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ACCELERATING AND DECELERATING COSMOLOGICAL MODELS WITH PERFECT FLUID AND DARK ENERGY FOR KASNER TYPE METRIC

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ABSTRACT

We consider a self-consistent system of Kasner Universe cosmology and binary mixture of perfect fluid and dark energy. The perfect fluid is taken to be one obeying the usual equation of state $p = \gamma\rho$ *with* $\gamma \in [0, 1]$*. The dark energy is considered to be either the quintessence or Chaplygin gas. Exact solutions to the corresponding Einstein equations are obtained as a quadrature. Models with power-law and exponential expansion have discussed in detail.*

Keywords: *- Cosmological models· Perfect fluid · Dark energy · Cosmological parameters.*

1. INTRODUCTION:

In the frame work of general relativity, the acceleration in the expansion of the universe during recent cosmological times ,first indicated by supernova observations [1] and also supported by the astrophysical data obtain from WMAP indicates the existence of a exotic fluid with negative pressure, which constitutes about the 70 percent of the total energy of the Universe. To know the nature and the behavior of this dark energy is one of the great challenges of cosmology and, indeed, of fundamental physics. Dark energy is a very useful concept since it encodes all our ignorance on the acceleration of the universe in a single cosmic component. Furthermore, Dark energy can also be used as an effective description of other mechanisms of the acceleration of the Universe [2]. A traditional candidate for the role of Dark energy is the cosmological Constant. The dynamical Dark energy, however, incorporate the advantage that they approach the modeling of the mysterious dark energy component in a more general way, allowing its properties to vary with the expansion.

But selection of the cosmological constant as dark energy faces a serious fine-tuning problem which demands that the value of cosmological constant must be 123 orders of magnitude and 5.5 orders of magnitude large on the Planck Scale $T \approx 10^{19} GeV$ and the electroweak Scale $T \approx 10^{2} GeV$, respectively, than it's presently observed value. Moreover, the matter and radiation energy densities of the expanding Universe fall off as a^{-3} and a^{-4}, respectively, where a is the scale factor of the Universe, while Λ remains constant.

Several Candidates to present dark energy have been suggested with observations , Quistessence [3] , Phantom [4] , Brane-world Models [5] , Pure Chaplygin gas model [6] , Generalised Chaplygin Gas (GCG) model [7] , modified Chaplygin Gas (MCG) model [8] and many others .Most recently , a new dark energy model , dubbed agegraphic dark energy has been proposed [9] , which takes into account the Heisenberg Uncertainty relation of quantum mechanics together with the gravitational effect in general relativity . Because the holographic energy density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of general relativity.

Kremer [10] has considered the universe containing a binary mixture whose constituents are described by a Van der Walls fluid and a dark energy density. In these studies the authors considered mainly a spatially flat, homogeneous and isotropic universe described by a FRW metric. Further Khalatnikov and Kamenshchik [11] and Saha [12] have studies Bianchi type-I cosmological model in the presence of perfect fluid and dark energy given by cosmological con stant. Saha [13] has considered Bianchi type-I model of the universe with a binary mixture of perfect fluid and dark energy. T. Singh and R. Chaubey [14] has considered Bianchi type-V model of the universe with a binary mixture of perfect fluid and dark energy. Very recently, Katore et. al. [15,16] have investigated Plane Symmetric and Bianchi Type VI cosmological models with perfect fluid and dark energy .The dark energy is considered to be either the quintessence or the Chaplygin gas.

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In this paper, we consider Kasner Universe gravitational field and a binary mixture of perfect fluid and dark energy. The perfect fluid has equation of state $p = \gamma\rho$ with $\gamma \in [0, 1]$ and the dark energy is given by either a quintessence or a Chaplygin gas. Exact solutions have been obtained in quadrature form. The cases of power-law and exponential expansion have been discussed in detail.

2. METRIC AND SOLUTIONS OF FIELD EQUATIONS:

We consider anisotropic [Bianchi type I] metric in Kasner form

$$ds^2 = dt^2 - t^{2q_1}dx^2 - t^{2q_2}dy^2 - t^{2q_3}dz^2, \tag{2.1}$$

where q_1, q_2, q_3 are three parameters that we shall required to be constant .

The Einstein field equations for the metric (2.1) are written in the form

$$\left[q_1(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = kT_1^1 \ , \tag{2.2}$$

$$\left[q_2(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = kT_2^2, \tag{2.3}$$

$$\left[q_3(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = kT_3^3, \tag{2.4}$$

$$\left[(s-\theta) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = kT_4^4. \tag{2.5}$$

Here k is the gravitational constant and overhead dot denotes differentiation with respect to t .

The energy-momentum tensor of the source is given by

$$T_i^j = (\rho + p)u_i u^j - p\delta_i^j \ , \tag{2.6}$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1. \tag{2.7}$$

In a co-moving system of coordinates , from equation (2.6) we find

$$T_1^1 = T_2^2 = T_3^3 = -p \text{ and } T_4^4 = \rho. \tag{2.8}$$

Now using equation (2.8) in equation (2.2)-(2.5), we get

$$\left[q_1(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = -kp, \tag{2.9}$$

$$\left[q_2(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = -kp \ , \tag{2.10}$$

$$\left[q_3(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = -kp, \tag{2.11}$$

$$\left[(s-\theta) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = k\rho. \tag{2.12}$$

Using equations (2.9) and (2.10), we get

$$(q_1 - q_2)(s-1)t^{-2} = 0. \tag{2.13}$$

Equations (2.13) can be written as

$$\frac{d}{dt}\left(\frac{q_1}{t} - \frac{q_2}{t}\right) + \left(\frac{q_1}{t} - \frac{q_2}{t}\right)\frac{s}{t} = 0. \tag{2.14}$$

Let V be a function of t defined by

$$V = t^{(q_1+q_2+q_3} = t^s. \tag{2.15}$$

Then from equation (2.15), we obtain

$$\frac{d}{dt}\left(\frac{q_1}{t} - \frac{q_2}{t}\right) + \left(\frac{q_1}{t} - \frac{q_2}{t}\right)\frac{\dot{V}}{V} = 0. \tag{2.16}$$

Integrating the above equation, we get

$$\frac{q_1}{q_2} = d_1 \exp\left(x_1 \int \frac{1}{V} dt\right), \tag{2.17}$$

where d_1 and x_2 are constants of integrations .

In view of $V = t^s$, we write $t^{q_1}, t^{q_2}, t^{q_3}$ in the explicit form

$$t^{q_1} = D_1 V^{\frac{1}{3}} \exp\left(X_1 \int \frac{1}{V} dt\right), \tag{2.18}$$

$$t^{q_2} = D_2 V^{\frac{1}{3}} \exp\left(X_2 \int \frac{1}{V} dt\right), \tag{2.19}$$

$$t^{q_3} = D_3 V^{\frac{1}{3}} \exp\left(X_3 \int \frac{1}{V} dt\right), \tag{2.20}$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Using equations (2.9), (2.10), (2.11) and (2.12) , we get

$$s(s-1)t^{-2} = \frac{3k}{2}(\rho - p). \tag{2.21}$$

In view of $V = t^s$, from equation (2.18) we obtain

$$\frac{\ddot{V}}{V} = \frac{3k}{2}(\rho - p), \tag{2.22}$$

The conservational law for the energy-momentum tensor gives

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + p) . \tag{2.23}$$

From equations (2.22) and (2.23), we have

$$\dot{V} = \pm\sqrt{2\left(C_1 + \frac{3k}{2} V^2 \rho\right)} , \tag{2.24}$$

where C_1 being an integration constant.

Rewriting equation (2.23) in the form

$$\frac{\dot{\rho}}{\rho_{WDF} + p_{WDF}} = -\frac{\dot{V}}{V}, \tag{2.25}$$

and taking into account that the pressure and the energy density obeying an equation of state of type $p = f(\rho)$, we conclude that ρ and p , hence the right-hand side of equation (2.22) is a function of V only .

$$\ddot{V} = \frac{3k}{2}(\rho - p)V \equiv F(V). \tag{2.26}$$

From the mechanical point of view, equation (2.26) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then

$$\dot{V} = \sqrt{2(\in - U(V))}. \tag{2.27}$$

Here $\in$ can be viewed as energy and $U(V)$ as the potential of the force F .

Comparing equations (2.24) and (2.27) , we find $\in = C_1$ and

$$U(V) = -\frac{3}{2}kV^2\rho . \tag{2.28}$$

Finally, we write the solution to equation (2.24) in quadrature form

$$\int \frac{dV}{\sqrt{2\left(C_1 + \frac{3k}{2}V^2\rho\right)}} = t + t_0 , \tag{2.29}$$

where the integration constant t_0 can be zero , since it only gives a shift in time .

3. UNIVERSE AS A BINARY MIXTURE OF PERFECT FLUID AND DARK ENERGY

We consider the evolution of the Plane Symmetric Universe filled with perfect fluid and dark energy. Taking into account that the energy density ρ and pressure p in this case comprise those of perfect fluid and dark energy .

$$\rho = \rho_{PF} + \rho_{DE}, \; p = p_{PF} + p_{DE} . \tag{3.1}$$

The energy momentum tensor can be decomposed as

$$T_i^j = (\rho_{PF} + \rho_{DE} + p_{PF} + p_{DE})u^j u_i - (p_{DE} + p_{PF})\delta_i^j . \tag{3.2}$$

In the above equation ρ_{DE} is the dark energy density, p_{DE} its pressure. We also use the notations ρ_{PF} and p_{PF} to denote the energy density and the pressure of the perfect fluid respectively. Here

we consider the case when the perfect fluid obeys the following equation of state

$$p_{PF} = \gamma\rho_{PF} . \tag{3.3}$$

Here γ is a constant and lies in the interval $\gamma \in [0,1]$.

Depending on its numerical value γ describes the following type of universe

$$\gamma = 0 \text{ (Dust universe) }, \tag{3.4a}$$

$$\gamma = \frac{1}{3} \text{ (Radiation universe) }, \tag{3.4b}$$

$$\gamma \in (\frac{1}{3}, 1) \text{ (Hard universe)}, \tag{3.4c}$$

$\gamma = 1$ (Zeldovich universe or Stiff matter). (3.4d)

In a co-moving frame the conservation law of energy momentum tensor leads to the balance equation for the energy density

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}\left(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}\right). \tag{3.5}$$

The dark energy is supposed to interact with itself only and it is minimally coupled to the gravitational field. As a result the evolution equation for the energy density decouples from that of the perfect fluid and from equation (3.5) we obtain two balance equations

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}\left(\rho_{DE} + p_{DE}\right) = 0, \tag{3.6a}$$

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}\left(\rho_{PF} + p_{PF}\right) = 0. \tag{3.6b}$$

From equations (3.3) and (3.6b), we write

$$\rho_{PF} = \frac{\rho_0}{V^{(1+\gamma)}}, \quad p_{PF} = \frac{\rho_0 \gamma}{V^{(1+\gamma)}}, \tag{3.7}$$

where ρ_0 is an integration constant .

In the absence of the dark energy, equation (2.26) reduces to

$$\int \frac{dV}{\sqrt{2\left(C_1 + \frac{3k}{2}\rho_0 V^{1-\gamma}\right)}} = t. \tag{3.8}$$

For $C_1 = 0$, equation (3.8) reduces to

$$V = Ct^{\frac{2}{1+\gamma}}, \tag{3.9}$$

where C being some integration constant .

4. MODELS WITH A QUINTESSENCE AND CHAPLYGIN GAS

4.1. Case with a quintessence

Let us consider the case when the dark energy is given by a quintessence which obeys the equation of state

$$p_q = w_q \rho_q, \tag{4.1}$$

where the constants w_q varies between -1 and zero i.e. $w_q \in [-1,0]$.

From equations (4.1) and (3.6a) we get

$$\rho_q = \frac{\rho_{0_q}}{V^{1+w_q}}, \quad p_q = \frac{w_q \rho_{0_q}}{V^{1+w_q}} \tag{4.2}$$

with ρ_{0_q} being an integration constant .

Now, the evolution of equation (2.19) for *V* can be written as

$$\ddot{V} = \frac{3k}{2}\left(\frac{(1-\gamma)\rho_0}{V^{\gamma}} + \frac{(1-w_q)\rho_{0_q}}{V^{w_q}}\right). \tag{4.3}$$

As it was mentioned earlier equation (4.3) admits exact solution that can be written in quadrature as

$$\int \frac{dV}{\sqrt{2\left(C_1 + \frac{3k}{2}\left(\rho_0 V^{(1-\gamma)} + \rho_{0_q} V^{(1-w_q)}\right)\right)}} = t + t_0 . \tag{4.4}$$

Here t_0 is a constant of integration that can be taken to be zero .

4.2. Case with Chaplygin gas

Let us now consider the case when the dark energy is represented by Chaplygin gas

$$p_c = -\frac{A}{\rho_c} , \tag{4.5}$$

with A being a positive constant .

From equations (4.5) and (3.6a) we get

$$\rho_c = \sqrt{\frac{\rho_{0_c}}{V^2} + A}, \; p_c = \frac{-A}{\sqrt{\frac{\rho_{0_C}}{V^2} + A}} \tag{4.6}$$

with ρ_{0c} being an integration constant .

Now , the evolution of equation (2.19) for V can be written as

$$\ddot{V} = \frac{3k}{2}\left(\frac{(1-\gamma)\rho_0}{V^\gamma} + \sqrt{\rho_{0_c} + AV^2} + \frac{A}{\sqrt{\rho_{0_c} + AV^2}}\right). \tag{4.7}$$

The corresponding solution in quadrature form is

$$\int \frac{dV}{\sqrt{2\left(C_1 + \frac{3k}{2}\left(\rho_0 V^{(1-\gamma)} + \sqrt{\rho_{0_c} V^2 + AV^4}\right)\right)}} = t , \tag{4.8}$$

where the second integration has been taken to be zero .

5. MODELS WITH CONSTANT DECELERATION PARAMETER

Case I . Power – Law

Here we take

$$V = at^b , \tag{5.1}$$

where a and b are constants .

From equations (2.18), (2.19), (2.20) and (5.1), we get

$$t^{q_1} = D_1 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_1}{a(1-b)} t^{1-b}\right\}, \tag{5.2}$$

$$t^{q_2} = D_2 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_2}{a(1-b)} t^{1-b}\right\} , \tag{5.3}$$

$$t^{q_3} = D_3 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{ \frac{X_3}{a(1-b)} t^{1-b} \right\}, \tag{5.4}$$

where $D_i \,(i = 1,2,3)$ and $X_i \,(i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From equation (1.3) and (5.1) , we have

$$\rho = \rho_0 a^{-(1+\gamma)} t^{-(1+\gamma)b}, \tag{5.5}$$

From equation (1.1) and (5.5), we get

$$p = \gamma \rho_0 a^{-(1+\gamma)} t^{-(1+\gamma)b}. \tag{5.6}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ , the mean anisotropy parameter A , the shear scalar σ^2 and the deceleration parameter q . They are defined as

$$\theta = 3H, \tag{5.7}$$

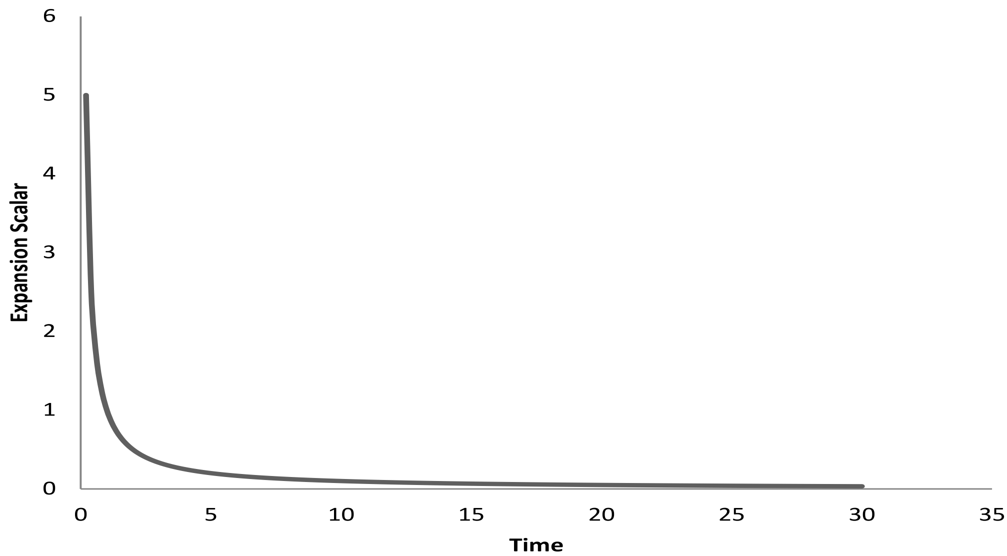
$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{5.8}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2} A H^2, \tag{5.9}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{5.10}$$

By using equations (5.78)-(5.10), we can express the physical quantities as

$$\theta = \frac{b}{t}, \tag{5.11}$$



$$A = \frac{3X^2}{a^2 b^2 t^{2(b-1)}}, \tag{5.12}$$

$$\sigma^2 = \frac{X^2}{2a^2 t^{2b}}, \tag{5.13}$$

$$q = \frac{3}{b} - 1, \tag{5.14}$$

where $X^2 = X_1^2 + X_2^2 + X_3^3$ is a constant .

For large *t*, the model tends to be isotropic when b>1.

Case II. Exponential Type

Here we take

$$V = \alpha e^{\beta t} \,, \tag{5.15}$$

where α, β are constants .

From equations (2.18),(2.19) ,(2.20) and (5.15) , we get

$$t^{q_1} = D_1 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{ \frac{-X_1}{\alpha\beta} e^{-\beta t} \right\}, \tag{5.16}$$

$$t^{q_2} = D_2 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{ \frac{-X_2}{\alpha\beta} e^{-\beta t} \right\}, \tag{5.17}$$

$$t^{q_3} = D_3 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{ \frac{-X_3}{\alpha\beta} e^{-\beta t} \right\}, \tag{5.18}$$

where $D_i\,(i=1,2,3)$ and $X_i\,(i=1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From equation (1.3) and (5.15) , we have

$$\rho = \rho_0 \alpha^{-(1+\gamma)} e^{-(1+\gamma)\beta t}, \tag{5.19}$$

From equation (1.1) and (5.19), we get

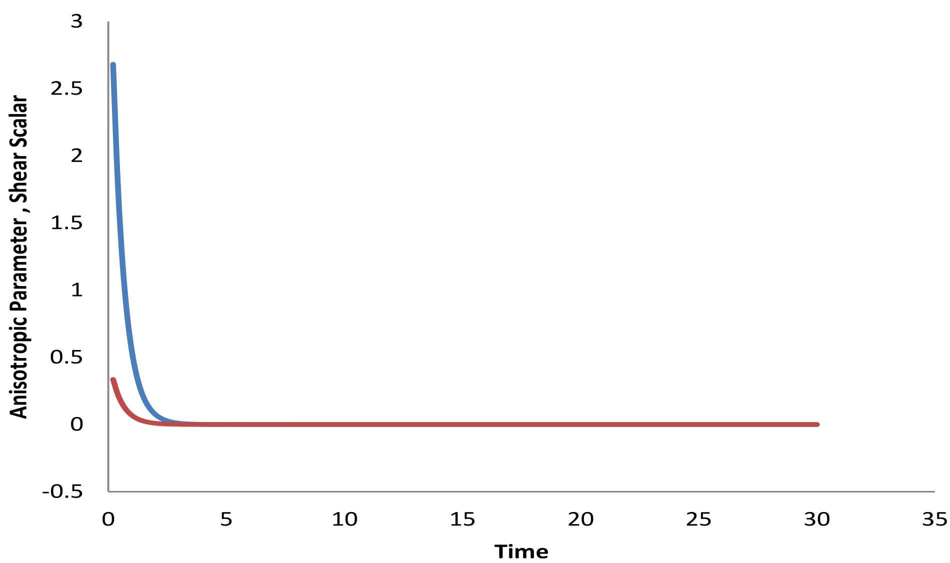
$$p = \gamma \rho_0 \alpha^{-(1+\gamma)} e^{-(1+\gamma)\beta t}, \tag{5.20}$$

By using equations (5.7)-(5.10), we can express the physical quantities as

$$\theta = \beta \,, \tag{5.21}$$

$$A = \frac{3X^2 e^{-2\beta t}}{\alpha^2 \beta^2}, \tag{5.22}$$

$$\sigma^2 = \frac{X^2 e^{-2\beta t}}{2\alpha^2}, \tag{5.23}$$



$$q = -1 \,, \tag{5.24}$$

where $X^2 = X_1^2 + X_2^2 + X_3^3$ is a constant .

For large t , the model tends to be isotropic.

6. CONCLUSION

A self-consistent system of Kasner Universe gravitational field filled with a perfect fluid and dark energy has been considered. The exact solution to the corresponding field equations are obtained in quadrature form. The solution for constant deceleration parameter have been studied in detail for power-law and exponential forms. It is interesting to note that our model resembles to the investigations of [14] .The inclusion of the dark energy into the system gives rise to an accelerated expansion of the model. As a result V approaches to infinity quicker than it does when the universe is filled with perfect fluid alone .In case of ordinary quintessence, following Jacobs [17] we can conclude that the initial anisotropy of the model dies away rather quickly. The solution for constant deceleration parameter have been studied in detail for power-law and exponential forms. The introduction of the dark energy does not eliminate the initial singularity.

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