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ARITHMETIC OPERATIONS OF FOCAL ELEMENTS AND THEIR CORRESPONDING BASIC PROBABILITY ASSIGNMENTS IN EVIDENCE THEORY

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ABSTRACT

Dempster-Shafer theory of evidence is a very important tool widely used in many fields such as Information Fusion and decision-making. In evidence theory when focal elements and their corresponding basic probability assignments (bpa) of variables are given then focal elements and their corresponding probability assignment (bpa) can be combined under arithmetic operation. In [5] authors proposed methods to combine fuzzy focal elements and their basic probability assignments of two variables. Here, we make further investigation for interval focal elements.

Keywords: *Evidence Theory, Basic Probability Assignment, Focal elements*

1. INTRODUCTION

Probability theory is intended only for aleatory uncertainty (i.e., uncertainty arises from heterogeneity or the random character of natural processes) and it is inappropriate to represent epistemic uncertainty (i.e., uncertainty arises from the partial character of our knowledge of the natural world). To overcome the limitation of probabilistic method, Dempster put forward a theory and now it is known as evidence theory or Dempster- Shefer theory (1976). This theory is now a days widely used for the epistemic and aleatory uncertainty analysis. The use of Dempster-Shefer theory in risk analysis has many advantages over the conventional probabilistic approach. It provides convenient and comprehensive way to handle engineering problems including, imprecisely specified distributions, poorly known and unknown correlation between different variables, modeling uncertainty, small sample size, and measurement uncertainty. R. R. Yager (1986) had shown that when focal elements and corresponding basic probability assignments of variables are given they can be combined under arithmetic operations such as addition. He considered all the arithmetic operations between focal elements but the operation for the corresponding basic probability assignment (bpa) of the resulting focal elements was considered as multiplication (product). In [5], authors have also considered all the arithmetic operations between fuzzy focal elements by taking the operation for the corresponding basic probability assignment (bpa) of the resulting focal elements based on the operation between the focal elements. In this paper, we study the proposed methods for interval focal elements.

2. BASIC CONCEPT OF DEMPSTER-SHAFER THEORY OF EVIDENCE [7]

A frame of discernment (or simply a frame), usually denoted as Θ is a set of mutually exclusive and exhaustive propositional hypotheses, one and only one of which is true.

Evidence theory is based on two dual non-additive measure, i.e. belief measure and plausibility measure. There is one important function in Dempster-Shefer theory to define belief measure and plausibility measure which is known as basic probability assignment (bpa).

A function $m : 2^{\Theta} \to [0,1]$ is called basic probability assignment (bpa) on the set Θ if it satisfies the following two conditions:

$$m(\phi) = 0$$

$$\sum_{A \subseteq \Theta} m(A) = 1$$

Where ϕ is an empty set and A is any subset of Θ .

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The Basic Probability Assignment function (or mass function) is a primitive function. Given a frame, Θ , for each source of evidence, a mass function assigns a mass to every subset of Θ , which represents the degree of belief that one of the hypotheses in the subset is true, given the source of evidence.

The subset A of frame Θ is called the focal element of m, if $m(A) > 0$.

Using the basic probability assignment (bpa), belief measure and plausibility measure are respectively determined as

$$Bel(A) = \sum_{B \subseteq A} m(B), A \subseteq \Theta \text{ and } Pl(A) = \sum_{B \cap A \neq \Phi} m(B)$$

Here $m(B)$ is the degree of evidence in the set B alone, whereas $Bel(A)$ is the total evidence in set A and all subset B of A and the plausibility of an event A is the total evidence in set A, plus the evidence in all sets of the universe that intersect with A.

Where $Bel(A)$ and $Pl(A)$ represent the lower bound and upper bound of belief in A. Hence, interval $[Bel(A) , Pl(A)]$ is the range of belief in A .

Given two mass functions m_1 and m_2, Dempster-Shafer theory also provides Dempster's combination rule for combining them, which is defined as follows:

$$m(C) = \frac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \phi} m_1(A) m_2(B)}$$

3. INTERVAL ARITHMETIC

For the intervals A = $[a_1, a_2]$ and B = $[b_1, b_2]$ the arithmetic operations are defined as below:

3.1 Addition of intervals

$A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$

3.2 Subtraction of intervals

$A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$

3.3 Multiplication of intervals

$A. B = [a_1, a_2]. [b_1, b_2]$

$= [\min (a_1b_1,a_1b_2,a_2b_1,a_2b_2), \max(a_1b_1,a_1b_2,a_2b_1,a_2b_2)]$

where min (.) and max (.). produce the smallest and the largest number in the brackets correspondingly.

3.4 Inverse of an interval

$A^{-1} = [a_1, a_2]^{-1} = [1/a_2, 1/a_1], \ 0 \notin [a_1, a_2]$

3.5 Division of intervals

$A/B = [a_1, a_2] / [b_1, b_2]$

$= [a_1, a_2] / [1/b_2, 1/b_1], \ 0 \notin [b_1, b_2]$

$= [\min (a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max (a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)]$

4. PROPOSED COMBINATION OF FOCAL ELEMENTS [5]

Let X_1 and X_2 be two variables whose values are represented by Dempser-Shafer structure with focal elements $A_1, A_2, A_3, \ldots, A_n$ and $B_1, B_2, B_3, \ldots, B_m$ which are considered as intervals and their corresponding basic probability assignment (bpa) are as follows

$m(A_i) = a_i$ and $m(B_j) = b_j$, $i = 1,2,3,...,n$; $j = 1,2,3,\ldots,m$ respectively.

Where $\sum_{i=1}^{n} a_i = 1$ and $\sum_{j=1}^{m} b_j = 1$

Initially we combine all the focal elements using interval arithmetic which will produce *nm* number of focal elements and thereafter the corresponding basic probability assignment of resulting focal elements will be calculated as follows:

4.1 Addition of Focal Elements:

$$m(c_{ij}) = m(A_i + B_j) = \frac{m(A_i) + m(B_j)}{\sum_i \sum_j (m(A_i) + m(B_j))} \tag{4.1}$$

4.2 Subtraction of Focal Elements:

$$m(c_{ij}) = m(A_i - B_j) = \frac{m(A_i)(1 - m(B_j))}{\sum_i \sum_j (m(A_i)(1 - m(B_j)))} \tag{4.2}$$

4.3 Multiplication of Focal Elements:

$$m(c_{ij}) = m(A_i B_j) = \frac{(m(A_i) m(B_j))}{\sum_i \sum_j (m(A_i) m(B_j))} \tag{4.3}$$

4.4 Division of Focal Elements:

$$m(c_{ij}) = m(A_i / B_j) = \frac{(m(A_i) / m(B_j))}{\sum_i \sum_j (m(A_i) / m(B_j))} \tag{4.4}$$

Finally, we arrange all the focal elements in increasing order of the left end point.

5. NUMERICAL EXAMPLE:

Suppose basic probability assignment (bpa) of two parameters is assigned by an expert and which are given in the following tables:

Focal elements	Bpa
[5,9]	0.15
[7,15]	0.20
[16,20]	0.35
[18,22]	0.30

Table 1: Bpa of the first parameter

Focal elements	Bpa
[25,28]	0.05
[29,32]	0.12
[30,35]	0.43
[34,42]	0.25
[42,47]	0.15

Table 2: Bpa of the second parameter

5.1 Addition of Focal Elements:

Number of focal elements of the first parameter is 4 and second parameter is 5 respectively. After summing up all the focal elements using interval arithmetic we get 20 numbers of focal elements. Now, the corresponding basic probability assignments of resulting focal elements are calculated using (4.1) and arranging all the focal elements in increasing order of the left end point are given in the following table 3.

Table3: Basic probability assignment of resulting focal elements using algebraic addition.

Focal elements	Bpa
[30, 37]	0.022
[32, 43]	0.027
[34, 41]	0.03
[35, 44]	0.06
[36, 47]	0.035
[37, 50]	0.07
[39, 51]	0.044
[41, 48]	0.044
[41, 57]	0.05
[43, 50]	0.039

Focal elements	Bpa
[45, 52]	0.052
[46, 55]	0.087
[47, 54]	0.046
[47, 56]	0.033
[48, 57]	0.081
[49, 62]	0.038
[50, 62]	0.066
[52, 64]	0.061
[58, 67]	0.055
[60, 69]	0.06

5.2 Subtraction of focal elements:

Subtracting all the focal elements using interval arithmetic we get 20 numbers of focal elements (intervals). Now, the corresponding basic probability assignments of resulting focal elements are calculated using (4.2) and arranging all the focal elements in increasing order of the left end point are given in the following table 4.

Table4: Basic probability assignment of resulting focal elements using algebraic Subtraction.

Focal elements	Bpa
[-42,-33]	0.032
[-40,-27]	0.042
[-37,-25]	0.028
[-35,-19]	0.038
[-31,-22]	0.074
[-30,-21]	0.021
[-29,-20]	0.064
[-28,-15]	0.028
[-27,-20]	0.033
[-26,-14]	0.066

Focal elements	Bpa
[-25,-14]	0.044
[-24,-12]	0.056
[-23,-16]	0.036
[-21,-10]	0.048
[-19,-10]	0.05
[-17,-8]	0.043
[-16,-9]	0.066
[-14,-7]	0.077
[-12,-5]	0.083
[-10.-3]	0.071

5.3 Multiplication of focal elements:

Multiplying all the focal elements using interval arithmetic we get 20 numbers of focal elements (intervals). Now, the corresponding basic probability assignments of resulting focal elements are calculated using (4.3) and arranging all the focal elements in increasing order of the left end point are given in the following table 5.

Table5: Basic probability assignment of resulting focal elements using algebraic multiplication.

Focal elements	Bpa
[125,252]	0.0075
[145,288]	0.018
[150,315]	0.0645
[170,378]	0.0375
[175,420]	0.01
[203,480]	0.024
[210,423]	0.0225
[210,525]	0.086
[238,630]	0.05
[294,705]	0.03

Focal elements	Bpa
[400,560]	0.0175
[450,616]	0.015
[464,640]	0.042
[480,700]	0.1505
[522,704]	0.036
[540,770]	0.129
[544,840]	0.0875
[612,924]	0.075
[672,940]	0.0525
[756,1034]	0.045

5.4 Division of focal elements:

Dividing all the focal elements using interval arithmetic we get 20 numbers of focal elements (intervals). Now, the corresponding basic probability assignments of resulting focal elements are calculated using (4.4) and arranging all the focal elements in increasing order of the left end point are given in the following table 6.

Table6: Basic probability assignment of resulting focal elements using algebraic Division.

Focal elements	Bpa
[0.10638,0.21429]	0.0242
[0.11904,[0.26471]	0.0145
[0.14285,0.3]	0.0084
[0.14893,0.35715]	0.0322
[0.15625,0.31035]	0.0302
[0.1666,0.44118]	0.0194
[0,17857,0.36]	0.0726
[0.2,0.5]	0.0113
[0.21875,0.51725]	0.0403
[0.25,0.6]	0.0968

Focal elements	Bpa
[0.34042,0.4762]	0.0565
[0.38095,0.58824]	0.0339
[0.38297,0.52381]	0.0484
[0.42857,0.64706]	0.0290
[0.45714,0.6667]	0.0.0197
[0.5,0.68966]	0.0706
[0.51428,0.7334]	0.0169
[0.5625,0.75863]	0.0605
[0.57142,0.8]	0.1694
[0.64285,0.88]	0.1452

CONCLUSION

Evidence theory based uncertainty quantification is a recent trend, as it can possess the computation with imprecise information. Probabilistic methods can handle only aleatory uncertainty. Evidence theory can handle both aleatory and epistemic uncertainty. Three important functions in evidence theory: the basic probability assignment function (bpa), Belief function (Bel), and Plausibility function (Pl) are used to quantify the given variable. One of the advantages of evidence theory is that focal elements and their corresponding basic probability assignments of variables can be combined. Here, we found that that methods proposed by authors in [5] are also suitable to combine interval focal elements and their basic probability assignments of two variables.

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