant is taken as negative so that it is an accelerating model of the universe.

Therefore the solution of equation (21) is

$$R=\left(\alpha t+\beta\right)^{1/1+q} , \tag{22}$$

where $\alpha \neq 0$ and $\beta$ are constants of integration. This equation implies that the condition of expansion is $\left(1+q\right)>0$ .

 

Also, the set of equations being highly non-linear, we assume a relation between metric coefficients given by

$$B = \mu C \ , \tag{23}$$

where $\mu$ is constant .

The field equation (16)-(19) admit solutions given by

$$A = B = \mu^{\frac{1}{3}} e^{\frac{-4}{3}} (\alpha t + \beta)^{1/1+q}, \tag{24}$$

$$C = \mu^{\frac{-2}{3}} e^{\frac{-x}{3}} (\alpha t + \beta)^{1/1+q}, \tag{25}$$

and $$\phi = \left[ C_3 (\alpha t + \beta)^{\frac{4+q}{1+q}} + C_4 \right]^{\frac{2}{n+2}}. \tag{26}$$

After a suitable choice of coordinates and constants, the dark energy model corresponding to equations (16)-(19) can be written as

$$ds^2 = dT^2 - T^{\frac{2}{1+q}} [\mu^{\frac{2}{3}} e^{\frac{-2x}{3}} dx^2 - \mu^{\frac{2}{3}} e^{\frac{-4x}{3}} dy^2 - \mu^{\frac{-4}{3}} e^{\frac{-2x}{3}} dz^2], \tag{27}$$

The model (27) represent an exact Bianchi type-III radiating cosmological model in the presence of Dark energy with negative constant deceleration parameter in the frame work of Saez-Ballester scale tensor theory of gravitation . The model has no initial singularity.

## 4. PHYSICAL PROPERTIES

The model (27) represent Bianchi type-III Dark energy cosmological model with variable EoS parameter in Saez-Ballester scale tensor theory of gravitation, this model has no initial singularity.

The physical quantities that are imparted in cosmology are special volume $v^3$, expansion scalar $\theta$, shear scalar $\sigma^2$, and Hubble's parameter $H$ which have the following expansion for the model (27).

Proper volume: $$V^3 = T^{\frac{3}{1+q}}. \tag{28}$$

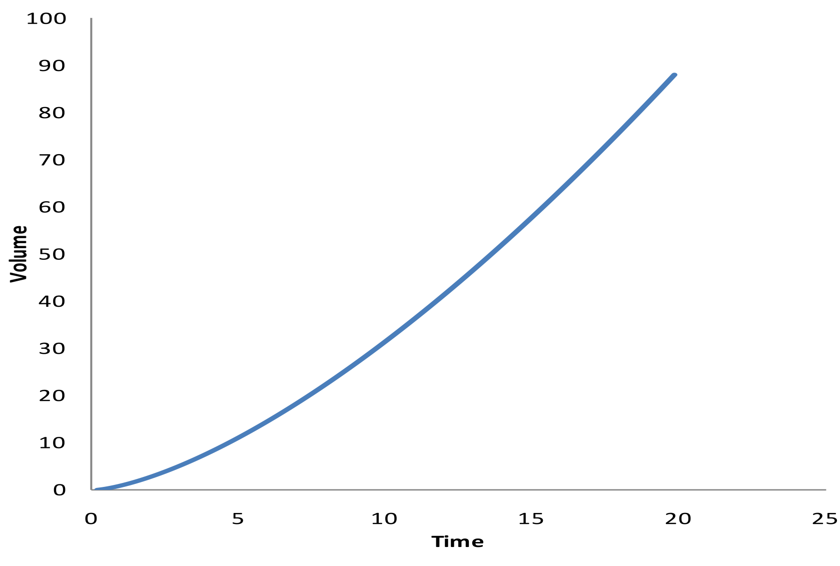


**Figure: 1.** Spatial volume versus cosmic time.

Expansion Scalar : $$\theta = \frac{\alpha}{(1+q)T}. \tag{29}$$

 

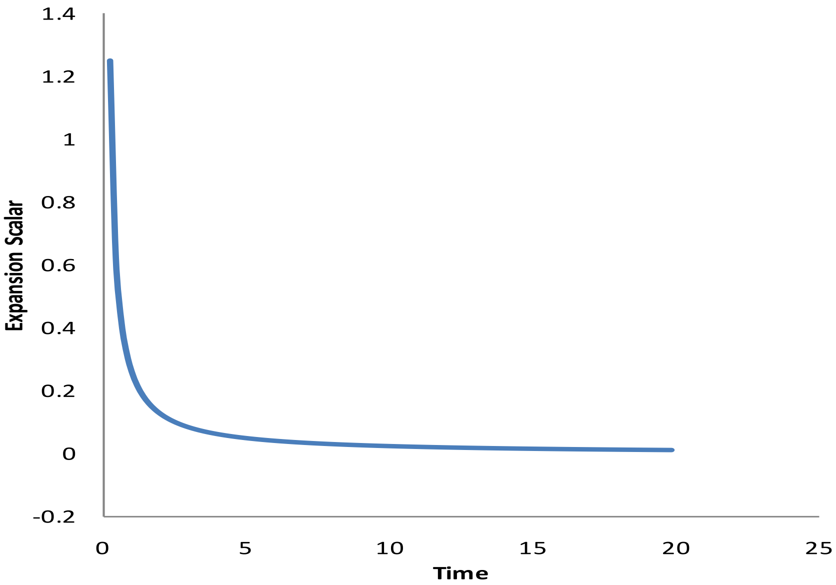


**Figure: 2.** Expansion scalar versus cosmic time.

Shear scalar: $$\sigma^2 = \frac{\alpha^2}{6\left[(1+q)^2 T\right]^2}. \tag{30}$$

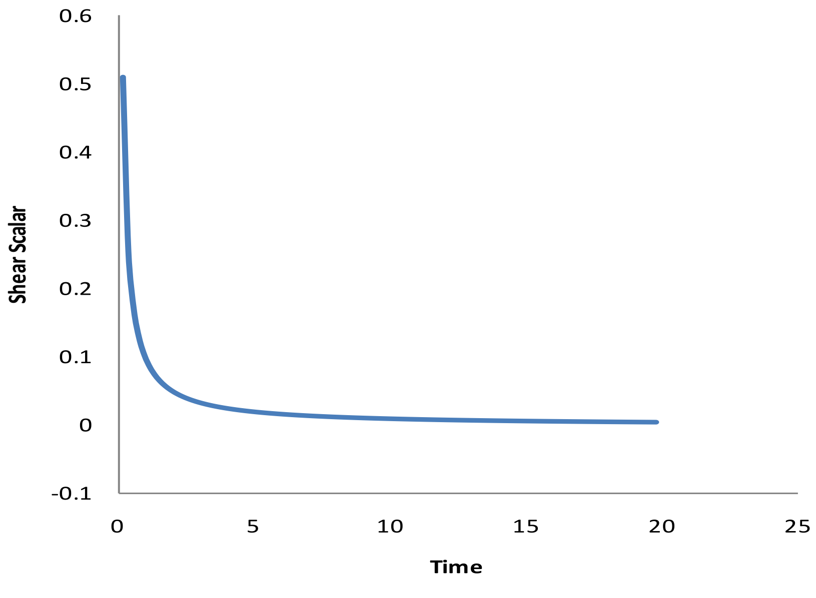


**Figure: 3.** Shear scalar versus cosmic time.

Hubble parameter: $$H = \frac{a}{(1+q)T}. \tag{31}$$

 

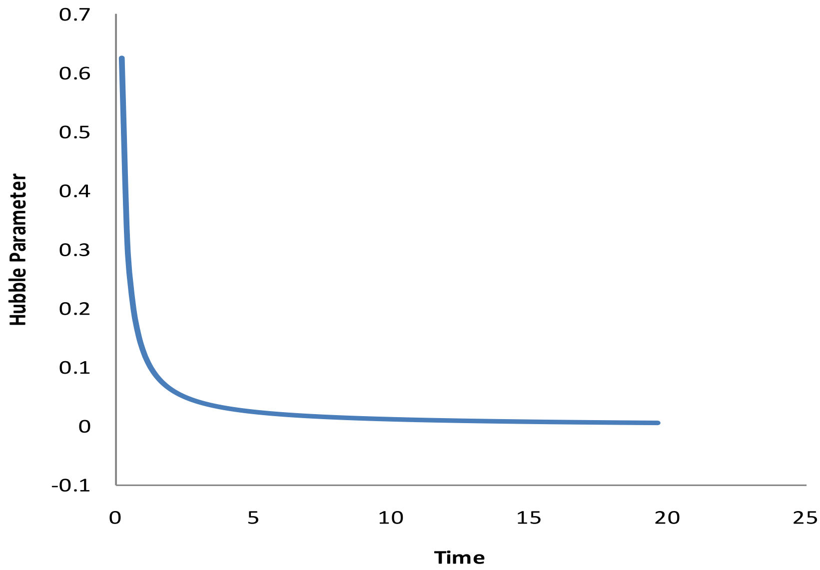


**Figure: 4.** Hubble parameter versus cosmic time.

The energy density $\rho$, the skew ness parameter $\delta$ and the EoS parameter $w$ are given by

$$\rho = \frac{3\alpha^2}{(1+q)^2 T^2} - \frac{1}{\alpha^2 \mu^{\frac{2}{3}} e^{\frac{-8}{3}} T^{\frac{2}{1+q}}} + \frac{2w\alpha^2 C_3 \phi^n (C_1+q)^{2\,2}}{(q+1)^2 (n+2)^2} T^{\frac{6}{1+q}} \left[ C_3 (\alpha t + \beta)^{\frac{4+q}{1+q}} + C_4 \right]^{\frac{-2n}{n+2}} \tag{32}$$

$$w = \frac{1}{\rho} \left\{ \frac{2\alpha^2 q}{(1+q)^2 T^{\frac{2+q}{1+q}}} - \frac{\alpha^2}{(1+q)^2 T^2} + \frac{2w\alpha^2 C_3 \phi^n (4+q)^2}{(n+2)^2 (1+q)^2 T^{\frac{6}{1+q}}} \left( C_3 T^{\frac{4+q}{1+q}} + C_4 \right)^{\frac{-2n}{n+2}} \right\}, \tag{33}$$

and

$$\delta = 0\,. \tag{34}$$

From the above results, we observe that spatial volume is zero at $T = 0$ and it increases with increase of $T$. This shows that the universe starts evolving with zero volume at $T = 0$ and expands with cosmic time $T$. It can be seen that the Hubble's parameter is zero as $T \to \infty$. The scalar expansion $\theta$, the shear scalar $\sigma^2$, the energy density $\rho$, the EoS parameter $w$ diverge at $T = 0$ while they become zero as $T \to \infty$.

Also, since $1+q > 0$ the model is accelerating. It, therefore, follows that our dark energy model in Saez–Ballester theory is consistent with the recent observations of Type-I a super novae (Perlmutter et al. [47]; Reiss et al. [ 48]).

## 5. CONCLUSIONS

An anisotropic Bianchi type-III dark energy model with variable EOS parameter w has been investigated in Saez–Ballester [19] scalar–tensor theory of gravitation. The model obtained represents accelerating model of the universe which is consistent with the recent observations of type-Ia supernovae. The proposed model in this paper presents a new dark energy model in Saez–Ballester theory with variable EoS parameter.

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