International Journal of Mathematical Archive-3(3), 2012, Page: 1222-1230
Available online through www.ijma.info ISSN 2229 – 5046

δg^* - Closed sets in topological spaces

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(Received on: 27-02-12; Accepted on: 21-03-12)

ABSTRACT

In this paper a new class of sets, namely δg^*-closed sets and δg^*-open sets are introduced and studied in topological spaces. We prove that the class of δg^*-closed sets lies between the class of δ-closed sets and the class of δg -closed sets. Also we find some relations between δg^*-closed sets and already existing closed sets.

Keywords and phrases: *generalized closed sets, δ-closure, δg-closed sets and g-open sets.*

Mathematics Subject classification: *54A05.*

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1. INTRODUCTION:

The concept of generalized closed sets plays a significant role in topology. There are many research papers which deal with different types of generalized closed sets. Levine [10] introduced generalized closed (briefly g-closed) sets and studied their basic properties. Bhattacharya and Lahiri [3], Arya and Nour [2], Maki et al [12,13], Dontchev and Ganster [5], Maragathavalli et al [16] and VeeraKumar [28] introduced semi-generalized closed sets, generalized semi-closed sets, generalized α-closed sets, α-generalized closed sets, δ-generalized closed sets, $s\alpha g^*$-closed sets and $g^{\#}s$-closed sets respectively. VeeraKumar [30] introduced g^*-closed sets in topological spaces. The purpose of this present paper is to define a new class of closed sets called δg^*-closed sets and also we obtain the basic properties of δg^*-closed sets in topological spaces. Applying this set, we obtain the new spaces which are called $T_{\delta g^*}$-spaces and $\#T_{\delta g^*}$-spaces.

2. PRELIMINARIES:

Throughout this paper (X,τ) (or simple X) represents topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition: 2.1. A subset A of a space (X,τ) is called a

(i) semi-open set [9] if $A \subseteq cl(int(A))$.
(ii) α-open set [18] if $A \subseteq int(cl(int(A)))$.
(iii) regular open set [22] if $A = int(cl(A))$.
(iv) Pre-open set [15] if $A \subseteq int(cl(A))$.

The complement of a semi open (resp. α-open, regular open, pre-open) set is called semi-closed (resp. α-closed, regular closed, pre-closed).

The semi-closure [4] (resp.α-closure [18], pre-closure [15]) of a subset A of (X,τ), denoted by scl(A) (resp. $cl_\alpha(A)$, pcl(A)) is defined to be the intersection of all semi-closed (resp.α-closed, pre-closed) sets containing A. It is known that scl(A) (resp. $cl_\alpha(A)$, pcl(A)) is a semi-closed (resp.α-closed, pre-closed) set.

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Definition: 2.2. The δ-interior [31] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $int_\delta(A)$. The subset A is called δ-open [31] if A = $int_\delta(A)$. i.e., a set is δ-open if it is the union of regular open sets, the complement of a δ-open is called δ-closed. Alternatively, a setA ⊆ X is called δ-closed [29] if A = $cl_\delta(A)$, where $cl_\delta(A) = \{x \in X ; int(cl(U)) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$. Every δ-closed set is closed [31].

Definition: 2.3. A subset A of (X, τ) is called

1) δ-generalized closed (briefly δg-closed) [5] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) $\hat{g}$-closed [27] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X,τ).
3) g^*- closed [30] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
4) generalized closed (briefly g-closed) [10] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
5) semi-generalized closed (briefly sg-closed) [3] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
6) generalized semi-closed (briefly gs-closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
7) α-generalized closed (briefly αg-closed) [13] if αcl(A)⊆U whenever A⊆U and U is open in (X, τ).
8) generalized α closed (briefly gα-closed) [12] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
9) αg^*-closed [17] if cl(A) ⊆ U whenever A ⊆ U and U is α - open in (X, τ).
10) $\alpha\hat{g}$-closed [1] if αcl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$ open in (X, τ).
11) $g^{\#}s$-closed [28] if scl(A) ⊆ U whenever A ⊆ U and U is αg-open in (X, τ).
12) $s\alpha g^*$-closed [16] if αcl(A) ⊆ U whenever A ⊆ U and U is g^*-open in (X,τ).
13) $\ddot{g}$-closed [21] if cl(A) ⊆ U whenever A ⊆ U and U is sg-open in (X,τ).
14) g''-closed [19] if cl(A) ⊆ U whenever A ⊆ U and U is gs-open in (X,τ).
15) gp- closed [14] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
16) g^*p- closed [26] if pcl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
17) $^*g$- closed [30] if cl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open in (X, τ).
18) $^{\#}gs$-closed [29] if scl(A) ⊆ U whenever A ⊆ U and U is $^*g$-open in (X, τ).
19) $\tilde{g}$- closed [6] if cl(A) ⊆ U whenever A ⊆ U and U is $^{\#}gs$-open in (X, τ).
20) $\tilde{g}_\alpha$- closed [7] if αcl(A) ⊆ U whenever A ⊆ U and U is $^{\#}gs$-open in (X, τ).
21) $\tilde{g}_s$- closed [24] if scl(A) ⊆ U whenever A ⊆ U and U is $^{\#}gs$-open in (X, τ).
22) $\tilde{g}_p$- closed [8] if pcl(A) ⊆ U whenever A ⊆ U and U is $^{\#}gs$-open in (X, τ).
23) g^*s-closed [20] if scl(A) ⊆ U whenever A ⊆ U and U is gs-open in (X, τ).
24) $\delta\hat{g}$-closed [11] if $cl_\delta(A) \subseteq U$ whenever A ⊆ U and U is $\hat{g}$-open in (X, τ).
25) $s\delta g^*$ - closed [23] if $cl_\delta(A) \subseteq U$ whenever A ⊆ U and U is g^*-open in (X, τ).

The complements of the above mentioned sets are called their respective open sets.

Definition: 2.4. A space X is called

1) a $T_{1/2}$-space [10] if every g-closed set in it is closed.
2) a $T_{3/4}$-space [5] if every δg-closed set in it is δ-closed.

3. δg^* -CLOSED SETS

We now introduce δg^* - closed sets in topological spaces and study some relations between δg^*- closed sets and other existing closed sets.

Definition: 3.1. A subset A of a space X is called δg^*- closed if $cl_\delta(A) \subseteq U$ whenever A ⊆ U and U is a g-open set in X.

Proposition: 3.2. Every δ-closed set is δg^*-closed

Proof: Let A be an δ-closed set and U be any g-open set containing A. Since A is δ-closed, $cl_\delta(A)$ = A. Therefore $cl_\delta(A)$ = A ⊆ U and hence A is δg^*-closed.

Remark: 3.3. The converse of the above theorem is not true as shown in the following example.

Example: 3.4. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}}. Here, g-open sets with respect to τ are open sets. Then the set {b, c} is δg^*-closed but not δ-closed, since the only non-trivial δ-closed sets are {a, c} and {b}.

Proposition: 3.5. Every δg^*-closed set is g-closed.

Proof: Let A be a δg^*-closed set and U be an any open set containing A in X. Since every open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$. Since $cl(A) \subseteq cl_\delta(A) \subseteq U$, we have cl(A) ⊆ U and hence A is g-closed.

Remark: 3.6. A g-closed set need not be δg^*-closed as shown in the following example.

Example: 3.7. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. Then the set {b} is g-closed but not δg^*-closed.

Proposition: 3.8. Every δg^*-closed set is g^*-closed.

Proof: Let A be a δg^*-closed set and U be an any g-open set containing A in X. Since A is δg^*-closed, $cl_\delta(A) \subseteq U$. But $cl(A) \subseteq cl_\delta(A) \subseteq U$, we have $cl(A) \subseteq U$ and hence A is g^*-closed.

Remark: 3.9. A g^*-closed set need not be δg^*-closed as shown in the following example.

Example: 3.10. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. Then the set {b} is g^*-closed but not δg^*-closed.

Proposition: 3.11. Every δg^*-closed set is gs-closed.

Proof: Let A be δg^*-closed and U be any open set containing A in X. Since every open set is g-open, $cl_\delta(A) \subseteq U$ for every subset A of X. Since $scl(A) \subseteq cl_\delta(A) = U$, we have $scl(A) \subseteq U$ and hence A is gs-closed.

Remark: 3.12. A gs-closed set need not be δg^*-closed as shown in the following example.

Example: 3.13. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. Then the set {b} is gs-closed but not δg^*-closed.

Proposition: 3.14. Every δg^*-closed set is αg-closed.

Proof: It is true from the fact that $\alpha cl(A) \subseteq cl_\delta(A)$ for every subset A of X.

Remark: 3.15. A αg-closed set need not be δg^*-closed as shown in the following example.

Example: 3.16. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. Then the set {c} is αg-closed but not δg^*-closed.

Proposition: 3.17. Every δg^*-closed set is $s\alpha g^*$-closed.

Proof: Let A be δg^*-closed and U be any g*-open set containing A in X. Since every g^*-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$, for every subset A of X. Since $\alpha cl(A) \subseteq cl_\delta(A) \subseteq U$, we have $\alpha cl(A) \subseteq U$ and hence A is $s\alpha g^*$-closed.

Remark: 3.18. A $s\alpha g^*$-closed set need not be δg^*-closed as shown in the following example.

Example: 3.19. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. Then the set {c} is $s\alpha g^*$-closed but not δg^*-closed.

Proposition: 3.20. Every δg^*-closed set is $\delta\hat{g}$-closed.

Proof: Let A be δg^*-closed and U be any $\hat{g}$-open set containing A in X. Since every $\hat{g}$-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$. Hence A is $\delta\hat{g}$-closed.

Remark: 3.21. A $\delta\hat{g}$-closed set need not be δg^*-closed as shown in the following example.

Example: 3.22. Let X = {a, b, c}, τ = {X, ϕ, {a}}. Then the set {a, b} is $\delta\hat{g}$-closed but not δg^*-closed.

Proposition: 3.23. Every δg^*-closed set is $\alpha\hat{g}$-closed.

Proof: Let A be δg^*-closed and U be any $\hat{g}$-open set containing A in X. Since every $\hat{g}$-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$. Since $\alpha cl(A) \subseteq cl_\delta(A) \subseteq A$, we have $\alpha cl(A) \subseteq U$ and hence A is $\alpha\hat{g}$-closed.

Remark: 3.24. $\alpha\hat{g}$-closed set need not be δg^*-closed as shown in the following example.

Example: 3.25. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. Then the set {b} is $\alpha\hat{g}$-closed but not δg^*-closed.

Proposition: 3.26. Every δg^*-closed set is $s\delta g^*$-closed.

Proof: Let A be δg^*-closed and U be any g^*-open set containing A in X. Since every g^*-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$, for every subset A of X. Hence A is $s\delta g^*$-closed.

Remark: 3.27. A $s\delta g^*$-closed set need not be δg^*-closed as shown in the following example.

Example: 3.28. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b, c}}. Then the set {a, c} is $s\delta g^*$-closed but not δg^*-closed.

Proposition: 3.29. Every δg^*-closed set is δg-closed.

Proof: Let A be δg^*-closed and U be any open set containing A in X. Since every open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$, for every subset A of X. Hence A is δg-closed.

Remark: 3.30. A δg-closed set need not be δg^*-closed as shown in the following example.

Example: 3.31. Let X = {a, b, c}, τ = {X, ϕ, {a}}. Then the set {c} is δg-closed but not δg^*-closed.

Proposition: 3.32. Every δg^*-closed set is $^{\#}$gs-closed.

Proof: Let A be δg^*-closed and U be any *g-open set containing A in X. Since every *g-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$, for every subset A of X. But $scl(A) \subseteq cl_\delta(A) \subseteq A$, we have $scl(A) \subseteq U$ Hence A is $^{\#}$gs-closed.

Remark: 3.33. A $^{\#}$gs-closed set need not be δg^*-closed as shown in the following example.

Example: 3.34. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b, c}}. Then the set {a, b} is $^{\#}$gs-closed but not δg^*-closed.

Proposition: 3.35. Every δg^*-closed set is *g-closed.

Proof: Let A be δg^*-closed and U be any $\hat{g}$-open set containing A in X. Since every $\hat{g}$-open set is g-open and A is δg^*-closed, $cl_\delta(A) \subseteq U$ for every subset A of X. Since $cl(A) \subseteq cl_\delta(A) \subseteq A$, we have $cl(A) \subseteq U$ and hence A is *g-closed.

Remark: 3.36. A *g-closed set need not be δg^*-closed as shown in the following example.

Example: 3.37. Let X = {a, b, c}, τ = {X, ϕ, {a}}. Then the set {a, c} is *g-closed but not δg^*-closed.

Proposition: 3.38. Every δg^*-closed set is g^*p-closed.

Proof: Let A be δg^*-closed and U be any g-open set containing A in X. Since A is δg^*-closed, $cl_\delta(A) \subseteq U$ for every subset A of X. Since $pcl(A) \subseteq cl_\delta(A) \subseteq A$, we have $pcl(A) \subseteq U$ and hence A is g^*p-closed.

Remark: 3.39. A g^*p-closed set need not be δg^*-closed as shown in the following example.

Example: 3.40. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. Then the set {b} is g^*p-closed but not δg^*-closed.

Proposition: 3.41. Every δg^*-closed set is gp-closed.

Proof: It follows from the fact that every open set is g-open.

Remark: 3.42. A gp-closed set need not be δg^*-closed as shown in the following example.

Example: 3.43. Let X = {a, b, c}, τ = {X, ϕ, {a, b}}. Then the set {a} is gp-closed but not δg^*-closed.

Remark: 3.44. The following examples show that δg^*-closeness is independent from $\tilde{g}$- closeness, $\tilde{g}_\alpha$- closeness and $\tilde{g}_s$- closeness.

Example: 3.45. Let X = {a, b, c}, τ = {X, ϕ, {a, b}}. In this topology the set {b, c} is δg^*-closed but not $\tilde{g}$- closed, $\tilde{g}_\alpha$- closed and $\tilde{g}_s$- closed.

Example: 3.46. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. In this topology the set {b} is $\tilde{g}$-closed, $\tilde{g}_\alpha$- closed and $\tilde{g}_s$- closed but not δg^*-closed.

Remark: 3.47. The following examples show that δg^*-closeness is independent from gα-closeness, $g^\#$s-closeness, g^*s-closeness and α-closeness

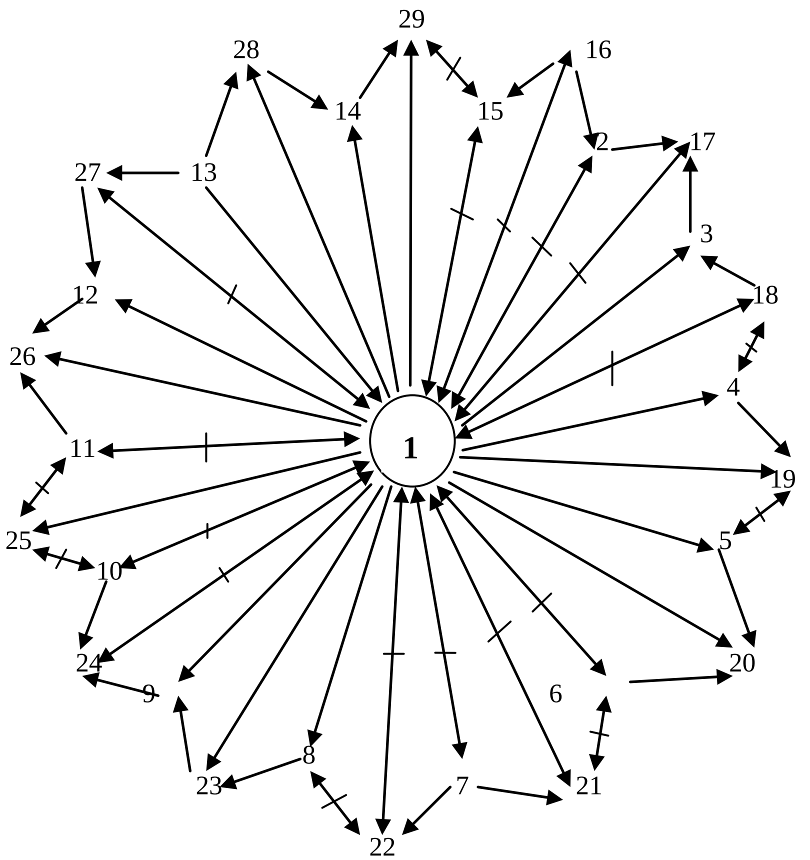
Example: 3.48. Let X = {a, b, c}, τ = {X,ϕ, {a}, {a, b}}. In this topology the set {b} is gα-closed, $g^\#$s-closed, g^*s-closed and α-closed but not δg^*-closed. The set {a, c} is δg^*-closed but not gα-closed, $g^\#$s-closed, g^*s-closed and α-closed.

Remark: 3.49. The following examples show that δg^*-closeness is independent from αg^*-closeness.

Example: 3.50. Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}}. In this topology the set {b} is αg^*- closed but not δg^*-closed.

Example: 3.51. Let X = {a, b, c}, τ = {X,ϕ, {a}, {a, b}}. In this topology the set {a, c} is δg^*-closed but not αg^*- closed.

Remark: 3.52. The following diagram has shown the relationship of δg^*-closed sets with other known existing sets. A $\longrightarrow$ B represents A implies B but not conversely and A $\not\longleftrightarrow$ B represents A and B are independent to each other,



1. δg^*-closed 2.g''-closed 3. g-closed 4. g*closed 5. g*p-closed 6. $\tilde{g}_s$-closed 7. $\tilde{g}$-closed 8. $g^\#$s-closed 9. gs-closed 10. g^*s-closed 11. gα-closed 12. α$\hat{g}$-closed 13. δ-closed 14. sδg^*-closed 15. αg*-closed 16. closed 17. $\ddot{g}$-closed 18. $\hat{g}$-closed 19. *g- closed 20. gp-closed 21. $\tilde{g}_\alpha$- closed 22. $\tilde{g}_p$- closed 23. #gs-closed 24. sg-closed 25 δ$\hat{g}$-closed 26. αg-closed 27. α-closed 28. δg-closed 29. sαg*-closed.

4. CHARACTERISATION

Theorem: 4.1. The finite union of δg^*-closed sets is δg^*-closed.

Proof: Let $\{A_i \,/\, i = 1, 2, 3,.., n\}$ be a finite class of δg^*-closed subsets of X. Let $A = \bigcup_{i=1}^{n} A_i$. Let V be a g-open set containing A which implies $\bigcup_{i=1}^{n} A_i \subseteq V$.This implies $A_i \subseteq V$, for every i. By assumption $cl_\delta(A_i) \subseteq V$, for every i. Which implies $\bigcup_{i=1}^{n} cl_\delta(A_i) \subseteq V$. Then $cl_\delta(\bigcup_{i=1}^{n} A_i) \subseteq V$. Thus $cl_\delta(A) \subseteq V$. Hence finite union of δg^*-closed sets is δg^*-closed.

Remark: 4.2. The following example shows that intersection of any two δg^*-closed sets in X need not be δg^*-closed.

Example: 4.3. Let X = {a, b, c}, τ = {X, ϕ, {a}}. Then the set {a, b} and {a, c} are δg^*-closed but their intersection {a} is not δg^*- closed.

Proposition: 4.4. Let A be a δg^*-closed set of X. Then $cl_\delta(A)$ - A does not contain a non empty g-closed set.

Proof: Suppose that A is δg^*-closed, let F be a g-closed set contained in $cl_\delta(A)$ - A. Now F^c is a g-open set in X such that $A \subseteq F^c$. Since A is a δg^*-closed set of X, then $cl_\delta(A) \subseteq F^c$. Thus $F \subseteq (cl_\delta(A))^c$. Also $F \subseteq cl_\delta(A) - A$. Therefore $F \subseteq (cl_\delta(A))^c \cap cl_\delta(A) = \emptyset$. Hence F = ϕ.

Proposition: 4.5. If A is a g-open set and δg^*-closed subset of X then A is an δ-closed subset of X.

Proof: Since A is g-open and δg^*-closed, $cl_\delta(A) \subseteq A$. Hence A is δ-closed.

Theorem: 4.6. The intersection of a δg^*-closed set and a δ-closed set is always δg^*-closed.

Proof: Let A be δg^*-closed and F be δ-closed. Let $V = A \cap F$. Let U be g-open such that $V \subseteq U$ implies $A \cap F \subseteq U$ which implies $A \subseteq U \cap F^c$. Here F^c is δ-open, so F^c is open. Thus F^c is g-open. Hence $U \cup F^c$ is g-open and by assumption $A \subseteq U \cap F^c \Rightarrow cl_\delta(A) \subseteq U \cap F^c$, $cl_\delta(V) = cl_\delta(A \cap F) \subseteq cl_\delta(A) \cap cl_\delta(F) = cl_\delta(A) \cap F$ which is contained in U. Therefore $cl_\delta(V) \subseteq V$. Hence $A \cap F$ is δg^*-closed.

Theorem: 4.7. In a $T_{3/4}$-space every δg^*-closed set is δ-closed.

Proof: Let X be a $T_{3/4}$-space. Let A be a δg^*-closed set of x. We know that every δg^*-closed set is δg-closed. Since X is $T_{3/4}$-space, A is δ-closed.

Proposition: 4.8. If A is an δg^*-closed set in a space X and $A \subseteq B \subseteq cl_\delta(A)$, then B is also a δg^*-closed.

Proof: Let U be a g-open set of X such that $B \subseteq U$. Then $A \subseteq U$. Since A is δg^*-closed set, $cl_\delta(A) \subseteq U$. Also since $B \subseteq cl_\delta(A)$, $cl_\delta(B) \subseteq cl_\delta(cl_\delta(A)) = cl_\delta(A)$. Hence $cl_\delta(B) \subseteq U$. Therefore B is also a δg^*-closed set.

Theorem: 4.9. Let A be a δg^*-closed set of X. Then A is δ-closed iff $cl_\delta(A)$ - A is g-closed.

Proof: Necessity. Let A be a δ-closed subset of X. Then $cl_\delta(A) = A$ and so $cl_\delta(A) - A = \emptyset$, which is g-closed.

Sufficiency. Let $cl_\delta(A)$ - A be g-closed. Since A is δg^*-closed, by Proposition 4.4., $cl_\delta(A)$ - A does not contain a non empty g-closed set which implies $cl_\delta(A) - A = \emptyset$. That is $cl_\delta(A) = A$. Hence A is δ-closed.

Proposition: 4.10. For each $a \in X$ either {a} is g-closed or $\{a\}^c$ is δg^*-closed in X.

Proof: Suppose that {a} is not g-closed in X, then $\{a\}^c$ is not g-open and the only g-open set containing $\{a\}^c$ is the space X itself. That is $\{a\}^c \subseteq X$. Therefore $cl_\delta(\{a\}^c) \subseteq X$ and so $\{a\}^c$ is δg^*-closed.

DEFINTION: 4.11. The intersection of all g-open subsets of X containing A is called the g-kernel of A and is denoted by g-ker(A).

Lemma: 4.12. A subset A of X is δg^*-closed if and only if $cl_\delta(A) \subseteq g\text{-}ker(A)$.

Proof: Suppose that A is δg^*-closed in X, then $cl_\delta(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open on X. Let $x \in cl_\delta(A)$. If $x \notin g\text{-}ker(A)$, then there is a g-open set U such that $x \notin U$. Since U is a g-open set containing A, $x \notin cl_\delta(A)$ a contradiction.

Conversely let $cl_\delta(A) \subseteq g\text{-}ker(A)$. If U is any g-open set containing A, then $cl_\delta(A) \subseteq g\text{-}ker(A) \subseteq U$. Then A is δg^*-closed.

5. δg^*-OPEN SETS IN TOPOLOGICAL SPACES:

In this section we introduce the concept of δg^*-open sets in topological spaces and study some of their properties.

Definition: 5.1. A subset A of a topological space (X, τ) is called δg^*-open if its complement A^c is δg^*-closed in (X, τ).

Theorem: 5.2. If a subset A of a topological space (X,τ) is δ -open, then it is δg^*-open in X.

Proof: Let A be an δ -open set in a topological space (X, τ). Then A^c is δ -closed in X. By Theorem 3.2, A^c is δg^*-closed in (X, τ). Hence A is δg^*-open in (X,τ).

The converse of Theorem 5.2 need not be true as seen in the following example.

Example: 5.3. Let X = {a, b, c} & τ = {X,ϕ, {a, b}}, then the subset {a} is δg^*-open but not δ -open in (X, τ).

Proposition: 5.4. Every δg^*-open set is
1. g-open, 2. g*-open, 3. gs-open, 4. αg-open, 5. sαg*-open, 6. sδg^*-open, 7. $\delta\hat{g}$-closed, 8. $\alpha\hat{g}$-closed, 9. δg-open, 10.#gs -open, 11. *g-open, 12. gp-open, 13. g*p-open.

Theorem: 5.5. A subset A of a topological space (X,τ) is δg^*-open if and only if $G \subseteq int_\delta(A)$ whenever $A \supseteq G$ & G is g-closed.

Proof: Assume that A is δg^*-open. Then A^c is δg^*-closed. Let G be a g-closed set in (X,τ) contained in A. Then G^c is a g-open set in (X,τ) containing A^c . Since A^c is δg^*-closed, $cl_\delta(A^c) \subseteq G^c$, equivalently $G \subseteq int_\delta(A)$.

Conversely assume that G is contained in $int_\delta(A)$, whenever G is contained in A & G is g-closed in (X, τ). Let A^c be contained in F, where F is g-open. Then $F^c \subseteq A$. By criteria, $F^c \subseteq int_\delta(A)$. This implies $cl_\delta(A^c) \subseteq F$. Thus A^c is δg^*-closed. Hence A is δg^*-open.

Remark: 5.6. For a subset A of X, $cl_\delta(X - A) = X - int_\delta(A)$.

Proposition: 5.7. If $int_\delta(A) \subseteq B \subseteq A$ and A is δg^*-open in (X, τ), then B is δg^*-open in (X, τ).

Proof: Let $int_\delta(A) \subseteq B \subseteq A$ which implies that $X - A \subseteq X - B \subseteq X - int_\delta(A)$ By Remark 5.6., $X - A \subseteq X - B \subseteq cl_\delta(X - A)$. Since X – A is δg^*-closed, by Proposition 4.8, X – B is δg^*-closed and hence B is δg^*-open in (X, τ).

Theorem: 5.8. If A and B are δg^*-open sets in X then A ∩ B is δg^*-open in X.

Theorem: 5.9. If A and B are δg^*-open in X if and only if G = X whenever G is g-open and $X - A \subseteq X - B \subseteq int_\delta(A) \cup A^c \subseteq G$.

Proof: Necessity. Let A be a δg^*-open set and G be g-open and $int_\delta(A) \cup A^c \subseteq G$. This gives $G^c \subseteq (int_\delta(A) \cup A^c)^c = (int_\delta(A))^c \cap A = (int_\delta(A))^c - A^c = cl_\delta(A^c) - A^c$. Since A^c is δg^*-closed and G^c is g-closed, it follows that $G^c = \emptyset$. Therefore G = X.

Sufficiency. Suppose that F is δg^*-closed and $F \subseteq A$. Then $int_\delta(A) \cup A^c \subseteq int_\delta(A) \cup F^c$. It follows by hypothesis that $int_\delta(A) \cup F^c = \emptyset$ and hence $F \subseteq int_\delta(A)$. Therefore by Theorem 5.5., A is δg^*-closed in X.

6. APPLICATIONS:

Definition: 6.1. A space X is called
1) a $T_{\delta g^*}$-space if every δg^*-closed set in it is δ-closed.
2) a $\#T_{\delta g^*}$-space if every δg^*-closed set in it is closed.

Theorem: 6.2. For a space X the following conditions are equivalent
(i) X is a $T_{\delta g^*}$-space.
(ii) Every singleton of X is either g-closed or δ-open.

Proof: (i) ⇒(ii) Let x ∈ X. Suppose that {x} is not a g-closed set of X. Then X - {x} is not a g-open set. So X is the only g-open set containing X - {x}. Then X - {x} is an δg^*-closed set of X. Since X is a $T_{\delta g^*}$-space, X - {x} is an δ-closed set of X and hence {x}is an δ-open set of X.

(ii) ⇒(i) Let A be an δg^*-closed set of X, $A \subseteq cl_\delta(A)$. Let $x \in cl_\delta(A)$. *B*y (ii) {x} is either g-closed or δ-open.

Case (a) Suppose that {x} is g-closed. If $x \in cl_\delta(A) - A$ then by Theorem 4.4., we arrive at a contradiction. Thus $x \in A$.

Case (b) Suppose that {x} is δ-open. Since $x \in cl_\delta(A)$, $\{x\} \cap A \neq \phi$. This implies $x \in A$..

Thus in any case $x \in A$., so $cl_\delta(A) \subseteq A$. Therefore $cl_\delta(A) = A$. The or equivalently A is δ-closed.

Theorem: 6.3. Every $T_{\delta g^*}$-space is $\#T_{\delta g^*}$-space.

Proof: Let X be a $T_{\delta g^*}$-space. Let A be a δg^*-closed set in X. By hypothesis, A is δ-closed. Since every δ-closed set is closed, $\#T_{\delta g^*}$-space.

Remark: 6.4. A $\#T_{\delta g^*}$-space need not be $T_{\delta g^*}$-space as shown in the following example.

Example: 6.5. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b}, {a, c}, {a, b}}. Then the space (X, τ) is $\#T_{\delta g^*}$-space but not cosed but not a $T_{\delta g^*}$-space.

Remark: 6.6. The following examples show that $T_{\delta g^*}$-space is independent from $T_{1/2}$ space.

Example: 6.7. Let X = {a, b, c}, τ ={X, ϕ, {a}, {b, c}}. Then the space (X, τ) is $T_{\delta g^*}$-space but not $T_{1/2}$ space.

Example: 6.8. Let X = {a, b, c}, τ ={X, ϕ, {a}, {b}, {a, b}, {a, c}}. Then the space (X,τ) is $T_{1/2}$ space but not $T_{\delta g^*}$-space.

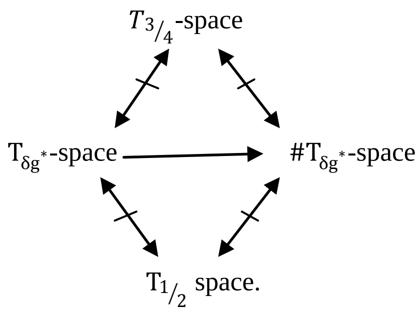
Theorem: 6.9. Every $T_{3/4}$-space is $T_{\delta g^*}$-space.

Proof: Let X be a $T_{\delta g^*}$-space. Let A be a δg^*-closed set in X. Since every δg^*-closed set is δg-closed, A is δg-closed. By hypothesis, A is δ-closed. Hence X is $T_{\delta g^*}$-space.

Remark: 6.10. A $T_{\delta g^*}$-space need not be $T_{3/4}$-space as shown in the following example.

Example: 6.11. Let X = {a, b, c}, τ = {X, ϕ, {a}, {b, c}}. Then the space (X, τ) is $T_{\delta g^*}$-space but not $T_{3/4}$-space.

Remark: 6.12. The following diagram shows the relationships between $T_{\delta g^*}$-space and $\#T_{\delta g^*}$-space with other known existing sets. A ⟶ B represents A implies B but not conversely and A ⟷ B represents A and B are independent to each other.



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