International Journal of Mathematical Archive-3(3), 2012, Page: 1254-1256
IJMA Available online through www.ijma.info ISSN 2229 – 5046

A NOTE ON MATRIX NEAR - RINGS CONTAINING ALL DIAGONAL MATRICES

S. Bhavani Durga

Chalamaiah junior college, Guntur, India

Tumurukota venkata Pradeep Kumar*

Asstt. Prof. in Mathematics, ANU college of engineering and technology, Acharya Nagarjuna University

E-mail: *pradeeptv7@rediffmail.com*

(Received on: 23-11-11; Accepted on: 13-12-11)

ABSTRACT

In this paper we studied the structure being the same in particular for all fields R, Matrix near-rings between $D_n(R)$ and $M_n(R)$ correspond to certain 0 – 1 matrices of size n x n via the incidence algebra construction, independently of the field R. The Noetherian and Artinian properties of R are also reflected in the lattice structure of intermediate rings.

Key words: *Near ring, Matrix ring, matrix near ring, matrix sub near ring.*

AMS Classification: *16 Y 30.*

1. INTRODUCTION

In recent decades Near – Rings play a vital role in the development of abstract algebra. Near-rings seems to have reluctantly concluded that in most cases it does not make sense to speak about a near ring of matrices over an arbitrary Near – Ring.

In 1986, JDP Meldrum and APJ Van Der Walt [5] introduced the concept of Matrix near rings. The n x n matrix near – ring over a near ring R is denoted by $M_n(R)$.

In this paper matrix near-rings containing all diagonal matrices over any coefficient near ring R, correspond bijectively to certain matrices whose entries are ideals of R. The characteristic property of these matrices with ideal entries involves a generalization of idempotency and transitivity. As a consequence, The order structure of the segment of the sub-ring lattice of the full matrix near ring $M_n(R)$ which is above the Near – ring $D_n(R)$ of diagonal matrices, depends only on the ordered monoid structure of all ideals of R.

2. PRELIMINARIES

In this section we have given the definitions, examples and required literature which is used in the later sections.

2.1 Definition: A Near – Ring is a set R together with two binary operations "+" and "." Such that (i) (R, +) is a Group not necessarily abelian; (ii) (R,) is a semi Group and (iii) for all n_1, n_2, $n_3 \in$R, $(n_{1+} n_2).n_3=$ $(n_{1.} n_{3\,+}\, n_{2.}\, n_3)$ i.e. right distributive law.

2.2 Example: Let Z be the set of positive and negative integers with 0. (Z, +) is a group. Define '.' on Z by a . b = a for all a, b $\in$ Z. Clearly (Z, +.) is a near-ring.

2.3 Example*:* Let Z12 = {0, 1, 2 ,…, 11}. (Z12, +) is a group under '+' modulo 12.Define '.' on Z12 by a. b = a for all a $\in$ Z12. Clearly (Z12, +,) is a near-ring.

2.4 Definition: Let R be a ring with identity 1$\neq$ 0. The ring of n x n matrices over ring R is denoted by $M_n(R)$ is known as matrix ring.

***Corresponding author:** *Tumurukota venkata Pradeep Kumar**, ***E-mail:** *pradeeptv7@rediffmail.com*

2. 5 Examples: Let M_{2x2} ={ {(aij) / Z ; Z is treated as a near-ring}. M_{2x2} under the operation of matrix addition '+' and matrix multiplication

Now we give the definition of a matrix near-ring according to JDP Meldrum and APJ Vander walt [5].

Let R be a Near-ring and R^n the direct sum of n-copies of the group(R, +). Then R^n is also a Near – ring as usual way.

We denote M (R^n), the set of all mappings from R^n to it self. Then M (R^n) is a Near-ring with point-wise addition and composition of mappings.

Let ε_j = {0,0,0,…,1, 0,0,..,0} (jth position is '1' and the other positions are zero). We define the following mappings:
(i) f^r: R → R by $f^r(s)$ = rs for all s ∈ R;
(ii) f^r_{ij}: $R^n \to R^n$ by $f^r_{ij} = t_i f^r \pi_j$ for all i, j = 1, 2, …,n and all r ∈ R.

2.6 Definition: The near – ring of n x n matrices over R, denote by $M_n(R)$, is the sub near-ring of M (R^n) generated by the set {f^r_{ij} / r ∈ R , i, j = 1,2,…,n}. Here the elements of $M_n(R)$ will be referred as n x n matrices over R.

The next two propositions are immediate:

2.7 Proposition [5]: $M_n(R)$ is a right near-ring with identity.

2.8 Proposition [5]: If R is a ring with identity, then $M_n(R)$ is isomorphic to the ring of n x n matrices over R.

3. DIAGONAL MATRICES

Let R be a ring with unit, not necessarily commutative, I(R) the set of its ideals, M_n(R) the near - ring of n x n matrices with coefficients in R, n ≥ 1 and D_n(R) its sub near - ring of diagonal matrices.

Define Inc P = {J ∈ $M_n(R)$: J(i, j) ∈ P(I, j) for all i, j}, where P be a matrix and Inc P is an additive sub group of M_n(R) and it is a sub- near ring if and only if for all I, j we have

P (I, k). P (k, j) ⊆ P (I, j).

This property is equivalent to the condition that P^2 (i, j) ⊆ P (i, j) for all I, j where P^2 is the matrix product P.P computed with respect to the ideal sum and product.

We call such matrices P aresub- idempotent.

The set Inc P contains $D_n(R)$ if and only if P (i, j) = (1) for all 1 ≤ i ≤ n. Such matrices are called Unit – diagonal.

3.1 Proposition: For every ring R with unity, the sub-rings are of $M_n(R)$ containing $D_n(R)$ are precisely the rings IncP for n x n unit diagonal sub – idempotent matrices P with entries in I(R).

Proof: Let IncP for n x n unit diagonal sub idempotent matrices P with entries in I(R). So it is clear that if P is Unit – diagonal and sub idempotent, then IncP is a sub near-ring of $M_n(R)$ containing D_n (R).

Conversely, for an intermediate ring T, such that $D_n(R) \subseteq T \subseteq M_n(R)$.

Now consider for each couple of indices I, j the set {A (i, j): A∈ T} which is an ideal of R because T is a sub near – ring containing all diagonal matrices.

Let the n x n matrix P with ideal entries be defined by

P (i, j) = {A (i, j): A∈ T}

Clearly P is a matrix sub near – ring of $M_n(R)$.

Let A, B ∈ T with A (i, k) = a and B (k, j) = b.

Define E_t is an n x n diagonal matrix which has all elements entries equal to 0 except that the entry E(t, t) is 1 for t = 1, 2, …, n.

Clearly the element in (i, j) is ab and all the other entries are 0's in the matrix $E_iAE_k^2BE_j$.

Since T contains the diagonal matrices E_i, E_j, E_k, so we have the element ab in P(i, j).

Hence P (i, k) P (k, j) $\subseteq$ P (i, j) and P is sub – idempotent.

Also P is unit diagonal, since T contains all n x n diagonal matrices.

So it is clear that IncP = T.

Hence the proof.

From the above proposition we get the following corollaries.

3.1 Corollary: Let R_1 and R_2 be rings with unit such that there is an inclusion – preserving monoid isomorphism between I(R_1) and I(R_2). Then the indices [D_n(R_1), M_n(R_1)] and [D_n(R_2), M_n(R_2)] are isomorphic for all n $\geq$1.

The above corollary is also true if R_1, and R_2 are both division rings.

3.3 Corollary: Let n $\geq$1. For all division rings R, [D_n (R), M_n(R)] has the same lattice structure.

REFERENCES

[1] Atsushi Nakajima "Derivations on Matrix Near-ring", J. of the faculty of environmental science and Technology, Okayama University, Vol. 3, No. 1 (1998), PP 1 – 3.

[2] E. Stanley, Enumerative Combinatorics, Volume 1, Cambridge U. Press 1997

[3] G. Pilz "Near – Rings", Amsterdam, 1983.

[4] G.D. Abrams, The recovery question for local incidence rings, Stud. Sci. Hung. 23, No1-2, (1988), PP 195-201.

[5] JDP Meldrum and APJ Vander walt "Matrix Near-rings", Arch. Math., Vol.47, (1986), PP 312 - 319.

****

