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PARAMETRIC GENERALIZED NONLINEAR QUASI -VARIATIONAL INCLUSION PROBLEMS

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ABSTRACT

In this work, we intend to introduce a new class of parametric generalized nonlinear quasi-variational inclusions. We prove the existence of solutions for our inclusions and study the sensitivity analysis of the solution set. The continuity and the Lipschitz continuity of the solution set with respect to the parameter are proved under the suitable assumptions

Key Words: *Generalized parametric quasi-variational inclusions, sensitivity analysis, resolvent operator, Hausdorff metric.*

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1. INTRODUCTION

In recent years, much attention has been devoted to develop general methods for the sensitivity analysis of solution set for variational inequalities and variational inclusions. From the mathematical and engineering point of view, sensitivity properties of various variational inequalities can provide new insight concerning the problem being studied and can stimulate ideas for solving problems. The sensitivity analysis of solution set for variational inequalities have been studied extensively by many authors using quite different methods. By using the projection technique, Dafermos [3], Mukherjee and Verma [19], Noor [22] and Yen [29] dealt with the sensitivity analysis for variational inequalities with single valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [28] dealt with the sensitivity analysis for variational inequalities in finite-dimensional spaces. By using resolvent operator technique, Adly[1], Noor and Noor[25,21], and Aggarwal et al.[2] study the sensitivity analysis for quasi-variational inclusions with single valued mappings. Recently, by using projection technique and the property of fixed point set of multi-valued contractive mappings, Ding and Lou [9], Liu et al.[18],Ding[11] study the behavior and sensitivity analysis of solution set In this paper, we intend to introduce a new class of parametric generalized nonlinear quasi-variational inclusions and prove the existence of solutions for our inclusions and study the sensitivity analysis of the solution set by using resolvent operator technique.

2. PRELIMINARIES

Let H be a real Hilbert space with a norm $\|.\|$ and an inner product $\langle . , . \rangle$. Let C(H) denote the family of all nonempty compact subsets of H and H(. , .) denote the Hausdorff metric on C(H) defined by

$$H(A, B)= \max\{\sup_{a\in A} d(a, B), \sup_{b\in B} d(A, b) \}, \ \forall\ A, B \in C(H),$$

where $d(a,B)= \inf_{b\in B} \| a-b \|$ and $d(A,b)= \inf_{a\in A} \|a-b \|$.

We now consider the following parametric generalized nonlinear quasi-variational inclusion problem. To this end, let Ω be a nonempty open subset of H in which the parameter Ω takes the values, $N:H \times H\times \Omega \to H$ and f, m: $H\times \Omega \to H$ be a single valued mappings and A, B, C, D, E, G: $H \times \Omega\to C(H)$ be multi-valued mappings. Let $M: H \times H \times \Omega \to 2^H$ be a multi-valued mappings such that for each given $(z, \lambda)\in H\times \Omega$, $M(.,z, \lambda):H\to 2^H$ is a maximal monotone mappings with $(G(H, \lambda) - m(H, \lambda)) \cap \text{dom}\, M(. , z, \lambda) \neq \phi$. Throughout in this paper, unless otherwise stated, we will consider the following parametric generalized nonlinear quasi-variational inclusion problem (In short, PGNQVIP):

For each fixed $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that

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$$0 \in M(s(\lambda) - m(w(\lambda), \lambda), z(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda) \tag{2.1}$$

Special Cases:

(1) If $f(t(\lambda), \lambda) = 0$, then the PGNQVIP(2.1) is equivalent to the following parametric generalized nonlinear implicit quasi-variational inclusion problem: for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that

$$0 \in M(s(\lambda) - (w(\lambda), \lambda), z(\lambda), \lambda) + N(u(\lambda), v(\lambda), \lambda). \tag{2.2}$$

(2) If $G = g: H \times \Omega \to H$ is a single valued mapping, then the PGNQVIP (2.1) is equivalent to the following parametric generalized nonlinear implicit quasi-variational inclusion problem: for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$ such that

$$0 \in M(g(x(\lambda), \lambda) - m(w(\lambda), \lambda), z(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda). \tag{2.3}$$

(3) If $m(x, \lambda) = 0$ for all $(x, \lambda) \in H \times \Omega$, then problem(2.3) reduces to the following parametric problem: for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$ such that

$$0 \in M(g(x(\lambda), \lambda), z(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda). \tag{2.4}$$

(4) Let $\varphi : H \times H \times \Omega \to R \cup \{+\infty\}$ be such that for each fixed $(z, \lambda) \in H \times \Omega$, $\varphi(., z, \lambda)$ is a proper convex lower semi-continuous functional satisfying

$G(H, \lambda) - m(H, \lambda) \cap \mathrm{dom}\,\partial\varphi(., z, \lambda) \neq \Phi$, where $\partial\varphi(., z, \lambda)$ is the sub differential of $\varphi(., z, \lambda)$. By [27], $\partial\varphi(., \lambda): H \times 2^H$ is a maximal monotone mapping. Let $M(. \lambda) = \partial\varphi(., z, \lambda)$, $\forall (z, \lambda) \in H \times \Omega$. for given $(z, \lambda) \in H \times \Omega$, by the definition of the sub differential of $\varphi(., z, \lambda)$, it is easy to see that problem (2.1) reduces to the following.

parametric problem: for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that

$$\langle f(t(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda), y - s(\lambda) \rangle \geq \varphi(s(\lambda) - m(w(\lambda), v), z(\lambda), \lambda) - \varphi(y, z(\lambda), \lambda), \quad \forall\, y \in H. \tag{2.5}$$

(5) If $G = g: H \times \Omega \to H$ is a single valued mapping and $m(x, \lambda) = 0$ for all $(x, \lambda) \in H \times \Omega$, then problem (2.5) reduces to the following parametric : for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$ such that

$$\langle f(t(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda), y - g(x(\lambda), \lambda) \rangle \geq \varphi(g(x(\lambda), \lambda), z(\lambda), \lambda) - \varphi(y, z(\lambda), \lambda), \quad \forall\, y \in H. \tag{2.6}$$

(6) If $K: H \times \Omega \to 2^H$ is a multi-valued mapping such that for each $(x, \lambda) \in H \times \Omega$, $K(x, \lambda)$ is a closed convex subset of H and for each fixed $(z, \lambda) \in H \times \Omega$, $\varphi(., z, \lambda) = I_K(z, \lambda)(.)$ is the indicator function of $K(z, \lambda)$,

$$I_{K(z, \lambda)}(x) = \begin{cases} 0, if x \in K(x, \lambda) \\ +\infty, otherwise \end{cases}$$

then problem (2.6) reduces to the following parametric generalized strongly nonlinear implicit quasi-variational inequality problem:

for each $\lambda \in \Omega$, find $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$ such that

$$g(x(\lambda), \lambda) \in K(z(\lambda), \lambda) \text{ and } \langle f(t(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda), y - g(x(\lambda), \lambda) \rangle \geq 0, \ \forall\, y \in K(z(\lambda), \lambda). \tag{2.7}$$

In brief, for appropriate and suitable choices of A, B, C, D, E, G, N, m, f and M, it is easy to see that the PGNQVIP (2.1) includes a number of (parametric) quasi-variational inclusions, (parametric) generalized quasi-variational inclusions, (parametric) quasi-variational inequalities, (parametric) generalized implicit quasi-variational inequalities studied by many authors as special cases, for example, see [1-5, 11-13] and the references therein. Furthermore, these types of (parametric) generalized quasi-variational inclusions can enable us to study the behavior and sensitivity analysis of the solution sets of many important nonlinear problems arising in mechanics, physics, optimization and control, nonlinear programming, economics, finance, regional structural, transportation, elasticity, and various applied science in a general and unified framework.

Now for each fixed $\lambda \in \Omega$, the solution set S (λ) of the PGNQVIP(2.1) is denoted as $S(\lambda) = \{ x(\lambda) \in H$: there exists $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that

$$0 \in M(s(\lambda) - m(w(\lambda), \lambda), z(\lambda), \lambda) - N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda) \}$$

In this paper, our main aim is to study the behavior of the solution set $S(\lambda)$, and the conditions on these mappings A, B, C, D, E, G, M, N, f and m under which the function $S(\lambda)$ is continuous or Lipschitz continuous with respect to the parameter $\lambda \in \Omega$.

Now, we give the following concepts and known results.

Definition 2.1: (See [27].) Let H be a Hilbert space and let $M: H \to 2^H$ be a maximal monotone mapping. For any fixed $\rho > 0$, the mapping $J_\rho^M : H \to H$, defined by

$$J_\rho^M(x) = (I+\rho M)^{-1} x), \ \forall x \in H,$$ is said to be the resolvent operator of M where I is the identity mapping on H.

Lemma 2.1: (See [27].) Let $M: H \to 2^H$ be a maximal monotone mapping. Then the resolvent operator $J^M_\rho: H \to H$ of M is non expansive, i.e.,

$$\| J_\rho^M(x) - J_\rho^M(y) \| \le \| x - y \|, \quad \forall x, y \in H.$$

Lemma 2.2: (See [17].) Let (X, d) be a complete metric space and $T_1, T_2: X \to C(X)$ be two multi-valued contractive mappings with same contractive constant $\theta \in (0,1)$, i.e.,

$$H(T_i(x), T_i(y)) \le \theta d(x, y), \forall x, y \in X, i = 1, 2. \text{ Then}$$
$$H(F(T_1), F(T_2)) \le \frac{1}{1-\theta} \sup_{x \in X} H(T_1(x), T_2(x)),$$

Where $F(T_1)$ and $F(T_2)$ are fixed point sets of T_1 and T_2, respectively.

Definition 2.2: A multi-valued mapping $G: H \times \Omega \to C(H)$ is said to be
(i) δ - *strongly monotone* if there exists a constant $\delta > 0$ such that

$$\langle s_1 - s_2, x - y \rangle \ge \delta \|x - y\|^2, \ \forall (x, y, \lambda) \in H \times H \times \Omega, s_1 \in G(x, \lambda), s_2 \in G(y, \lambda).$$

(ii) λ_G- *Lipschitz continuous* if there exists a constant $\lambda_G > 0$ such that

$$H(G(x, \lambda), G(y, \lambda)) \le \lambda_G \|x - y\| \quad \forall (x, y, \lambda) \in H \times H \times \Omega.$$

Definition 2.3: $A: H \times \Omega \to C(H)$ be a multi-valued mapping and $N: H \times H \times \Omega \to H$ be a single valued mapping:
(i) $N(u, v, \lambda)$ is said to be α - *strongly monotone* with respect to A such that

$$\langle s_1 - s_2, x - y\rangle \ge \alpha \| x - y \|^2, \ \forall (x, y, \lambda) \in H \times H \times \Omega, s_1 \in G(x, \lambda), s_2 \in G(y, \lambda).$$

(ii) $N(u, v, \lambda)$ is said to be β - *Lipschitz continuous* in the first argument if there exists a constant $\beta > 0$ such that

$$\| N(u_1, v, \lambda) - N(u_2, v, \lambda) \| \le \beta \| u_1 - u_2 \|, \forall (u_1, u_2, v, \lambda) \in H \times H \times H \times \Omega.$$

In a similar way, we can define the ξ- Lipschitz continuity of $N(u, v, \lambda)$ in the second argument.

3.1 SENSITIVITY ANALYSIS OF SOLUTION SET

We first transfer the PGNQVIP (2.1) into a parametric fixed point problem.

Theorem 3.1: For each fixed $\lambda \in \Omega$ $x(\lambda) \in S(\lambda)$ is a solution of the PGNQVIP(2.1) if and only if there exists $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that the following relation holds:

$$s(\lambda) = m(w(\lambda), \lambda) + J_\rho^{M(.,Z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda), \lambda) - \rho N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda)),$$

where $\rho > 0$ is a constant.

Proof: For each fixed $\lambda \in \Omega$, by the definition of the resolvent operator $J_{\rho}^{M(.,z(\lambda),\lambda)}$ of $M(.,z(\lambda),\lambda)$, we have that there exists $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda),\lambda)$, $v(\lambda) \in B(x(\lambda),\lambda)$, $w(\lambda) \in C(x(\lambda),\lambda)$, $z(\lambda) \in D(x(\lambda),\lambda)$, $t(\lambda) \in E(x(\lambda),\lambda)$, and $s(\lambda) \in G(x(\lambda),\lambda)$ such that(3.1) holds if and only if

$$s(\lambda) - m(w(\lambda),\lambda) - \rho N(u(\lambda),v(\lambda),\lambda) + f(t(\lambda),\lambda) \in s(\lambda) - m(w(\lambda),\lambda)\,) + \rho M(\, s(\lambda) - m(w(\lambda),\lambda), z(\lambda), \lambda. \qquad (3.1)$$

The above relation holds if and only if

$$0 \in M(\, s(\lambda) - m(w(\lambda),\lambda), z(\lambda),\lambda) - N(u(\lambda), v(\lambda), \lambda\,) + f(t(\lambda),\lambda\,).$$

By the definition of $S(\lambda)$, we obtain that $x(\lambda) \in S(\lambda)$ is a solution of the PGNQVIP(2.1) if and only if there exist $x(\lambda) \in H$, $u(\lambda) \in A(x(\lambda),\lambda)$, $v(\lambda) \in B(x(\lambda),\lambda)$, $w(\lambda) \in C(x(\lambda),\lambda)$, $z(\lambda) \in D(x(\lambda),\lambda)$, $t(\lambda) \in E(x(\lambda),\lambda)$, $s(\lambda) \in G(x(\lambda),\lambda)$ such that(3.1) holds.

Theorem 3.2: Let A, B, C, D, E, G : $H\times \Omega \to C(H)$ be multi-valued mappings such that A, B, C, D, E, and G are λ_A- Lipschitz continuous, λ_B - Lipschitz continuous, λ_C- Lipschitz continuous, λ_D - Lipschitz continuous, λ_E - Lipschitz continuous and λ_G- Lipschitz continuous, respectively; and G: $H\times \Omega \to C(H)$ be -δ strongly monotone. Let N: $H\times H \times \Omega \to H$ be α - strongly monotone with respect to A and β - Lipschitz continuous in the first argument and ξ- Lipschitz continuous in the second argument. Let m: $H\times \Omega \to H$ be η- Lipschitz continuous and f: $H \times \Omega \to H$ be ε- Lipschitz continuous. Let M: $H\times \Omega \to 2^H$ be such that for each fixed $(z, \lambda) \in H\times\Omega$, $M(.,z,\lambda):H\to 2^H$ is a maximal monotone mapping satisfying $G(H,\lambda) - m(H,\lambda)\cap \mathrm{dom}\, M(.,z,\lambda) \neq \Phi$. Suppose that for any $(x, y, z, \lambda) \in H\times H\times H\times \Omega$,

$$\| J_{\rho}^{M(.,x,\lambda)}(z) - J_{\rho}^{M(.,y,\lambda)}(z) \| \le \mu \| x - y \| \qquad (3.2)$$

and there exists a constant $\rho > 0$ such that

$$k = 2\sqrt{1-2\delta + \lambda_G^2 + \mu\lambda_D + 2\eta\lambda_C + \varepsilon\lambda_E}, \quad k+ \rho\, \xi\, \lambda_B < 1,\ \xi\lambda_B < \alpha \le \lambda_A\beta,$$

$$\alpha > (1-k)\, \xi\, \lambda_B + \sqrt{(\lambda_A^2\beta^2 - \xi^2\lambda_B^2)(2k-k)^2}\ , \qquad (3.3)$$

$$|\rho - \frac{\alpha-(1-k)\xi\lambda_B}{\lambda_A^2\beta^2 - \xi^2\lambda_B^2}| < \frac{\sqrt{\alpha-(1-k)\xi\lambda_B - (\lambda_A^2\beta^2-\xi^2\lambda_B^2)(2k-k^2)}}{\lambda_A^2\beta^2 - \xi^2\lambda_B^2}.$$

Then, for each $\lambda\in \Omega$, we have the following

(1) The solution set $S(\lambda)$ of the PGNQVIP(2.1) is nonempty.
(2) $S(\lambda)$ is a closed subset in H.

Proof: Define a multi-valued mapping F: $H\times \Omega \to 2^H$ by
$F(x,\lambda) = \cup u\in A(x,\lambda),\ v\in B(x,\lambda),\ w\in C(x,\lambda),\ z\in D(x,\lambda),\ t\in E(x,\lambda),\ s\in G(x,\lambda)\ [x-s + m(w,\lambda) + J_{\rho}^{M(.,.)}(s - m(w,\lambda) - \rho N(u, v, \lambda) + f(t, \lambda))],\ \forall\ (x,\lambda) \in H \times \Omega$.

For any $(x,\lambda) \in H \times \Omega$, since $A(x,\lambda), B(x,\lambda), C(x,\lambda), D(x,\lambda), E(x,\lambda), G(x,\lambda) \in C(H)$, and m,f and $J^{M(.,z,\lambda)}$ are continuous, we have $F(x,\lambda) \in C(H)$. Now for each fixed $\lambda \in \Omega$, we prove that $F(x,\lambda)$ is a multi-valued contractive mapping. For any $(x,\lambda), (y,\lambda) \in H\times \Omega$ and $a\in F(x,\lambda)$, there exist

$u_1 \in A(x,\lambda)$, $v_1\in B(x,\lambda)$, $w_1 \in C(x,\lambda)$, $z_1 \in D(x,\lambda)$, $t_1 \in E(x,\lambda)$, $s_1 \in G(x,\lambda)$ such that
$a = x-s1 + m(w1,\lambda)+ J_{\rho}^{M(.,.)}(s1- m(w1,\lambda)- \rho N(\, u1, v1, \lambda)+ f(t1,\lambda))$.

Note that $A(y,\lambda), B(y,\lambda), C(y,\lambda), D(y,\lambda), E(y,\lambda), G(y,\lambda) \in C(H)$, there exists $u_2 \in A(y,\lambda)$, $v_2 \in B(y,\lambda)$, $w_2 \in C(y,\lambda)$, $z_2 \in D(y,\lambda)$, $t_2 \in E(y,\lambda\,)$ and $s_2\in G(y,\lambda)$ such that

$$\begin{aligned}
&\| u_1 - u_2 \| \le H\,(A(x,\lambda), A(y,\lambda)),\\
&\| v_1 - v_2 \| \le H\,(B(x,\lambda), B(y,\lambda)),\\
&\| w_1 - w_2 \| \le H(C(x,\lambda), C(y,\lambda)), \qquad (3.4)\\
&\| z_1 - z_2 \| \le H\,(D(x,\lambda), D(y,\lambda)),
\end{aligned}$$

$$\| t_1 - t_2 \| \le H(E(x, \lambda), E(y, \lambda)),$$

$$\| s_1 - s_2 \| \le H(G(x, \lambda), G(y, \lambda)).$$

Let $b = y - s_2 + m(w_2, \lambda) + J_\rho^{M(.,.)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda))$, then we have $b \in F(y, \lambda)$. It follows that

$$\| a - b \| = \| x - s_1 + m(w_1, \lambda) + J_\rho^{M(.z_1,\lambda)}(s_1 - m(w_1, \lambda) - \rho N(u_1, v_1, \lambda) + f(t_1, \lambda)) - [y - s_2 + m(w_2, \lambda) + J_\rho^{M(.z_2,\lambda)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda))] \|$$

$$\le \| x - y - (s_1 - s_2) \| + \| m(w_1, \lambda) - m(w_2, \lambda) \| + \| J_\rho^{M(.,z_1,\lambda)}(s_1 - m(w_1, \lambda) - \rho N(u_1, v_1, \lambda) + f(t_1, \lambda)) - J_\rho^{M(.z_2,\lambda)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)) \|. \quad (3.5)$$

Since G is δ - strongly monotone and λ_G- Lipschitz continuous, we have

$$\| x - y - (s_1 - s_2) \|^2 = \| x - y \|^2 - 2 \langle x - y, s_1 - s_2 \rangle + \| s_1 - s_2 \|^2$$

$$\le \| x - y \|^2 - 2\delta \| x - y \|^2 + [H(G(x, \lambda), G(y, \lambda))]^2$$

$$\le \| x - y \|^2 - 2\delta \| x - y \|^2 + \lambda_G^2 \| x - y \|^2$$

and hence,

$$\| x - y - (s_1 - s_2) \| \le \sqrt{1 - 2\delta + \lambda_G^2}\, \| x - y \|. \quad (3.6)$$

By Lemma 2.1 and condition (3.2), we have

$$\| J_\rho^{M(.,z_1,\lambda)}(s_1 - m(w_1, \lambda) - \rho N(u_1, v_1, \lambda) + f(t_1, \lambda)) - J_\rho^{M(.,z2,\lambda)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)) \|$$

$$\le \| J_\rho^{M(.,z1,\lambda)}(s_1 - m(w_1, \lambda) - \rho N(u_1, v_1, \lambda) + f(t_1, \lambda)) - J_\rho^{M(.,z1,\lambda)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)) \| + \| J^{M(.,z1,\lambda)}\rho(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)) - J_\rho^{M(.,z2,\lambda)}(s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)) \|$$

$$\le \| s_1 - m(w_1, \lambda) - \rho N(u_1, v_1, \lambda) + f(t_1, \lambda) - [s_2 - m(w_2, \lambda) - \rho N(u_2, v_2, \lambda) + f(t_2, \lambda)] \| + \mu \| z_1 - z_2 \|$$

$$\le \| x - y - (s_1 - s_2) \| + \| x - y - \rho(N(u_1, v_1, \lambda) - N(u_2, v_1, \lambda)) \| + \rho \| N(u_2, v_1, \lambda) - N(u_2, v_2, \lambda) \| + \| m(w_1, \lambda) - m(w_2, \lambda) \| + \| f(t_1, \lambda) - f(t_2, \lambda) \| + \mu \| z_1 - z_2 \|. \quad (3.7)$$

Since $N(u, v, \lambda)$ is α- strongly monotone with respect to A and β- Lipschitz continuous in the first argument and A is λ_A - Lipschitz continuous , we have

$$\| x - y - \rho(N(u_1, v_1, \lambda) - N(u_2, v_1, \lambda)) \|^2 = \| x - y \|^2 - 2\rho \left\langle N(u_1, v_1, \lambda) - N(u_2, v_1, \lambda), x - y \right\rangle + \rho^2 \| N(u_1, v_1, \lambda) - N(u_2, v_1, \lambda) \|^2$$

$$\le \| x - y \|^2 - 2\rho\alpha \| x - y \|^2 + \rho^2\beta^2 [H(A(x, \lambda), A(y, \lambda))]^2$$

$$\le (1 - 2\rho\alpha + \rho^2 \beta^2 \lambda_A^2) \| x - y \|^2. \quad (3.8)$$

Using ξ- Lipschitz continuity of $N(u, v, \lambda)$ in the second argument and λ_B- Lipschitz continuity of B, we have

$$\| N(u_2, v_1, \lambda) - N(u_2, v_2, \lambda) \| \le \xi \| v_1 - v_2 \| \le \xi H(B(x, \lambda), B(y, \lambda))$$

$$\le \xi \lambda_B \| x - y \|. \quad (3.9)$$

Using ε- Lipschitz continuity of $f(t, \lambda)$ and λ_E -Lipschitz continuity of E, we have

$$\| f(t_1, \lambda) - f(t_2, \lambda) \| \le \varepsilon \| t_1 - t_2 \| \le \varepsilon H(E(x, \lambda), E(y, \lambda))$$

$$\le \varepsilon\lambda_E \| x - y \|. \quad (3.10)$$

By the λ_D - Lipschitz continuity of D, we have

$$\| z_1 - z_2 \| \le H(D(x, \lambda), D(y, \lambda))$$

$$\le \lambda_D \| x - y \|. \tag{3.11}$$

By the η-Lipschitz continuity of m and λ_C- Lipschitz continuity of C, we have

$$\| m(w_1, \lambda) - m(w_2, \lambda) \| \le \eta \| w_1 - w_2 \|$$

$$\le \eta H(C(x, \lambda), C(y, \lambda))$$

$$\le \eta\lambda_C \| x - y \|. \tag{3.12}$$

By (3.5) - (3.12), we have

$$\| a - b \| \le [2\sqrt{1-2\delta+\lambda_G^2} + \sqrt{1-2\rho\alpha+\rho^2\beta^2\lambda_A^2} + \rho\xi\lambda_B + (\mu\lambda_D + 2\eta\lambda_C + \varepsilon\lambda_E)] \| x - y \|$$

$$= (k + t(\rho)) \| x - y \|$$

$$= \theta \| x - y \|. \tag{3.13}$$

where $k = 2\sqrt{1-2\delta+\lambda_G^2} + \mu\lambda_D + 2\eta\lambda_C + \varepsilon\lambda_E$, $t(\rho) = \sqrt{1-2\rho\alpha+\rho^2\beta^2\lambda_A^2} + \rho\xi\lambda_B$ and $\theta = k + t(\rho)$.

It follows from the condition (3.3) that $\theta < 1$. Hence, we have

$$d(a, F(y, \lambda)) = \inf_{b\in F(y,\lambda)} \| a - b \| \le \theta \| x - y \|.$$

Since $a \in F(x, \lambda)$ is arbitrary, we have

$$\sup_{a\in F(x,\lambda)} d(a, F(y, \lambda)) \le \theta \| x - y \|.$$

By using the same argument, we can prove

$$\sup_{b\in F(y,\lambda)} d(F(x, \lambda), b) \le \theta \| x - y \|.$$

By the definition of Hausdorff metric H on C (H), we obtain that for all $(x, y, \lambda) \in H \times H \times \Omega$,

$$H(F(x, \lambda), F(y, \lambda)) \le \theta \| x - y \|,$$

i.e., $F(x, \lambda)$ is a multi-valued contractive mapping which is uniform with respect to $\lambda \in \Omega$. By a fixed point theorem of Nadler[20], for each $\lambda \in \Omega$, $F(x, \lambda)$ has a fixed point $x(\lambda) \in H$, that is, $x(\lambda) \in F(x(\lambda), \lambda)$. By the definition of F, there exists $u(\lambda) \in A(x(\lambda), \lambda)$, $v(\lambda) \in B(x(\lambda), \lambda)$, $w(\lambda) \in C(x(\lambda), \lambda)$, $z(\lambda) \in D(x(\lambda), \lambda)$, $t(\lambda) \in E(x(\lambda), \lambda)$, $s(\lambda) \in G(x(\lambda), \lambda)$ such that

$x(\lambda) = x(\lambda) - s(\lambda) + m(w(\lambda), \lambda) + J_\rho^{M(.z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda), \lambda) - \rho N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda))$, and so

$$s(\lambda) = m(w(\lambda), \lambda) + J_\rho^{M(.z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda), \lambda) - \rho N(u(\lambda), v(\lambda), \lambda) + f(t(\lambda), \lambda)).$$

By Theorem 3.1; $x(\lambda) \in S(\lambda)$ is a solution of the PGNQVIP (2.1) and so $S(\lambda) \neq \phi$ for each $\lambda \in \Omega$.

(2) For each $\lambda \in \Omega$, let $x_n \subset S(\lambda)$ and $x_n \to x_0$ as $n \to \infty$. Then we have $x_n \in F(x_n, \lambda)$ for all n = 1, 2 ...By the proof of the conclusion 1, we have

$$H(F(x_n, \lambda), F(x_0, \lambda)) \le \theta \| x_n - x_0 \|$$

It follows that

$$d(x_0, F(x_0, \lambda)) \le \| x_0 - x_n \| + d(x_n, F(x_n, \lambda)) + H(F(x_n, \lambda), F(x_0, \lambda))$$

$$\leq (1+\theta)\,\|x_n - x_0\| \to 0, \text{ as } n\to\infty.$$

Hence, we have $x_0 \in F(x_0, \lambda)$ and $x_0 \in S(\lambda)$.

Therefore, $S(\lambda)$ is a nonempty closed subset of H.

Theorem 3.3: Under the hypothesis of Theorem 3.2, further assume

(i) for any $x \in H$, $\lambda \mapsto A(x, \lambda)$, $\lambda \mapsto B(x, \lambda)$, $\lambda \mapsto C(x, \lambda)$, $\lambda \mapsto D(x, \lambda)$, $\lambda \mapsto E(x, \lambda)$, $\lambda \mapsto G(x, \lambda)$, $\lambda \mapsto m(x, \lambda)$ are Lipschitz continuous (or continuous) with Lipschitz constants l_A, l_B, l_C, l_D, l_E, l_G and l_m, respectively;

(ii) for any u, v, z, w, $t \in H$, $\lambda \mapsto N(u, v, \lambda)$ and $\lambda \mapsto J^{M(.,z,\lambda)}_{\rho}(w)$ are Lipschitz continuous (or continuous) with Lipschitz constants l_N and l_j, respectively.

Then the solution set $S(\lambda)$ of the PGNQVIP(2.1) is a Lipschitz continuous (or continuous) mapping from Ω to H .

Proof: For each $\lambda, \bar{\lambda} \in \Omega$, by Theorem 3.2, $S(\lambda)$ and} $S(\bar{\lambda})$ are both nonempty closed subset. By the proof of Theorem 3.2, $F(x, \lambda)$ and $F(x, \bar{\lambda})$ are both multi-valued contractive mappings with same contraction constant $\theta \in (0, 1)$. By Lemma 2.2, we have

$$H(S(\lambda), S(\bar{\lambda})) \leq \frac{1}{1-\theta} \sup_{x\in H} H(F(x, \lambda), F(x, \bar{\lambda})). \tag{3.14}$$

Taking any $a \in F(x, \lambda)$, there exists $u(\lambda) \in A(x, \lambda)$, $v(\lambda) \in B(x, \lambda)$, $w(\lambda) \in C(x, \lambda)$, $z(\lambda) \in D(x, \lambda)$, $t(\lambda) \in E(x, \lambda)$, and $s(\lambda) \in G(x, \lambda)$ such that

$$a = x - s(\lambda) + m(w(\lambda),\lambda) + J_{\rho}^{M(.,z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda),\lambda) - \rho N(u << (\lambda <<), v(\lambda),\lambda) + f(t(\lambda),\lambda)).$$

Since $A(x, \lambda) \in C(H)$ and $A(x, \bar{\lambda}) \in C(H)$ there exists $u(\bar{\lambda}) \in A(x, \bar{\lambda})$ such that

$$\|u(\lambda) - u(\bar{\lambda})\| \leq H(A(x, \lambda), A(x, \bar{\lambda})).$$

Similarly there exists $v(\bar{\lambda}) \in B(x, \bar{\lambda})$, $w(\bar{\lambda}) \in C(x, \bar{\lambda})$, $z(\bar{\lambda}) \in D(x, \bar{\lambda})$, $t(\bar{\lambda}) \in E(x, \bar{\lambda})$, and $s(\bar{\lambda}) \in G(x, \bar{\lambda})$ such that

$$\|v(\lambda) - v(\bar{\lambda})\| \leq H(B(x, \lambda), B(x, \bar{\lambda})),$$
$$\|w(\lambda) - w(\bar{\lambda})\| \leq H(C(x, \lambda), C(x, \bar{\lambda})),$$
$$\|z(\lambda) - z(\bar{\lambda})\| \leq H(D(x, \lambda), D(x, \bar{\lambda})),$$
$$\|t(\lambda) - t(\bar{\lambda})\| \leq H(E(x, \lambda), E(x, \bar{\lambda})),$$
$$\|s(\lambda) - s(\bar{\lambda})\| \leq H(G(x, \lambda), G(x, \bar{\lambda})).$$

Let $b = x - s(\bar{\lambda}) + m(w(\bar{\lambda}),\bar{\lambda}) + J_{\rho}^{M(.,z(\lambda^{-}),\lambda^{-})}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}), v(\bar{\lambda}),\bar{\lambda}) + f(t(\bar{\lambda}),\bar{\lambda}))$,

Then it follows that

$$\begin{aligned}
\|a - b\| \leq{} & \|s(\lambda) - s(\bar{\lambda})\| + \|m(w(\lambda), \lambda) - m(w(\bar{\lambda}), \bar{\lambda})\| + \| j_{\rho}^{M(.,,Z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda),\lambda) - \rho N(u(\lambda),v(\lambda), \lambda) + f(t(\lambda), \lambda)) \\
& - j_{\rho}^{M(.,Z(\bar{\lambda}),\bar{\lambda})}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda}) + f(t(\bar{\lambda}),\bar{\lambda}))\| \leq \|s(\lambda) - s(\bar{\lambda})\| \\
& + \|m(w(\lambda), \lambda) - m(w(\bar{\lambda}),\bar{\lambda})\| + \| j_{\rho}^{M(.,,Z(\lambda),\lambda)}(s(\lambda) - m(w(\lambda), \lambda) - \rho N(u(\lambda),v(\lambda), \lambda) + f(t(\lambda), \lambda)) \\
& - j_{\rho}^{M(.,,Z(\lambda),\lambda)}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}), v(\bar{\lambda}),\bar{\lambda}) + f(t(\bar{\lambda}),\bar{\lambda})\| \\
& + \| j_{\rho}^{M(.,,Z(\lambda),\lambda)}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda}) + f(t(\bar{\lambda}),\bar{\lambda})) \\
& - j_{\rho}^{M(.,,Z(\lambda),\lambda)}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda}) \\
& + f(t(\bar{\lambda}),\bar{\lambda})) + \| j_{\rho}^{M(.,,Z(\bar{\lambda}),\lambda)}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda}) \\
& + f(t(\bar{\lambda}),\bar{\lambda})) - j_{\rho}^{M(.,,Z(\bar{\lambda}),\bar{\lambda})}(s(\bar{\lambda}) - m(w(\bar{\lambda}),\bar{\lambda}) - \rho N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda}) + f(t(\bar{\lambda}),\bar{\lambda})\|.
\end{aligned}$$

$$\leq 2\|s(\lambda)-s(\bar{\lambda})\| + 2\|m(w(\lambda),\lambda)- m(w(\bar{\lambda}),\bar{\lambda})\| + \rho\| N(u(\lambda),v(\lambda),\lambda) - N(u(\bar{\lambda}), v(\bar{\lambda}),\bar{\lambda})\|$$
$$+ \|f(t(\lambda),\lambda)- f(t(\bar{\lambda}),\bar{\lambda})\| + \mu\| Z(\lambda)- Z(\bar{\lambda})\| + l_j\|\lambda-\bar{\lambda}\|. \tag{3.15}$$

By the Lipschitz continuity of G in λ, we have

$$\| s(\lambda)- s(\bar{\lambda})\| \leq H(G(x,\lambda),G(x,\bar{\lambda})) \leq l_G\|\lambda - \bar{\lambda}\|. \tag{3.16}$$

By the Lipschitz continuity of m and C in $\lambda\in\Omega$, we have

$$\| m(w\lambda),\lambda) - m(w(\bar{\lambda}),\bar{\lambda})\| \leq \| m(w(\lambda),\lambda) - m(w(\bar{\lambda}),\lambda)\| + \| m(w(\bar{\lambda}),\lambda) - m(w(\bar{\lambda}),\bar{\lambda})\|$$
$$\leq \eta\|w(\lambda) - w(\bar{\lambda})\| + l_m\|\lambda-\bar{\lambda}\|$$
$$\leq \eta\, H(C(x,\lambda), C(x,\lambda)) + l_m\|\lambda-\bar{\lambda}\| \leq (\eta\, l_C+ l_m)\|\lambda - \bar{\lambda}\|. \tag{3.17}$$

By the Lipschitz continuity of N (u, v, λ), we have

$$\| N(u(\lambda),v(\lambda),\lambda) - N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda})\| \leq \| N(u(\lambda), v(\lambda),\lambda) - N(u(\bar{\lambda}),v(\lambda),\lambda)\|$$
$$\leq \| N(u(\bar{\lambda}),v(\lambda),\lambda) -N(u(\bar{\lambda}),v(\bar{\lambda}),\lambda)\|$$
$$\leq \|N(u(\bar{\lambda}),v(\bar{\lambda}),\lambda) - N(u(\bar{\lambda}),v(\bar{\lambda}),\bar{\lambda})\|$$
$$\leq \beta\|u(\lambda) - u(\bar{\lambda})\| + \xi\| v(\lambda) - v(\bar{\lambda})\| + l_N\|\lambda - \bar{\lambda}\|$$
$$\leq (\beta\lambda_A l_A + \xi\lambda_B l_B + l_N)\|\lambda - \bar{\lambda}\|. \tag{3.18}$$

By the Lipschitz continuity of f and E in $\lambda\in\Omega$, we have

$$\| f(t(\lambda),\lambda) - f(t(\bar{\lambda}),\bar{\lambda})\| \leq \|f(t(\lambda),\lambda) - f(t(\bar{\lambda}),\lambda)\|$$
$$\leq \|f(t(\bar{\lambda}),\lambda) - f(t(\bar{\lambda}),\bar{\lambda})\|$$
$$\leq \varepsilon\| t(\lambda) - t(\bar{\lambda})\| + l_f\|\lambda - \bar{\lambda}\|$$
$$\leq \varepsilon\, H(E(x,\lambda), E(x,\bar{\lambda})) + l_f\|\lambda - \bar{\lambda}\|$$
$$\leq (\varepsilon\, l_E + l_f)\|\lambda - \bar{\lambda}\|. \tag{3.19}$$

By the Lipschitz continuity of D, we have

$$\| z(\lambda) - z(\bar{\lambda})\| \leq H(D(x,\lambda), D(x,\bar{\lambda})) \leq l_D\|\lambda - \bar{\lambda}\|. \tag{3.20}$$

It follows from (3.15)-(3.20) that

$$\| a - b\| \leq [2(l_G+\eta\, l_C + l_m)+ \rho(\beta\lambda_A l_A + \xi\lambda_B l_B + l_N) + \mu\, l_D + l_J+ \varepsilon\, l_E+ l_f]\|\lambda - \bar{\lambda}\| = M\|\lambda - \bar{\lambda}\|,$$

where $M = 2(l_G+\eta\, l_C + l_m) + \rho(\beta\lambda_A l_A + \xi\lambda_B l_B + l_N) + \mu\, l_D+ l_J+ \varepsilon\, l_E+ l_f$

Hence, we obtain

$$\sup_{a\in F(x,\lambda)} d(a,F(x,\bar{\lambda})) \leq M\|\lambda-\bar{\lambda}\|.$$

By using a similar argument as above, we obtain

$$\text{Sup}_{\,b\in F(x,\,\bar{\lambda})}\, d(F(x, \lambda), b) \le M \,\|\, \lambda - \bar{\lambda} \,\|.$$

It follows that

$$H(F(x, \lambda), F(x, \bar{\lambda})) \le M \,\|\, \lambda - \bar{\lambda} \,\|.$$

By Lemma 2.2, we obtain

$$H(S(\lambda), S(\bar{\lambda})) \le \frac{1}{1-\theta} \,\|\, \lambda - \bar{\lambda} \,\|.$$

This proves that $S(\lambda)$ is Lipschitz continuous in $\lambda \in \Omega$. If, each mapping in conditions (i) and (ii) is assumed to be continuous in $\lambda \in \Omega$, then by similar argument as above, we can show that $S(\lambda)$ is also continuous in $\lambda \in \Omega$.

Remark.3.4: Since the PGNQVIP(2.1)includes many parametric(generalized) quasi-variational inclusions and parametric(generalized) nonlinear implicit quasi-variational inequalities as special cases, Theorem 3.2 and 3.3 improve and generalize the known results in [2 ,3, 9, 13, 17, 19, 22, 25, 27, 28, 29].

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