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On Texture Fuzzy α open sets and Texture Fuzzy α- Closedsets in Fuzzy Ditopological Texture Spaces

A. A. Nithya*[1] & I. Arockia Rani[2]

[1]Department of Mathematics, Nirmala College, Coimbatore-641 018, India
[2]Department of Mathematics, Nirmala College, Coimbatore-641 018, India

E-mail: aanithyajerry@gmail.com

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ABSTRACT

In this paper the two main pillars of mathematics such as Fuzzy and Di- topological texture spaces are united to give a new form as Fuzzy Ditopological Texture spaces. Here we characterize many features of Fuzzy α-T-open set and Fuzzy α-T-closed set in the light of Fuzzy Ditopological texture spaces. And also we analyse some properties and several characterizations of Fuzzy α-open sets and Fuzzy α-closed sets in the fuzzy ditopological texture spaces.

Keywords: *Fuzzy Texture spaces, Fuzzy Ditopology, Fuzzy Ditopological Texture spaces, Texture fuzzy α-open sets and Texture fuzzy α-closed sets.*

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1. Introduction

Textures were introduced by L. M. Brown [2] as a point-set in 1998. Textures offers a convenient setting for the investigation of complement-free concepts in general, so much of the recent work has proceeded independently of the fuzzy setting.

Definition.1.1: [5] Ditopological Texture Spaces: Let S be a set, a texturing T of S is a subset of P(S). If

(1) (T, ⊆) is a complete lattice containing S and φ, and the meet and join operations in (T,⊆) are related with the intersection and union operations in (P(S),⊆)by the equalities $\wedge_{i\in I} A_i = \cap_{i\in I} A_i$, $A_i \in T$, $i \in I$, for all index sets I, while $\vee_{i\in I} A_i = \cup_{i\in I} A_i$, $A_i \in T$, $i \in I$, for all index sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given $s_1 \neq s_2$ in S we have $A \in T$ with $s_1 \in A$, $s_2 \notin A$, or $A \in T$ with $s_2 \in A$, $s_1 \notin A$.

If S is textured by T we call (S, T) a texture space or simply a texture.

Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology or ditopology, namely a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies
1. S, φ ∈ τ,
2. $G_1; G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$ and
3. $G_i \in \tau$, $i \in I$ then $\vee_i G_i \in \tau$, and the set of closed sets κ satisfies
1. S, φ ∈ κ
2. $K_1; K_2 \in \kappa$ then $K_1 \vee K_2 \in \kappa$ and

***Corresponding author: A. A. Nithya*[1], *E-mail: aanithyajerry@gmail.com**

3. $K_i \in \kappa$, $i \in I$ then $\cap K_i \in \kappa$. Hence a ditopology is essentially a 'topology" for which there is no a priori relation between the open and closed sets.

In 1965, L.A. Zadeh [19] laid the foundation of fuzzy set theory. The subsequent de- velopment of it and its allied topics provide a natural frame-work for generalizing the concepts of general topology which is called fuzzy topological spaces. C.L. Chang [8] introduced the concept of fuzzy topological space in 1968. In later stage Lowen [9] modified the definition of Chang changing the first condition. Many authors like [8, 11, 12, 13, 14] made their researches in fuzzy topology.

Throughout this paper, X represents a nonempty set and fuzzy subset A of X, denoted by $A \leq X$, then is characterized by a membership function in the sense of Zadeh [19]. The basic fuzzy sets are the empty set, the whole set and the class of all fuzzy sets of X which will be denoted by 0, 1 and I^X, respectively. A subfamily τ of I^X is called a fuzzy topology due to Chang [8]. Moreover, the pair (X, τ) will be meant by a fuzzy topological space, on which no separation axioms are assumed un- less explicitly stated. The fuzzy closure, the fuzzy interior and the fuzzy complement of any set A in (X,τ) are denoted by Cl(A), Int(A) and 1-A, respectively. A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is $\lambda \in (0, 1]$ is denoted by x^λ where the point x is called its support. Also, for a fuzzy point x^λ and a fuzzy set A we shall write $x^\lambda \in A$ to mean that $\lambda \leq A(x)$. The value of a fuzzy set A for some $x \in X$ will be denoted by A(x).

Definition 1.2: For a Fuzzy topological space (S, T, τ, κ)

1. $A \in S$ is called fuzzy pre-open (resp.fuzzy semi-open, fuzzy β-open) if $A \leq$ intclA (resp.$A \leq$ clintA, $A \leq$ clintclA).

2. $B \in S$ is called fuzzy pre-closed (resp. fuzzy semi-closed, fuzzy β-closed) if clint$B \leq B$ (resp. intcl$B \leq B$; ntclint$B \leq B$)

Definition 1.3: 14.A fuzzy topological space (X, τ) is called fuzzy T_0 space if for every pair of fuzzy points x_t, y_r in X(x= y) there exists $U \in FO(X,\tau)$ such that $x_t q U \leq 1 - y_r$ or $y_r q U \leq 1 - x_t$.

2. Texture fuzzy α-open and Texture fuzzy α-closed sets

Definition 2.1: Fuzzy Texture Space: Let S be a set , a texturing T of S is a subset of P(S) which is complete, completely distributive lattice containing 1 and 0 functions and also point seperating, (i.e) For every pair of fuzzy points x_t, y_r in S (x = y) there exist $U \in T$ such that $x_t q U \leq 1 - y_r$ or $y_r q U \leq 1 - x_t$. Then the pair (S,T) is said to be Fuzzy texture space.

Definition 2.2: Fuzzy Ditopological texture space: Let (S,T) be the fuzzy tex- ture space, then τ be the collection of fuzzy sets from T which satisfies,
1. $1, 0 \in \tau$,
2. $G_1; G_2 \in \tau$ then $G_1 \wedge G_2 \in \tau$ and
3. $G_i \in \tau$, $i \in I$ then $\vee_i G_i \in \tau$, and κ be the set of fuzzy closed sets from T which satisfies
1. $1, 0 \in \kappa$
2. $K_1; K_2 \in \kappa$ then $K_1 \vee K_2 \in \kappa$ and
3. $K_i \in \kappa$, $i \in I$ then $\wedge K_i \in \kappa$. Then the four tupple (S, T, τ, κ) is said to be fuzzy ditopological texture space.

Definition 2.3: Let $(S; T; \tau, \kappa)$ be fuzzy ditopological texture space and $A \in T$. (1) If $A \leq int(cl(int(A)))$ then A is Texture Fuzzy α open(TF-α open).
(2) If $cl(int(cl(A))) \leq A$ then A is Texture Fuzzy α closed(TF-α closed).

We denote by TFαO (S, T, τ, κ), or when there can be no confusion by TFαO(S), the set of TF-α open sets in T. Likewise, TFαC $(S; T; \tau, \kappa)$, or TFαC(S) will denote the set of TF-α closed sets.

Proposition 2.4: For a given Fuzzy ditopological texture space $(S; T; \tau, \kappa)$:

1. $FO(S) \leq TF\alpha O(S)$ and $FC(S) \leq TF\alpha C(S)$

2. Arbitrary join of TF-α open sets is TF-α open.

3. Arbitrary intersection of TF-α closed sets is TF-α closed.

Proof: (1) Let G∈FO(S). Since intG = G we have G≤int(cl(int(G))). Thus G∈TFαO(S). Secondly, let K∈FC(S). Since clK = K we have cl(int(cl(K))) ≤ K and so K ∈ TFαC(S). (2) Let $\{A_j\}_{j\in J}$ be a family of TF-α open sets. Then for each j∈ J, Aj ≤ int(cl(int(A_j))). Now, $\vee A_j$ ≤ ∨ int(cl(int(A_j))) ≤ int∨ cl(int(Aj)) = int(cl∨int(Aj))=int(cl(int$\vee A_j$). Hence $\vee A_j$ is a TF-α open set. The result (3) is dual of (2).

Generally there is no relation between the TF-α open and TF-α closed sets.

Examples 2.5: The following is an example to show that a fuzzy α closed set in fuzzy topological space and TF-α closed set in fuzzy ditpological texture space are independent concepts even though if we take same topology.

Let X={a, b}, I: X → [0,1], T={0, 1, A, B} where 0(a)=0, 0(b)=0:1(a)=1,1(b)=1: A(a)=0,A(1)=1: B(a)=1, B(b)=0 then (X,T) be the texture space, let τ = {0, 1, A} and κ = {0, 1} then B is fuzzy α closed but B is not TF- α closed.

Similarly we can show that a fuzzy α open set in fuzzy topological space and TF-α open set in fuzzy ditpological texture space are independent concepts.

This is so since the closed set topology are different, but this will be solved when we take complemented fuzzy ditopological texture space.

Definition 2.6: A mapping σ:T → T is said to be complementation on (S,T) if κ = σ(τ), then (S, T, σ, τ, κ) is said to be a fuzzy complemented ditopological texture space. The ditopology (τ, τ^c) is clearly complemented for the complementation σ_X: P (X) → P (X) given by σ_X (Y) = X¥Y.

Proposition 2.7: For a complemented fuzzy ditopological texture space (S; T; σ, τ, κ): A ∈ T is TF-α-open if and only if σ(A) is TF-α-closed.

Proof: It is clear from the definition.

Definition 2.8: Let (S; T; τ, κ) be a fuzzy ditopological texture space. For A ∈ T, we define.

1. The TF-α closure clα(A) of A under (τ, κ) by the equality cl_α(A) = ∩{B/B ∈ TF-αC(S) and A≤B}.

2. The TF-α interior intα (A) of A under (τ, κ) by the equality intα (A) =∨ {B/B∈ TFαO(S) and B ≤ A}.

From the Proposition 2.4, it is obtained, int_α(A) ∈ TFαO(S), clα(A)∈ TFαC(S).

Proposition 2.9: Let (S; T; τ, κ) be a fuzzy ditopological texture space. Then the following are true.

[1] cl_α(φ) = φ
[2] cl_α(A) is TF-α closed, for all A ∈ T.
[3] If A ≤B then cl_α(A) ≤ cl_α(B), for every A,B∈ T
[4] cl_α(cl_α(A)) = cl_α(A)

Definition 2.10: For a fuzzy ditopological texture space (S; T; τ, κ):

1. A ∈T is called TF-preopen (resp. TF-semi open, TF-β open) if A ≤ int(cl(A)) (resp. A ≤ clintA; A ≤ cl(int(cl(A))).

2. B ∈ T is called TF-pre closed (resp. TF-semi closed, TF-β closed) if cl(int(B)) ≤B (resp. int(cl(B)) ≤ B;int(cl(int(B))) ≤ B

Theorem 2.11: For a fuzzy ditopological texture space (S; T; τ, κ): [1] Every TF-α open is TF-preopen.

[2] Every TF-α open is TF-semi open. [3] Every TF-preopen is TF-βopen.
[4] Every TF-semi open is TF-β open. [5] Every TF-α-open is TF-β-open.

Proof: The proof is obivious.

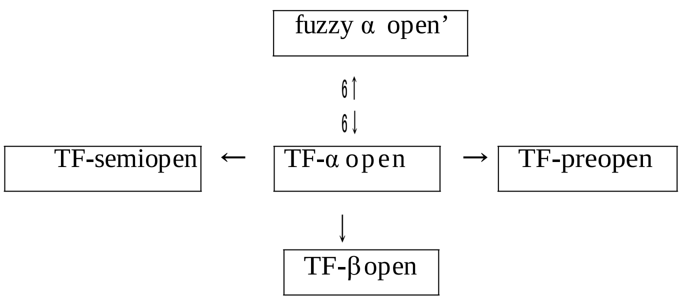
Remark 2.12: For a fuzzy ditopological texture space (S; T; τ, κ) the converse of the above results need not be always true.

Example 2.13: Let X={a, b}, I :X → [0,1], T= {0, 1, A, B} where 0(a)=0,0(b)=0:1(a)=1,1(b)=1: A(a)=0,A(1)=1: B(a)=1,B(b)=0 then T be the texture space, let τ = {0, 1, A} and κ = {0, 1} then B is TF-preopen but B is not TF-α open.

Remark.2.14: 1) If κ (the closed set topology) = indiscrete fuzzy topology, i.e) 0, 1 then every element in T is TF-α-open (i. e) T = TFαO(X).

2) If τ (the open set topology) = indiscrete fuzzy topology, i.e) 0, 1 then every ele- ment in T is TF-α-closed(i.e)T=TFαC(X).

Remark.2.15: The results in Proposition 2.8. is shown in the following figure.



Proposition 2.16: Every fuzzy texture space is fuzzy T_0 space.

But the converse need not be true always. This is because fuzzy texture space is completely distributive but fuzzy T_0 space is not completely distributive always.

Example 2.17: Let X={a, b}, τ = {0, 1, A, B, C, D, E} where 0(a)=0,0(b)=0; 1(a)=1,1(b)=1; A(a)=0,A(b)=1;B(a)=1,B(b)=0; C(a)=.5,C(b)=.5; D(a)=0,D(b)=.5; E(a)=1,E(b)=.5.

This is fuzzy T_0 space but fuzzy texture space. Because D∨ (C∧B) =D∧D=D, but (D∨C) ∧ (D∨B) = C∧E= C which are not equal.

Theorem 2.18: A fuzzy topological space (X, τ) is fuzzy T_0 space if and only if for every pair of fuzzy points x_t, y_r in X (x = y), cl(x_t) = cl(y_r).

Theorem 2.19: Every fuzzy ditopological space (X, τ, κ) is fuzzy T_0 space. But every pair of fuzzy points x_t, y_r in X (x = y), cl(x_t) = cl(y_r) is not true.

Proof: Suppose that (X, τ, κ) is fuzzy T_0 space, then for every pair of fuzzy points x_t, y_r in X (x = y) there exist U ∈ FO(X, τ) such that x_t qU ≤ $1 - y_r$ or y_r qU ≤ $1 - x_t$. If x_t qU ≤$1 - y_r$ then U ≤$1 - y_r$. (i.e) y_r ≤ 1 − U, But here we cannot say 1-U is always closed, since it depends on closed topology.

3. TF-α continuous functions and TF-α cocontinuous functions

Definition 3.1: Let (Sj; Tj; τ j, κj), j = 1; 2; be fuzzy ditopological texture spaces and (f; F): (S1; T1,τ 1, κ1) → (S2; T2,τ 2, κ2) a difunction.

1. It is called TF-α continuous, if F (G) is TF-α open, for every G ∈ FO(τ 2).
2. It is called TF-α cocontinuous, if $f^{\leftarrow}$ (K) is TF-α closed, for every K ∈ FC(κ2).
3. It is called TF-α bicontinuous, if it is TF- α continuous and TF-α cocontinuous.

Definition 3.2: A difunction (f; F): (S1; T1; τ_1, κ_1) → (S2; T2; τ_2, κ_2) is called

1. TF-pre continuous (resp. TF-semi continuous, TF- β continuous) if $F^{\leftarrow}$ (G) ∈ TFPO(T) (resp. $F^{\leftarrow}$(G) ∈ TFSO(T); $F^{\leftarrow}$(G) ∈ TFβO(T)) for every G∈ FO(T).

2. It is called TF-pre cocontinuous (resp. TF-semi cocontinuous, TF-β cocontin- uous) if $f^{\leftarrow}$(K) ∈ TFPC(T) (resp. $f^{\leftarrow}$(K) ∈ TFSC(T); $f^{\leftarrow}$(K) ∈ TFβ C(T)) for every K ∈ FC(T).

Remark.3.3: Sufficient A difunction (f; F) : (S1; T1; τ_1, κ_1) → (S2; T2; τ_2, κ_2)

1. If we take κ1 to be indiscrete topology, then every function TF-α continuous.

2. If we take τ1 to be indiscrete topology, then every function TF-α cocontinuous.

3. If we take both τ1,κ1 to be indiscrete topology, then every function TF-α bi- continuous.

Theorem 3.4: Let (Sj; Tj; τj, κj; σj), j = 1; 2, complemented ditopology and (f; F): (S1; T1) → (S2; T2) be complemented difunction. The following are equivalent:

[1] (f; F) is TFα continuous.
[2] (f; F) is TFα cocontinuous.
[3] $f^{\rightarrow}$(clintclA) ≤ cl($f^{\rightarrow}$(A)) for each A ∈ T1 [4] clintcl($f^{\leftarrow}$(B)) ≤ $f^{\leftarrow}$(clB) for each B ∈ T2

(2) The following are equivalent:

[1] (f; F) is TF-α cocontinuous.
[2] (f; F) is TF-α continuous.
[3] int($F^{\rightarrow}$(A)) ≤ $F^{\rightarrow}$(intclintA) for each A ∈ T1.
[4] $F^{\leftarrow}$(intB) ≤ intclint($F^{\leftarrow}$(B)) for each B ∈ T2.

Proof: (1) ⇒ (2) Since (f; F) is complemented, (F '; f ') = (f; F). From [4, Lemma 2.20], σ1((f ') $^{\leftarrow}$(B)) = $f^{\leftarrow}$(σ(B)) and σ 1((F ')$^{\leftarrow}$(B)) = $F^{\leftarrow}$(σ(B)) for all B ∈ S2. Hence the proof is clear from these equalities.

(2) ⇒ (3) Let A ∈ T1. Then cl$f^{\rightarrow}$(A) is closed set in (S2; T2). From (2) $f^{\leftarrow}$(cl$f^{\rightarrow}$(A)) is TF-α closed in (S1; T1). Hence we have clintclA ≤ clintcl$f^{\leftarrow}$(cl$f^{\rightarrow}$A) ≤ $f^{\leftarrow}$cl($f^{\rightarrow}$A). So then $f^{\rightarrow}$(clintclA) ⊂ cl($f^{\rightarrow}$A).

(3) ⇒ (4) Let B ∈ T2. Then $f^{\leftarrow}$(B) ∈ T1 and by hypothesis we have $f^{\rightarrow}$(clintcl(f←(B)) ⊂ cl($f^{\rightarrow}$($f^{\leftarrow}$(B))). Thus, $f^{\rightarrow}$(clintcl($f^{\leftarrow}$(B))) ≤ clB and clintcl($f^{\rightarrow}$(B)) ≤ $f^{\leftarrow}$($f^{\rightarrow}$(clintcl($f^{\leftarrow}$(B)))) ≤ $f^{\leftarrow}$(clB).

(4) ⇒ (1) Let G ∈ FO(T2) and K = σ ∈(G). By (4) $f^{\rightarrow}$(clintcl($f^{\leftarrow}$(K))) ≤ clK = K, since K ∈ FC(T2). That is clintcl($f^{\leftarrow}$(σ ∈(G))) ∈ $f^{\leftarrow}$(σ ∈(G)) or by [4, Lemma 2.20], clintcl(σ1((f ')←(G)) ≤ σ 1((f '(←G). Thus (f ')$^{\leftarrow}$(G) ≤ intclint(f ')$^{\leftarrow}$(G) and we have $F^{\leftarrow}$(G) ≤ intclint$F^{\leftarrow}$(G) since (f; F) is complemented. Hence $F^{\leftarrow}$(G) is TF-α open set.

similarly we can prove the dual.

Corollary 3.6: Let (f; F): (S1; T1; τ 1; κ 1) → (S2; T2; τ 2; σ 2) be a difunction. (1) If (f; F) is TF-α continuous then:

[1] $f^{\rightarrow}$(clA) ≤ cl($f^{\rightarrow}$(A)), for every A ∈ TFPO(T1). [2] cl($f^{\leftarrow}$(B)) ≤ $f^{\leftarrow}$(clB), for every B ∈ FO(T2).

(2) If (f; F) is TF-α cocontinuous then:

[1] int($F^{\rightarrow}$(A))≤ $F^{\rightarrow}$(intA) for every A ∈ TFPC(T1). [2] $F^{\leftarrow}$(intB) ≤ int($F^{\leftarrow}$B), for every B ∈ FC(T2).

Proof: (1) Let A ∈ TFPO(T1). Then clA ≤ clintclA and so $f^{\rightarrow}$(clA) ≤ f →(clintclA). From Theorem 3.4(1)-(4), we have, $f^{\rightarrow}$(clA) ≤ cl($f^{\rightarrow}$(A)).

(b) Let B ∈ FO(S2). From the assumption, $f^{\leftarrow}$(B) is TF-α open and by Remark 2.3, $f^{\leftarrow}$(B)∈FPO(T1).

Hence, $f^{\leftarrow}$(B)≤intcl($f^{\leftarrow}$(B)) and so cl($f^{\leftarrow}$(B))≤clintcl($f^{\leftarrow}$(B)). From Theorem 3.4(1)-(4), we have cl($f^{\leftarrow}$(B)) ≤ $f^{\leftarrow}$(clB).

Theorem 3.7: For a fuzzy ditopological texture space (S; T; τ, κ):

[1] Every TF-α continuous is TF-pre continuous. [2] Every TF-α continuous is TF-semi continuous. [3] Every TF-pre continuous is TF-β continuous. [4] Every TF-semi continuous is TF-β continuous.

Proof: The proof is obivious from Theorem 2.8.

Remark: fuzzy contra-TF-α-continuity (fuzzy cocontra-TF-α-continuity) and fuzzy TF-α-continuity(fuzzy TF-α-cocontinuity) are independent.

Remark: Composition of two fuzzy contra-TF-α-continuity need not be fuzzy contra TF-α-continuity.

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