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A NEW APPROACH FOR QUEUE-INVENTORY NETWORK MODEL USING EXTENSION GRAPH ELIMINATION THEOREM WITH ENHANCED PERFORMANCE IN TIME

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ABSTRACT

To develop a supply network model for a service facility system with inventory which takes as input the number of customers and the on hand inventory. In return the model generates the steady state probabilities. The inter-arrival time to the service station is assumed to be exponentially distributed with mean $1/\lambda$. The maximum inventory level is S and the maximum capacity of the waiting space is N. The replenishment process is assumed to be instantaneous i.e. (0, S) policy, and the service time for each customer is exponentially distributed with mean $1/\mu$. The steady state probability distribution for the system states are obtained by graph elimination method and the classical iteration method. The elimination ordering of variables is found to greatly affect the time and space complexity of the solution process if M is sparse. A numerical example is presented to validate the model and to illustrate its various features.

Keywords: *Service Facility system, Inventory control, Graph Elimination Method, Time complexity, Queue-inventory Model.*

I. INTRODUCTION

Over the last two decades, research on complex integrated production-inventory systems or service inventory systems has found much attention, often in connection with the research on integrated supply chain management. Interaction of service processes with inventory management for associated inventories is usually described using queuing networks and multi-echelon inventory models. Mathematical methods used in the field are usually aggregation – disaggregation techniques or simulation or hybrid techniques. Analytical models are rare until now.

Berman et al., considered an inventory management system at a service facility which uses one item of inventory for each service provided. They assumed that both demand and service rates are deterministic and constant as such queues can form only during stock outs. They determined the optimal order quantity Q that minimizes the total cost rate. Berman et al. analyzed a problem in stochastic environment where customers arrive at service facilities according to Poisson process and service times are exponentially distributed with mean inter-arrival times greater than the mean service time and each service require one item from inventory. A logically related model was studied by, He, Q-M, E.M. Jewes and Buzzacott et al., who analyzed a completely Markovian production – inventory system, where demands arrive at a workshop and are processed by a simple machine in batch sizes of one. Berman and Sapna et al. studied extensively an inventory control problem at a service facility which uses one item of inventory for service provided.

A continuous review perishable inventory system at service facilities was studied by Elango et al., The importance of inventory management for the quality of service of today's service system is generally accepted and optimization of systems in order to maximize quality of service is therefore an important topic.

In this article we considered a service facility system with perishable inventory for service completion. The steady state probability distribution of inventory level and queue size is obtained by Graph elimination method. The content of the

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paper is presented as follows. Section II deals with a introduction on the problem selected. In section III the proposed model for service facility system is described in detail and the notations used in the paper are explained. The core part, section III deals with analysis of the system and solution procedure. In section IV a numerical example is provided to compare the efficiency of the two methods.

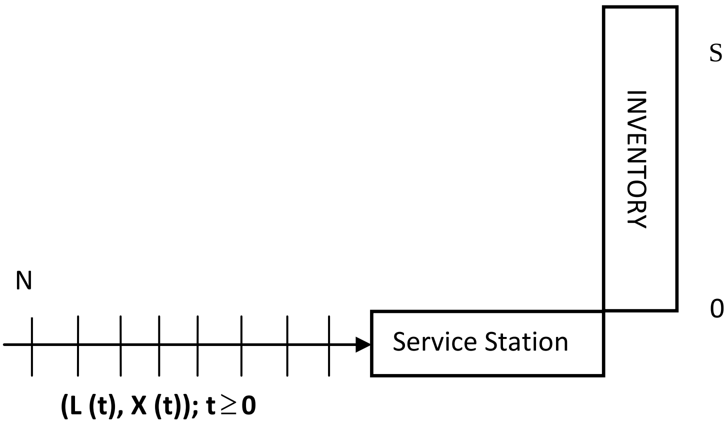
II. MODEL DESCRIPTION

Consider a service facility system in which inventory is maintained to perform service. The items are delivered to the demanding customers. The demand is for single item per customer. The maximum capacity of the inventory is S and the maximum number of customers allowed in the waiting space is N, i.e., an arriving customer who sees N customers already waiting in the system does not enter the system. The following assumptions are made:

- The time arrival of customer to the service station forms a Poisson process with parameter $\lambda (>0)$.
- The service time for each customer follows negative exponential distribution with parameter μ (>0).
- A (0, S) ordering policy is adopted with instantaneous supply of orders.

III. ANALYSIS

Steady State Analysis:



Let L (t) denote the inventory level and X (t), the number of customers (waiting and being served) in the system, at time t+. From the assumptions made on the input and output processes, it can be verified that (L, X)= $\{(L(t), X(t)); t \geq 0\}$ is a Markov process on the state space E.

where $E = E_1 \times E_2$, $E_1 = \{1,2,3,....,S\}$ & $E_2 = \{0,1,2,....,N\}$. The infinitesimal generator of this process can be obtained using the following arguments:

- The arrival of a customer makes a transition from (i, q) to (i, q+1) with intensity of transition λ.
- Completion of service makes one customer leave the system and decrease the inventory level by 1. Thus a transition takes place from (i, q) to (i-1, q-1) with intensity of transition μ.

Define

$$A = ((a(i,q,j,r)))_{(i,q),(j,r)\in E}$$

where

$$a(i,q,j,r) = \begin{cases} \lambda, i = S, S-1, \ldots, 1; q = 0, 1, \ldots, N-1; \ j = i; r = q+1; \\ \mu, \begin{cases} i = S, S-1, \ldots, 2; q = 1, 2, \ldots, N; \ j = i-1; r = q-1; \\ i = 1; q = 1, 2, \ldots, N; \ j = S; r = q-1; \end{cases} \\ -\lambda, i = S, S-1, \ldots, 1; q = 0; \ j = i; r = q; \\ -\mu, \ i = S, \ S-1, \ldots, 2, 1; q = N; \ j = i; r = q; \\ -(\lambda+\mu), i = S, S-1, \ldots, 2, 1; q = 1, 2, \ldots, N-1; \ j = i; r = q; \\ 0, otherwise. \end{cases}$$

The infinitesimal generator A can be written in terms of sub matrices A_{ij} , namely $A = \left(\left(A_{ij}\right)\right)$ where

$$A_{ij} = \begin{cases} A_{S-i+1},\ j = i;\ i = 1, 2, \ldots, S-1, S; \\ B_{S-i+1},\ i = 1,\ 2, \ldots,\ S-1;\ j = i+1; \\ B_1,\ i = S;\ j = 1; \\ 0,\ otherwise. \end{cases}$$

In which

$$A_i = \begin{pmatrix} -\lambda & \lambda & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & -(\lambda+\mu) & \lambda & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & -(\lambda+\mu) & \lambda & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & 0 & 0 & -(\lambda+\mu) & \lambda & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & -(\lambda+\mu) & \lambda & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & -(\lambda+\mu) & \lambda & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \vdots & \vdots & \vdots & \vdots & \cdots & 0 & -(\lambda+\mu) & \lambda \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -\mu \end{pmatrix}$$

and

$$B_i = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu & 0 & . & . & . & . & . & . & . & 0 \\ 0 & \mu & 0 & . & . & . & . & . & . & 0 \\ 0 & 0 & \mu & 0 & . & . & . & . & . & 0 \\ 0 & . & 0 & \ldots & 0 & . & . & . & . & 0 \\ \cdots\ 0 & . & \vdots & 0 & \ldots & 0 & . & . & . & 0 \\ 0 & . & . & . & 0 & \ldots & 0 & . & . & 0 \\ 0 & . & . & . & . & 0 & \mu & 0 & . & 0 \\ 0 & . & . & . & . & . & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \end{pmatrix}$$ for i=1,2,…,S-1,S.

The equilibrium distribution, is the unique solution to the equations $\pi Q=0$ and $\pi e=1$, where Q is the transition rate matrix and e is the column vector with all the components equal to 1.

From $\pi Q = 0$ we have,

$$If\quad i = S;\ q = 0;$$
$$-\lambda\,\pi^{(i,\,q)} + \mu\,\pi^{(1,\,q+1)} = 0,$$
$$If\quad i = S;\ q = 0, 1, \cdots, N-2;$$
$$\lambda\,\pi^{(i,\,q)} - (\lambda+\mu)\,\pi^{(i,\,q+1)} + \mu\,\pi^{(i,\,q+2)} = 0,$$
$$If\quad i = S;\ q = N-1;$$
$$\lambda\,\pi^{(i,\,q)} - \mu\,\pi^{(i,\,q+1)} = 0,$$
$$If\quad i = S-1, S-2, \cdots, 2, 1;\ q = 0;$$
$$-\lambda\,\pi^{(i,\,q)} + \mu\,\pi^{(i+1,\,q+1)} = 0,$$
$$If\quad i = S-1, S-2, \cdots, 2, 1;\ q = 0, 1, \cdots, N-2;$$
$$\lambda\,\pi^{(i,\,q)} - (\lambda+\mu)\,\pi^{(i,\,q+1)} + \mu\,\pi^{(i+1,\,q+1)} = 0,$$
$$If\quad i = S-1, S-2, \cdots, 1;\ q = N-1;$$
$$\lambda\,\pi^{(i,\,q)} - \mu\,\pi^{(i,\,q+1)} = 0.$$

Solving πe=1, and all the above equations by Classical iterative procedure the steady state probability of the system is given as $\pi = (\pi^{(i,q)})_{i=1,2,...,S;q=0,1,...,N}$ where,

$$\pi^{(i,0)} = \frac{1}{S}\left[\sum_{n=0}^{N}\left(\frac{\lambda}{\mu}\right)^{n}\right]^{-1};\ i=1,2,3,...,S$$

$$\&\pi^{(i,q)} = \frac{1}{S}\left[\sum_{n=0}^{N}\left(\frac{\lambda}{\mu}\right)^{n-q}\right]^{-1};\ i=1,2,3,...,S\ \&\ q=1,2,3,...,N$$

Graph Elimination Method:

In the proposed service facility model, the state transition diagram for the Markov chain (L, X) is a directed pseudo graph (Harary) with weight equal to transition rate from one state to another for each edge. This directed pseudo graph has both loops and directed edges.

Parter's (extension) Theorem:

Let G = (V_i, E_i), be the digraph corresponding to the rate transition matrix Q. Upon elimination of vertex i from (V_i, E_i), the new digraph G $_{i+1}$ representing the remaining system is obtained from G $_i$ by updating the weights of graph as follows:

i) $V_{i+1} = V_i - \{i\}$;

ii) $E_{i+1} = \{(x,z) \in E_i \,/\, x,z \in V_{i+1}\} \cup F_i(i)$;

iii) $M_{G_{i+1}}(e) = M_{G_i}(e)$ if e є E $_{i+1}$ - U $_i$ (i);

$M_{G_{i+1}}(x,z) = M_{G_i}(x,z) - M_{G_i}(x,i)[M_{G_i}(i,i)]^{-1}M_{G_i}(i,z)$ if (x, z) є H $_i$ (i);

$M_{G_{i+1}}(x,z) = -M_{G_i}(x,i)[M_{G_i}(i,i)]^{-1}M_{G_i}(i,z)$ if (x, z) є F $_i$ (i);

Where $F_i(y) = \{(x,z)/(x,y),(y,z) \in E_i, (x,z) \notin E_i, x \neq y, z \neq y\}$;

$H_i(y) = \{(x,z)/(x,y),(y,z) \in E_i, (x,z) \in E_i, x \neq y, z \neq y\}$

$U_i(y) = F_i(y) \cup H_i(y)$ for y є V $_i$, 2 ≤ i ≤ n.

ALGORITHM:

Step 1: Remove the vertex "i" and all edges connecting with it;

Step 2: If there exists edges (x, i) and (i, z) for x ≠ i, z ≠ i in G $_i$, then

a) If (x, z) is in G $_i$ then update the weight of (x, z) as $M_{G_i}(x,z) = M_{G_i}(x,z) - M_{G_i}(x,i)[M_{G_i}(i,i)]^{-1}M_{G_i}(i,z)$

b) If (x, z) is not in G $_i$, then form a new edge (x,z) with weight

$$M_{G_i}(x,z) = -M_{G_i}(x,i)[M_{G_i}(i,i)]^{-1}M_{G_i}(i,z).$$

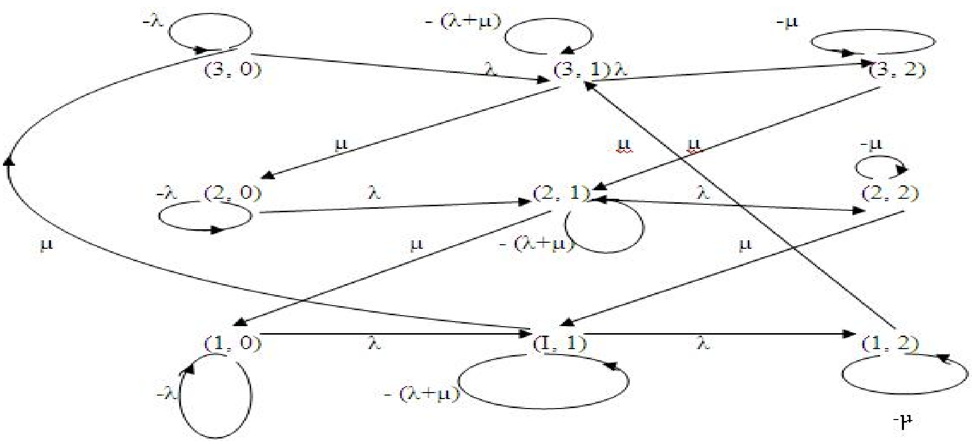
The vertex "i" is said to be eliminated from the digraph G $_i$. The above process of obtaining G $_{i+1}$ from G $_i$ for 1 ≤ i ≤ n-1, where G $_1$ = G is called the '***weighted graph eliminatio***n' process.

IV. NUMERICAL EXAMPLE:

Problem:

Consider a service facility system with S=3, N=2 and the arrival and service rate to be λ & μ. The transition rate pseudo digraph G and the transition matrix Q are depicted as follows:

A) Digraph G_1



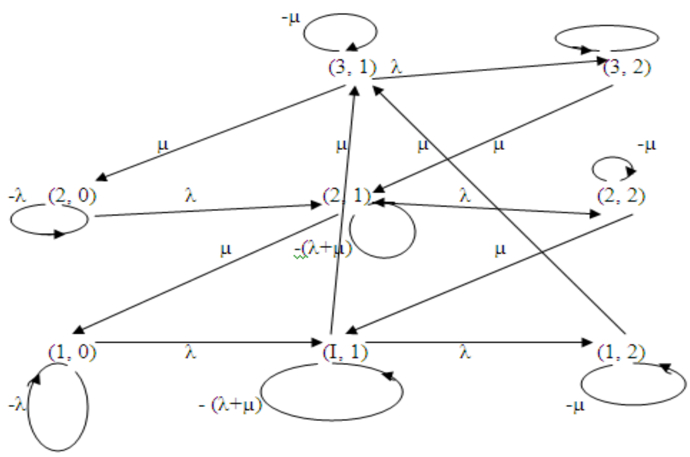
Transition Rate Matrix for G_1:

$$
\begin{array}{c}
 \\ (3,0) \\ (3,1) \\ (3,2) \\ (2,0) \\ (2,1) \\ (2,2) \\ 1,0) \\ (1,1) \\ (1,2)
\end{array}
\begin{array}{c}
\begin{array}{ccccccccc} (3,0) & (3,1) & (3,2) & (2,0) & (2,1) & (2,2) & (1,0) & (1,1) & (1,2) \end{array} \\
\begin{pmatrix}
-\lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -(\lambda+\mu) & \lambda & \mu & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu & 0 & \mu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\lambda & \lambda & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(\lambda+\mu) & \lambda & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mu & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda & 0 \\
\mu & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda+\mu) & \lambda \\
0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & -\mu
\end{pmatrix}
\end{array}
$$

Graph elimination Procedure:

Consider the digraph $\mathbf{G}_1$ in the previous section. By eliminating the vertex (3, 0) we obtain the digraph G_2 and the corresponding transition matrix as below.

B) Digraph G_2

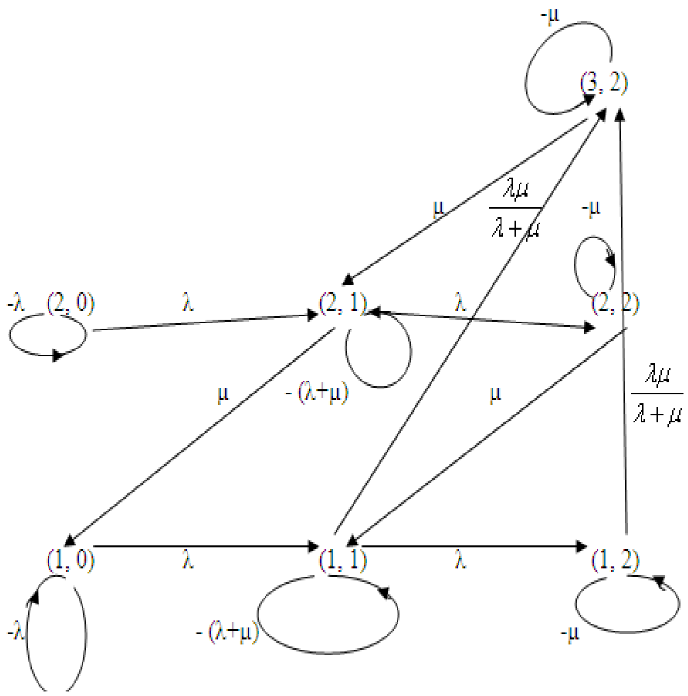


Transition Rate Matrix for G_2 :

$$Q_2=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\mu) & \lambda & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\lambda+\mu) & \lambda & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 & 0 & -(\lambda+\mu) & \lambda \\ 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix}$$

By eliminating the vertex (3, 1) we obtain the digraph G_3 and the corresponding transition matrix as below.

C) Digraph G_3

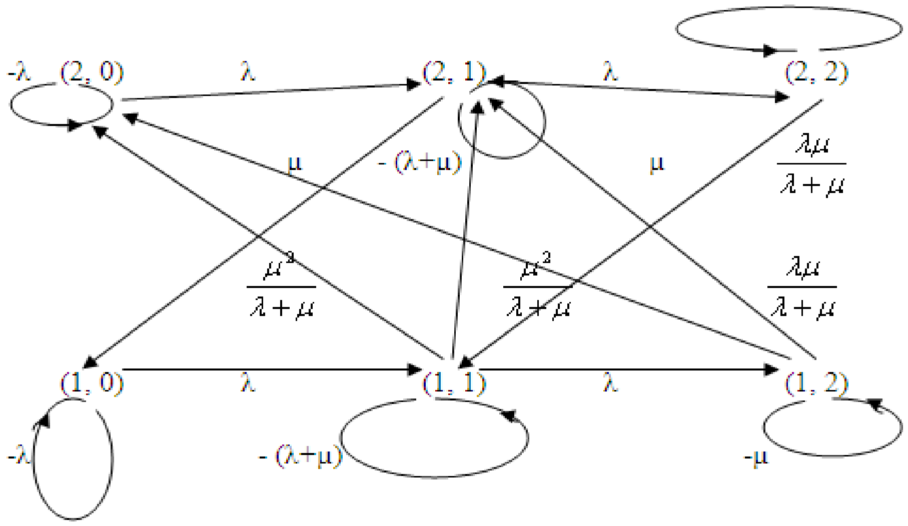


Transition Rate Matrix for G_3 :

$$Q_3=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\lambda+\mu) & \lambda & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda & 0 \\ 0 & 0 & \frac{\lambda\mu}{\lambda+\mu} & \frac{\mu^2}{\lambda+\mu} & 0 & 0 & 0 & -(\lambda+\mu) & \lambda \\ 0 & 0 & \frac{\lambda\mu}{\lambda+\mu} & \frac{\mu^2}{\lambda+\mu} & 0 & 0 & 0 & 0 & -\mu \end{pmatrix}$$

By eliminating the vertex (3, 2) we obtain the digraph G_4 and the corresponding transition matrix as below.

D) Digraph G_4



Transition Rate Matrix for G $_4$ **:**

$$
Q_4=\begin{pmatrix}
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&-\lambda&\lambda&0&0&0&0\\
0&0&0&0&-(\lambda+\mu)&\lambda&\mu&0&0\\
0&0&0&0&0&-\mu&0&\mu&0\\
0&0&0&0&0&0&-\lambda&\lambda&0\\
0&0&0&\frac{\mu^2}{\lambda+\mu}&\frac{\lambda\mu}{\lambda+\mu}&0&0&-(\lambda+\mu)&\lambda\\
0&0&0&\frac{\mu^2}{\lambda+\mu}&\frac{\lambda\mu}{\lambda+\mu}&0&0&0&-\mu
\end{pmatrix}.
$$

Similarly proceeding we obtain the remaining transition rate matrices for G_5, G_6, G_7, & G_8 as given below using the weighted graph elimination technique.

Transition Rate Matrix for G $_5$ **:**

$$
Q_5=\begin{pmatrix}
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&-(\lambda+\mu)&\lambda&\mu&0&0\\
0&0&0&0&0&-\mu&0&\mu&0\\
0&0&0&0&0&0&-\lambda&\lambda&0\\
0&0&0&0&\mu&0&0&-(\lambda+\mu)&\lambda\\
0&0&0&0&\mu&0&0&0&-\mu
\end{pmatrix}
$$

Transition Rate Matrix for G_6:

$$\mathbf{Q_6} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda\mu}{\lambda+\mu} & \frac{\mu^2}{\lambda+\mu} & -(\lambda+\mu) & \lambda \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda\mu}{\lambda+\mu} & \frac{\mu^2}{\lambda+\mu} & 0 & -\mu \end{pmatrix}$$

Transition Rate Matrix for G_7 & G_8:

$$\mathbf{Q_7} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu^2}{\lambda+\mu} & -\mu-\frac{\lambda^2}{\lambda+\mu} & \lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu^2}{\lambda+\mu} & \frac{\lambda\mu}{\lambda+\mu} & -\mu \end{pmatrix}$$

$$\mathbf{Q_8} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix}$$

Steady state Results:

A proceeding like this provides the following equations from the digraphs $G_8, G_7, G_6, G_5, G_4, G_3, G_2, \& G_1$.

$$-\lambda\,\pi^{(1,1)} + \mu\,\pi^{(1,2)} = 0,$$

$$-\lambda\,\pi^{(1,0)} + \frac{\mu^2}{\lambda+\mu}\pi^{(1,1)} + \frac{\mu^2}{\lambda+\mu}\pi^{(1,2)} = 0,$$

$$-\mu\,\pi^{(2,2)} + \frac{\lambda\mu}{\lambda+\mu}\pi^{(1,1)} + \frac{\lambda\mu}{\lambda+\mu}\pi^{(1,2)} = 0,$$

$$-(\lambda+\mu)\,\pi^{(2,1)} + \mu\pi^{(1,1)} + \mu\,\pi^{(1,2)} = 0,$$

$$-\lambda\,\pi^{(2,0)} + \frac{\mu^2}{\lambda+\mu}\pi^{(1,1)} + \frac{\mu^2}{\lambda+\mu}\pi^{(1,2)} = 0,$$

$$-\mu\,\pi^{(3,2)} + \frac{\lambda\mu}{\lambda+\mu}\pi^{(1,1)} + \frac{\lambda\mu}{\lambda+\mu}\pi^{(1,2)} = 0,$$

$$-(\lambda+\mu)\,\pi^{(3,1)} + \mu\,\pi^{(1,1)} + \mu\,\pi^{(1,2)} = 0, \quad and \quad -\lambda\,\pi^{(3,0)} + \mu\,\pi^{(1,1)} = 0.$$

Here we get the same type of recurrence relations as obtained from the matrix equation $\pi Q = 0$ and $\pi e = 1$. The stationary distribution vector is found by the backward substitution phase of the Markov Chain Reduction Principle (MCRP).

$$\pi = \left(\pi^{(i,j)}\right)_{i=1,2,3;\, j=0,1,2}$$

where

$$\pi^{(1,0)} = \pi^{(2,0)} = \pi^{(3,0)} = \frac{1}{3}\left[\sum_{0}^{2}\left(\frac{\lambda}{\mu}\right)^{n}\right]^{-1}$$

$$\pi^{(1,1)} = \pi^{(2,1)} = \pi^{(3,1)} = \frac{1}{3}\left[\sum_{0}^{2}\left(\frac{\lambda}{\mu}\right)^{n-1}\right]^{-1}$$

$$\pi^{(1,2)} = \pi^{(2,2)} = \pi^{(3,2)} = \frac{1}{3}\left[\sum_{0}^{2}\left(\frac{\lambda}{\mu}\right)^{n-2}\right]^{-1}$$

Thus we obtain the same steady state probability distribution from the weighted graph elimination as in classical iteration procedure.

Time and Space Complexity

The elimination ordering of variables is found to greatly affect the time complexity of the solution process if M is sparse. Graph elimination approach provides a fast algorithm for the solution of the queuing system $\pi\, Q = 0$ *and* $\pi\, e = 1$. The rate matrix Q is represented by a weighted digraph, and our work is based on a graph-theoretic approach which facilitates the identification of the repeated structure of the numerical values of the non-zero entries of the matrix Q.

Time Complexity: The time complexity for Gauss elimination or classical iteration procedure is $O(N^3 S^3 (N+1)^3)$ and that for Graph elimination is $O(N S^3 (N+1)^3)$. The space complexity for Gauss elimination procedure is $O(N^2 S^2 (N+1)^2)$ and that for Graph elimination is $O(N S^2 (N+1)^2)$.

V. CONCLUSION:

In this work we considered a service facility system with inventory for service completion and the replenishment is done instantaneously ((0, S) policy). This simplifies the system and reduces it logically equivalent to a queuing system. Future directions for research are wide open.

i. The ordering policy may be assumed to be (s, S) type.

ii. The items in stock may be of perishable type.

iii. The service time may depend on the number of customers in the system, say μ^n .

The authors are working in this direction for the improvement of the previous results.

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