International Journal of Mathematical Archive-3(3), 2012, Page: 1125-1135
Available online through www.ijma.info ISSN 2229 – 5046

STEADY FLOW OF A SLIGHTLY THERMO-VISCOUS FLUID IN A POROUS SLAB BOUNDED BETWEEN TWO FIXED POROUS PARALLEL PLATES

N. Pothanna[1*], N. Ch. Pattabhi Ramacharyulu[2] and P. Nageswararao[3]

[1]Asst.Prof. of Mathematics, (H&S Department), VNRVJIET, Hyderabad (A.P), India
***Email:** pothareddy81@gmail.com*

[2]Prof. of Mathematics,(Retd.) National Institute of Technology, Warangal (A.P), India
***Email:** pattabhi1933@yahoo.com*

[3]Prof. of Mathematics, (H&S Department), VNRVJIET, Hyderabad (A.P), India
***Email:** nageswararao_p@vnrvjiet.in*

(Received on: 14-02-12; Accepted on: 06-03-12)

ABSTRACT

In this paper, the problem of the steady flow of a second order thermo-viscous fluid through a porous slab bounded between two fixed parallel porous plates is examined. The two plates are kept at two different temperatures and the flow is generated by a constant pressure gradient. The solution for the differential equations for the velocity and temperature distribution with appropriate boundary conditions have been obtained by analytical method. Flow rate, shear stress and heat transfer coefficient are calculated and illustrated graphically. The flow in the absence of porosity of the plates and porosity of the medium has been deduced as the special cases. A special feature of thermo-viscous fluids is that a force is generated in the direction perpendicular to both the flow direction and the cross-stresses perpendicular to the plates.

Key Words: *Thermo-viscous fluids, Darcy's flux, strain thermal conductivity coefficient, porosity of the plates(V), porosity of the medium(S).*

1. INTRODUCTION

Koh and Eringin [2] introduced the concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences. For such a class of fluids, the stress-tensor 't' and heat flux bivector 'h' are postulated as polynomial functions of the kinematic tensor,viz.,the rate of deformation tensor 'd':

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal gradient bivector 'b'

$$b_{ij} = \in_{ijk} \theta,_k$$

where u_i is the i^{th} component of velocity and θ is the temperature field.

A second order theory of thermo-viscous fluids is characterized by the pair of thermo-mechanical constitutive relations:

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd)$$

and

$$h = \beta_1 b + \beta_3 (bd + db)$$

with the constitutive parameters α_i , β_i being polynomials in the invariants of d and b in which the coefficients depend on density(ρ) and temperature(θ) only. The fluid is Stokesian when the stress tensor depends only on the rate of deformation tensor and Fourier-heat-conducting when the heat flux bivector depends only on the temperature

***Corresponding author:** N. Pothanna[1]*, ***E-mail:** pothareddy81@gmail.com*

gradient-vector, the constitutive coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient of viscosity respectively and α_5 as that of cross-viscosity.

Fluid flows through a porous media has been a subject of both experimental and theoretical since a long time. Darcy, based on the findings of a large number of flows through porous media, proposed the empirical law $Q = -\frac{k^*}{\mu} A.\nabla P$, where Q is the total discharge of the fluid, k^* is the permeability of the medium, A is the cross-sectional area to flow the fluid, μ is the viscosity of the fluid and ∇P is the pressure gradient in the direction of the fluid. Dividing both sides of the equation by the area then the above equation becomes $q = -\frac{k^*}{\mu}\nabla P$, where q is known as Darcy's fluid flux and we know that the fluid velocity (u) is proportional to the fluid flux(q) by the porosity(k^*) , then $\nabla P = -\frac{\mu}{k^*} u$. The negative sign indicates that fluids flow from high pressure to low pressure.

The flow of incompressible thermo-viscous fluids satisfies the usual conservation equations.

Equation of Continuity

$$v_{i,i} = 0$$

Equation of Momentum

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i} - \frac{\mu}{k^*} v_i$$

and Energy equation

$$\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma - \frac{\mu}{k^*} v_i^{\,2}$$

where

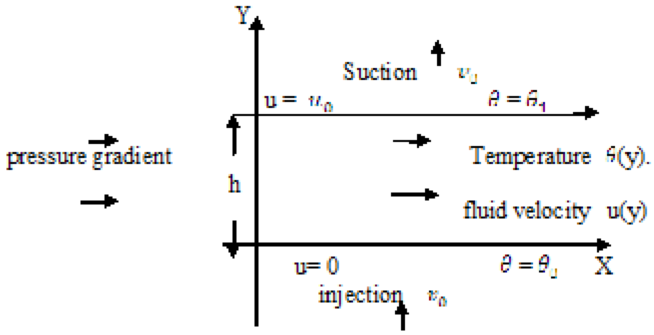
$F_k = k^{th}$ component of external force per unit mass,
$c =$ specific heat,
$\gamma =$ energy source per unit mass and
$q_i = i^{th}$ component of heat flux bivector.
$q_i = \in_{ijk} h_{jk} / 2$

Recently, the problem of thermo-viscous fluid flow between two non permeable fixed parallel plates was examined by Pattabhi Ramacharyulu and P.Nageswar Rao [8]. and the flow of thermo-viscous fluid between two non permeable parallel plates in relative motion was examined earlier by Pattabhi Ramacharyulu and K. Anuradha [9] . P.Nageswar.Rao and Srinivas Joshi [7] investigated the steady flow of thermo-viscous fluid between two parallel porous plates in relative motion.

2. MATHEMATICAL FORMATION OF THE PROBLEM

Consider the steady flow of a second order thermo-viscous fluid through a porous medium bounded between two porous parallel plates. The flow is generated by a constant pressure gradient in a direction parallel to the plates. Further, the plates are assumed to be porous allowing a constant injection at the lower plate and equal suction at the upper plate. Let v_0 be the injection/suction velocity.

With reference to a coordinate system O(XYZ) with origin on the plate, the X-axis in the direction of the fluid flow, Y-axis perpendicular to the plates. The plates are represented by y=0 and y=h. The two plates are maintained at constant temperatures θ_0 and θ_1 respectively.



Let the steady flow between the two plates is characterized by the velocity field [u(y), v_0, 0] and temperature field θ(y). This choice of velocity satisfies the continuity equation. The equations of motion in the absence of external forces and internal energy sources reduces to

in the X-direction

$$\rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial^2 \theta}{\partial y^2} + \rho F_x - \frac{\mu}{k^*} u \tag{1}$$

in the Y-direction :

$$0 = \mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho F_y \tag{2}$$

in the Z- direction :

$$0 = \alpha_8 \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}\right) + \rho F_z \tag{3}$$

and the energy equation

$$\rho c\left(u\frac{\partial \theta}{\partial x} + v_0 \frac{\partial \theta}{\partial y}\right) = \mu\left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} + k\frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x}\frac{\partial^2 u}{\partial y^2} + \rho\gamma - \frac{\mu}{k^*}u^2 \tag{4}$$

together with the boundary conditions u=0, $\theta = \theta_0$ at y=0

u =0, $\theta = \theta_1$ at y=h (5)

The fluid is assumed to be slightly thermo-viscous in such a way that the interaction between the mechanical stress and thermal gradients characterized by the coefficient (α_6) is of a lower order in magnitude compared to the magnitude of viscous dispensation $\mu^2 \left(\frac{du}{dy}\right)^2$ and non Fourier heat transfer coefficient β_3 i.e. terms containing α_6 in the momentum and energy balance equation taken to be smaller than the other terms in the energy equation. Under this assumption and in the absence of external forces and internal energy source, the equations (1) and (4) can be reduced to

$$\rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k^*} u \tag{6}$$

$$\rho c\left(u\frac{\partial \theta}{\partial x} + v_0 \frac{\partial \theta}{\partial y}\right) = \mu\left(\frac{\partial u}{\partial y}\right)^2 + k\frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k^*}u^2 \tag{7}$$

The following non-dimensional quantities are introduced

$$y = hY \ , \quad u = \left(\frac{\mu}{\rho h}\right) U \ , \quad u_0 = \left(\frac{\mu}{\rho h}\right) U_0 \ , \ T = \frac{\theta-\theta_0}{\theta_1-\theta_0} \ , \quad \frac{\partial \theta}{\partial x} = \frac{\theta_1-\theta_0}{h} C_2 \ , \quad \frac{\partial p}{\partial x} = \frac{\mu^2}{\rho h^3} C_1 \ , \ S = \frac{h^2}{k^*} \ ,$$

$$b_3 = \frac{\beta_3}{\rho h^2 c} \ , a_1 = \frac{\mu^2}{\rho h^2 c(\theta_1-\theta_0)} \ , \ a_6 = \frac{\alpha_6 \rho (\theta_1-\theta_0)^2}{\mu^2} \ , \ V = \frac{v_0 \rho h}{\mu}$$

where C_1 and C_2 are non- dimensional pressure and temperature gradients respectively and S is the porosity of the medium and V is the porosity of the plates

In terms of these non-dimensional quantities, the equations (6) and (7) can be written as

$$V\frac{dU}{dY} = -C_1 + \frac{d^2U}{dY^2} - SU \tag{8}$$

$$UC_2 + V\frac{dT}{dY} = a_1\left[\left(\frac{dU}{dY}\right)^2 - SU^2\right] + b_3 C_2 \frac{d^2U}{dY^2} + K\frac{d^2T}{dY^2} \tag{9}$$

together with boundary conditions

$U(0) = 0 \qquad U(1) = 0$ (10)

$T(0) = 0 \qquad T(1) = 1$ (11)

The equation (8) and the boundary conditions in (10) yields the velocity distribution

$$U(Y) = \frac{C_1}{S\sinh m_2}\left\{e^{m_1(Y-1)}\sinh m_2 Y - e^{m_1 Y}\sinh m_2(Y-1) - \sinh m_2\right\}$$

The equation (9) and the boundary conditions in (11) yields the temperature distribution

$$\begin{aligned}
T(Y) &= \frac{YC_1(a_1C_1-C_2)}{mS} + \left[\frac{e^{mY}-1}{e^m-1}\right]\left(1-\frac{C_1(a_1C_1-C_2)}{mS}\right) \\
&- \frac{a_1AB(m_1^2-m_2^2-S)}{m_1(2m_1-m)(e^m-1)}\left\{e^{2m_1}(1-e^{mY}) + (e^{mY}-e^m) - e^{2m_1Y}(1-e^m)\right\} \\
&+ \frac{A\left(C_2-2a_1C_1-b_3C_2(m_1+m_2)^2\right)}{(m_1+m_2)(m_1+m_2-m)(e^m-1)}\left\{e^{m_1+m_2}(1-e^{mY}) + (e^{mY}-e^m) - e^{(m_1+m_2)Y}(1-e^m)\right\} \\
&+ \frac{B\left(C_2-2a_1C_1-b_3C_2(m_1-m_2)^2\right)}{(m_1-m_2)(m_1-m_2-m)(e^m-1)}\left\{e^{m_1-m_2}(1-e^{mY}) + (e^{mY}-e^m) - e^{(m_1-m_2)Y}(1-e^m)\right\} \\
&- \frac{a_1A^2\left((m_1+m_2)^2-S\right)}{2(m_1+m_2)[(2(m_1+m_2)-m](e^m-1)}\left\{e^{2(m_1+m_2)}(1-e^{mY}) + (e^{mY}-e^m) - e^{2(m_1+m_2)Y}(1-e^m)\right\} \\
&- \frac{a_1B^2\left((m_1-m_2)^2-S\right)}{2(m_1-m_2)[(2(m_1-m_2)-m](e^m-1)}\left\{e^{2(m_1-m_2)}(1-e^{mY}) + (e^{mY}-e^m) - e^{2(m_1-m_2)Y}(1-e^m)\right\}
\end{aligned}$$

The flow rate Q is:

$$Q=\int_0^1 U(Y)dY = \frac{2C_1 m_2}{S(m_1^2-m_2^2)}\left[\frac{\cosh m_1+\cosh m_2}{\sinh m_2}\right]-\frac{C_1}{S}$$

The Shear Stress is: $\frac{dU}{dY}$

$$=\frac{C_1}{S\sinh m_2}\left\{e^{m_1(Y-1)}[m_2\cosh m_2 Y+m_1\sinh m_2 Y]-e^{m_1 Y}[m_2\cosh m_2(Y-1)+m_1\sinh m_2(Y-1)]\right\}$$

The Shear Stress on the Lower plate is: $\frac{dU}{dY}/(Y=0)$

$$=\frac{C_1}{S\sinh m_2}\left\{m_2(e^{-m_1}-\cosh m_2)-m_1\sinh m_2\right\}$$

The Shear Stress On the upper plate is: $\frac{dU}{dY}/(Y=1)$

$$=\frac{C_1}{S\sinh m_2}\left\{m_1\sinh m_2-m_2(e^{m_1}-\cosh m_2)\right\}$$

The heat transfer coefficient characterized by the Nussult number is: $\frac{dT}{dY}$

$$=p+me^{mY}\left\{\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\right\}$$
$$+(1-e^m)\left\{2p_1m_1e^{2m_1Y}-(m_1+m_2)e^{(m_1+m_2)Y}[p_2-2p_4e^{(m_1+m_2)Y}]-(m_1-m_2)e^{(m_1-m_2)Y}[p_3-2p_5e^{(m_1-m_2)Y}]\right\}$$

The heat transfer coefficient characterized by the Nussult number on the lower plate is: $\frac{dT}{dY}/(Y=0)$

$$=p+m\left\{\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\right\}$$
$$+(1-e^m)\left\{2p_1m_1-(m_1+m_2)[p_2-2p_4]-(m_1-m_2)[p_3-2p_5]\right\}$$

The heat transfer coefficient characterized by the Nussult number on the upper plate is: $\frac{dT}{dY}/(Y=1)$

$$=p+me^m\left\{\frac{p-1}{1-e^m}-p_1(1-e^{2m_1})+p_2(1-e^{m_1+m_2})+p_3(1-e^{m_1-m_2})-p_4(1-e^{2(m_1+m_2)})-p_5(1-e^{2(m_1-m_2)}\right\}$$
$$+(1-e^m)\left\{2p_1m_1e^{2m_1}-(m_1+m_2)e^{(m_1+m_2)}[p_2-2p_4e^{(m_1+m_2)}]-(m_1-m_2)e^{(m_1-m_2)}[p_3-2p_5e^{(m_1-m_2)}]\right\}$$

From the equation (2) the cross-viscous stress generated in the direction perpendicular to the flow is:

$$\rho F_y = -\mu_c\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2$$

$$=\frac{-2\mu_C C_1^2}{S^2\sin^2 hm_2}\left\{\begin{array}{l} e^{m_1(2Y-1)}\left\{\begin{array}{l}(m_1^2+m_2^2)[m_1\cosh m_2(2Y-1)+m_2\sinh m_2(2Y-1)]\\ -m_1\left[(m_1^2+m_2^2)\cosh m_2+2m_1m_2\sinh m_2\right]\end{array}\right\}\\ +\frac{e^{2m_1(y-1)}}{2}\left\{\begin{array}{l}m_2(m_1^2+m_2^2)\sinh 2m_2(Y-1)+2m_1^2m_2\sinh 2m_2Y\\ +m_1(m_1^2+3m_2^2)\cosh 2m_2Y-m_1(m_1^2-m_2^2)\end{array}\right\}\\ +\frac{e^{2m_1Y}}{2}\left\{\begin{array}{l}m_2(m_1^2+m_2^2)\sinh 2m_2Y-2m_1^2m_2\sinh 2m_2(Y-1)\\ +m_1(m_1^2-m_2^2)\cosh 2m_2(Y-1)-m_1(m_1^2+3m_2^2)\end{array}\right\}\end{array}\right\}$$

From this we observed that the viscous stress generated depends only on Riner-Rivilin cross viscosity coefficient (i.e. μ_c) and is independent of thermo viscous parameters.

Here

$$p = \frac{C_1(a_1C_1 - C_2)}{mS} - 1 \ , \quad p_1 = \frac{a_1 AB(m_1{}^2 - m_2{}^2 - S)}{m_1(2m_1 - m)(e^m - 1)} \ ,$$

$$p_2 = \frac{A[C_2 - 2a_1C_1 - b_3C_2(m_1 + m_2)^2]}{(m_1 + m_2)(m_1 + m_2 - m)(e^m - 1)} \ , \qquad p_3 = \frac{B[C_2 - 2a_1C_1 - b_3C_2(m_1 - m_2)^2]}{(m_1 - m_2)(m_1 - m_2 - m)(e^m - 1)}$$

$$p_4 = \frac{a_1A^2[(m_1 + m_2)^2 - S]}{2(m_1 + m_2)[2(m_1 + m_2) - m](e^m - 1)} \ , \ p_5 = \frac{a_1B^2[(m_1 - m_2)^2 - S]}{2(m_1 - m_2)[2(m_1 - m_2) - m](e^m - 1)} ,$$

$$m_1 = \frac{V}{2} \ , \ m_2 = \frac{\sqrt{V^2 + 4S}}{2} \ , \ m = \frac{V}{K} \ , \ A = \frac{C_1(e^{-m_1} - e^{-m_2})}{2S \sinh m_2} \ , B = \frac{C_1(e^{m_2} - e^{-m_1})}{2S \sinh m_2}$$

3. RESULTS AND DISCUSSION

The effects of large and small values of various parameters like thermal conductivity coefficient(b_3), porosity of the medium(S) and porosity of the plates(V)on velocity and temperature distributions have been discussed in the following figures by taking C_1=1.5, C_2 =1, K=1, a_1 =1

From Fig. (1) & Fig. (2), it is observed that, for low and high porosity of the medium (i.e. for S=0.01 and S=10) the velocity of the fluid is decreases up to the center of the channel and then it is increases so as to attain the velocity of the upper plate. From Fig. (1) & Fig (2), it is also observed that, as we increase the porosity of the medium the velocity profiles slowly increases and if we increase the porosity of the plates (i.e. V=0.1-2.0) the velocity profiles are increases upto the center of the channel then from the centre of the channel the velocity profiles slowly decreases near the upper plate then after they are coincident upto the upper plate. The parabolic profiles are realized in the following figures ((1) & (2)).

From Fig.(3), it is found that, for b_3=1 and if the porosity of the medium, porosity of the plates are small (i.e. S=0.01,V=0.1,0.5,1) the temperature profiles are sharply rises and attains the maximum temperature at the hotter plate and for large V (i.e.for V=1.5) the temperature profile decreases near the lower plate then it is increases to attain the temperature of the hotter plate. If the porosity of the plates are very large (i.e.for V=2) the temperature cools very fast upto the centre of the channel then it is increases to attain the maximum at the hotter plate.

Fig.(4) illustrates that, for high porosity of the medium (i.e. for S=10), as we increase the value of thermal conductivity coefficient (i.e b_3=3) and in the increasing the values of V the temperature cools very fast near the lower plate then the temperature profiles are increases near the hotter plate , after wards the temperature profiles slowly decreases so as to attain the temperature of the hotter plate.

From Fig.(5) &Fig.(6), for low and high porosity of the medium(i.e. for S=0.01 and S=10), it is observed that, for increasing the values of the porosity of the plates (i.e. V=0.1-2.0) the shear stress increases near the center of the channel then it is decreases near the upper plate , after wards it is increases and attains maximum at the upper plate and also from the Fig.(5) &Fig.(6), we observed that there are two inflection points one is near the center of the plates ,other one is near the upper plate.

From Fig.(7) , for low porosity of the medium(i.e. for S=0.01), it is observed that, for increasing the values of the porosity of the plates (i.e. V=0.1-2.0) the heat transfer coefficient slowly increases near the center of the channel then it is slowly decreases near the upper plate , after wards it is increases and attains maximum at the upper plate

From Fig.(8), for low porosity of the medium(i.e. for S=10), it is observed that, for increasing the values of the porosity of the plates (i.e. V=0.1-2.0) the heat transfer coefficient decreases upto the center of the channel then it is increases and coincides with the nussult number of the upper plate.

From Fig.(7)&Fig.(8), It is noticed that, for low porosity of the medium there are two inflection points one is near the center of the plates other one is near the upper plate but for the high porosity of the medium only one inflection point is observed.

Vel (U) with S=0.01

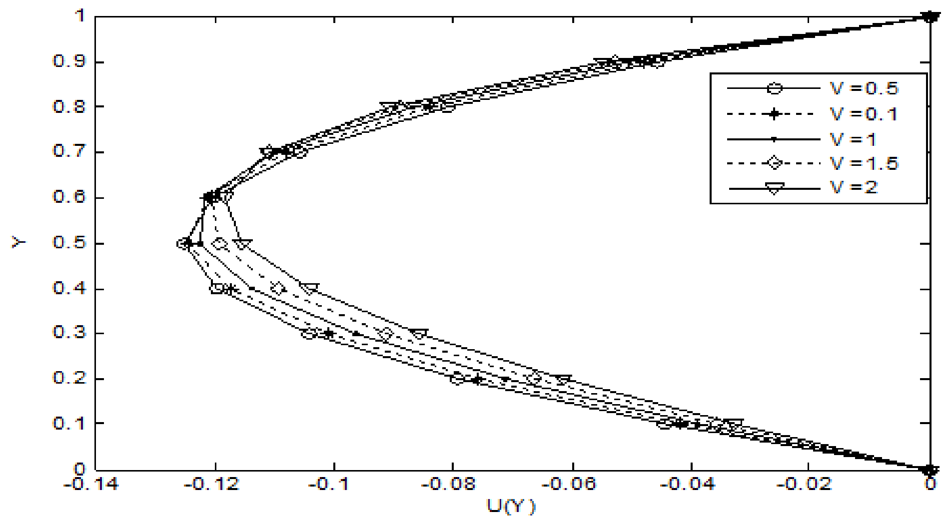


Fig. (1): Variations of the velocity (U) profiles with the porosity of the medium(S) and the porosity of the plates (V)

Vel (U) with S=10

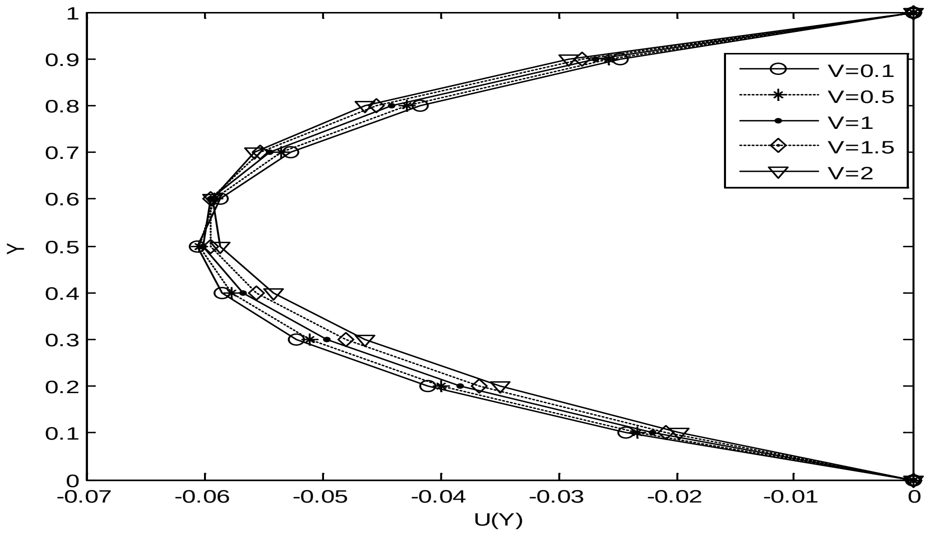


Fig. (2): Variations of the velocity (U) profiles with the porosity of the medium(S) and the porosity of the plates (V)

Temp (T) with S=0.01 and b_3=1

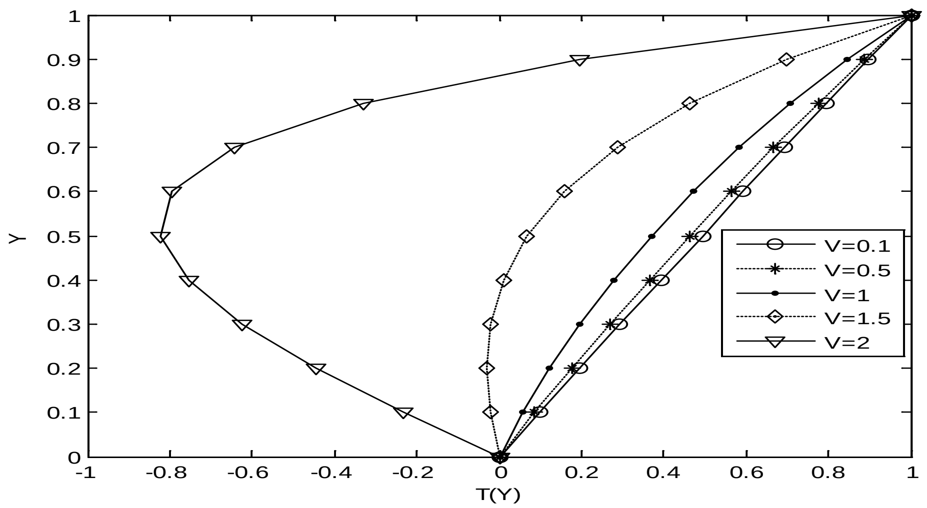


Fig. (3): Variations of the temperature (T) profiles with porosity of the medium (S), thermal conductivity coefficient (b_3) and the porosity of the plates (V)

Temp (T) with S=10 and b_3=3

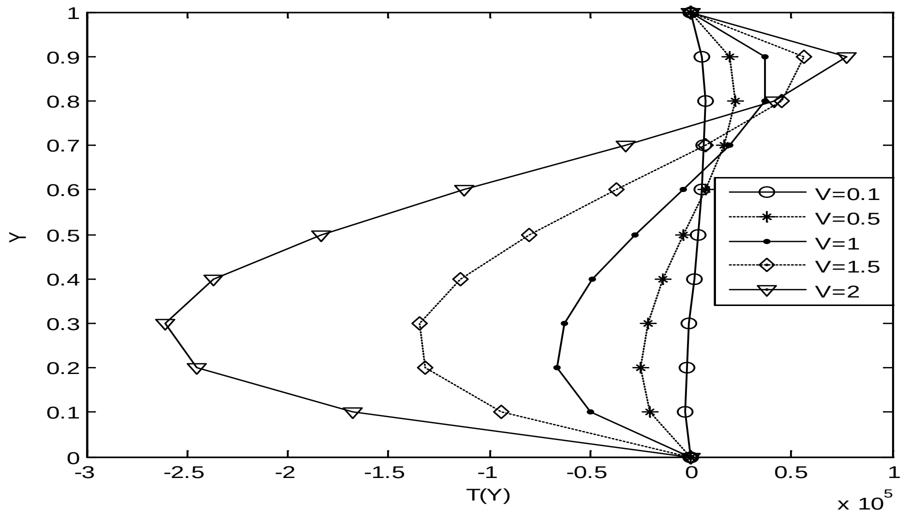


Fig.(4): Variations of the temperature (T) profiles with porosity of the medium (S), thermal conductivity coefficient(b_3) and the porosity of the plates (V)

Shear Stress with S=0.01

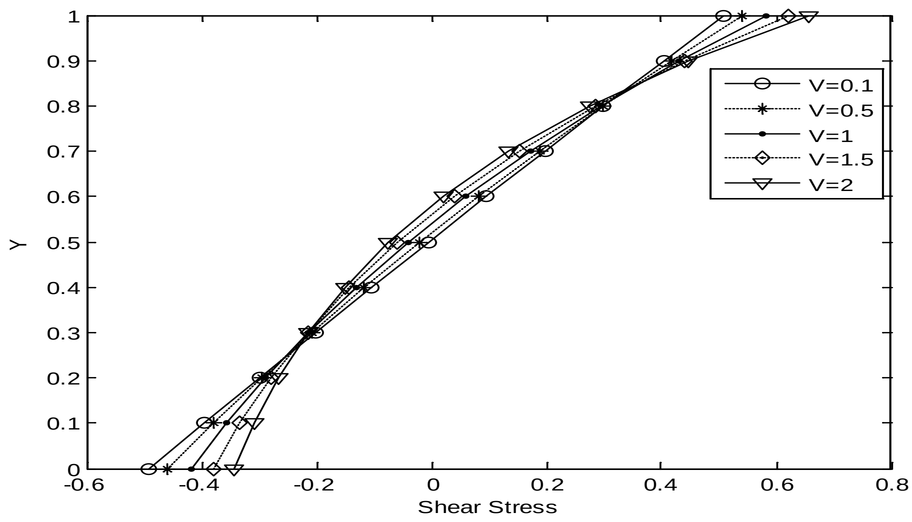


Fig. (5): Variations of the Shear Stress with the porosity of the medium(S) and the porosity of the plates (V)

Shear Stress with S=10

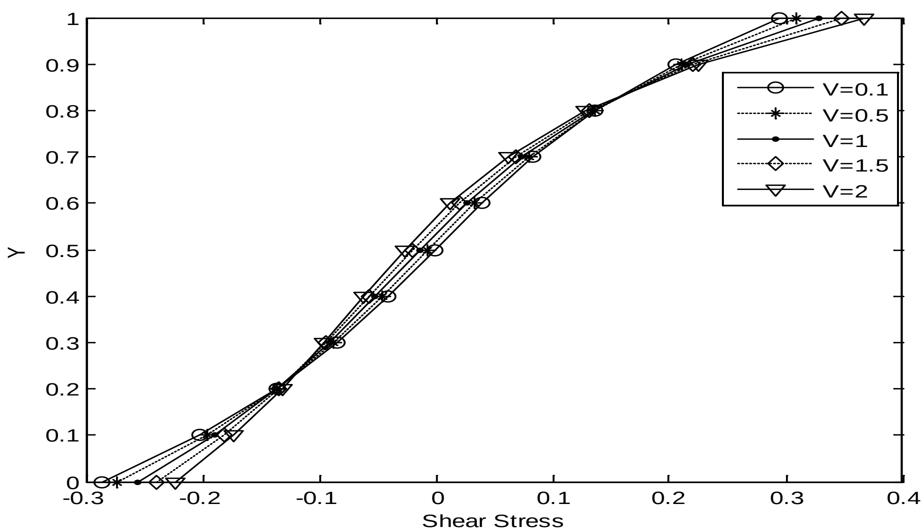


Fig. (6): Variations of the Shear Stress with the porosity of the medium(S) and the porosity of the plates (V)

Nussult Number with S=0.01, b_3=1

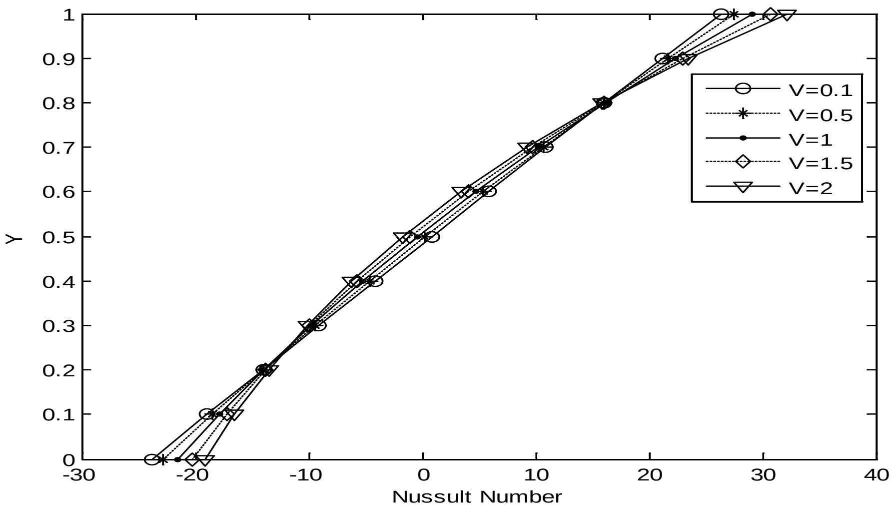


Fig. (7): Variations of the Nussult Number with the porosity of the medium(S), thermal conductivity coefficient (b_3) and the porosity of the plates (V)

Nussult Number with S=10, b_3=3

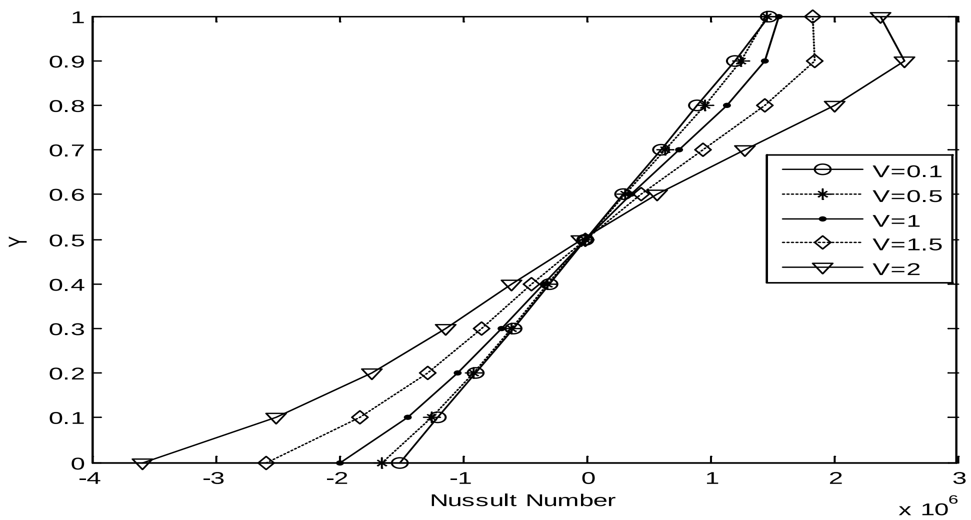


Fig. (8): Variations of the Nussult Number with the porosity of the medium(S), thermal conductivity coefficient (b_3) and the porosity of the plates (V)

4. SPECIAL CASES:

Special case-I: If the plates are not porous (i.e. $V_0 = 0$)

In this case, the velocity distribution is obtained as

$$U(Y) = \frac{C_1}{S \sinh\sqrt{S}} \left\{ \sinh\sqrt{S}Y - \sinh\sqrt{S}(Y-1) - \sinh\sqrt{S} \right\}$$

and the temperature distribution is obtained as

$$T(Y) = Y + Y(Y-1)\left\{ \frac{2a_1 C_1^{\,2}}{KS}\left(1+\cos^2 h\frac{\sqrt{S}}{2}\right)\right\} + 2\sinh\sqrt{S}Y \sinh\sqrt{S}(1-Y)\left\{ \frac{b_3 C_2 S + 8a_1 C_1 \cos^2 h\sqrt{S}}{S^2 \cosh\frac{\sqrt{S}}{2}} \right\}$$

The Shear Stress is: $\dfrac{dU}{dY}$

$$= \frac{C_1}{\sqrt{S}} \sec h \frac{\sqrt{S}}{2} \sinh \frac{\sqrt{S}(2Y-1)}{2}$$

The Nussult number is: $\dfrac{dT}{dY}$

$$= 1 + \frac{2(2Y-1)a_1 C_1^{\,2}}{KS}\left[1+\cos^2 h\frac{\sqrt{S}}{2}\right] + \frac{C_1}{S\sqrt{S}} \sinh \frac{\sqrt{S}(1-2Y)}{2}\left[b_3 C_2 S + 8a_1 C_1 \cos^2 h\sqrt{S}\right]$$

Special case-II: If the medium is not porous (i.e. $S = 0$)

In this case, the velocity distribution is obtained as

$$U(Y) = \frac{C_1}{V_0}\left\{\frac{1-e^{V_0 Y}}{1-e^{V_0}} - Y\right\}$$

and the temperature distribution is obtained as

$$T(Y) = \frac{1}{1-e^{\frac{V_0}{K}}}\left\{\begin{aligned} &1-e^{\frac{V_0 Y}{K}} + \frac{q_1 K}{V_0^{\,2}(K-1)}\left\{e^{\frac{V_0}{K}}\left(1-e^{V_0 Y}\right) - e^{\frac{V_0 Y}{K}}\left(1-e^{V_0}\right) + \left(e^{V_0 Y} - e^{V_0}\right)\right\} \\ &-\frac{q_2 K}{2V_0^{\,2}(2K-1)}\left\{e^{\frac{V_0}{K}}\left(1-e^{2V_0 Y}\right) - e^{\frac{V_0 Y}{K}}\left(1-e^{2V_0}\right) + \left(e^{2V_0 Y} - e^{2V_0}\right)\right\} \\ &-\left(1-e^{\frac{V_0 Y}{K}}\right)\left\{\frac{(m+2)q_3 - 2mq_4}{2m^2}\right\} + Y\left(1-e^{\frac{V_0}{K}}\right)\left\{\frac{(mY+2)q_3 - 2mq_4}{2m^2}\right\} \end{aligned}\right\}$$

The Shear Stress is: $\dfrac{dU}{dY} = \dfrac{C_1}{V_0\left(e^{V_0}-1\right)}\left[1-e^{V_0} + V_0 e^{V_0 Y}\right]$

The Nussult number is: $\dfrac{dT}{dY}$

$$= \frac{1}{\left(e^{\frac{V_0}{K}}-1\right)}\left\{\begin{aligned} &\frac{q_1}{V_0(k-1)}\left[\left(1-e^{V_0}\right)e^{\frac{V_0 Y}{K}} - K\left(1-e^{\frac{V_0}{K}}\right)e^{V_0 Y}\right] - \frac{q_2}{2V_0(2k-1)}\left[\left(1-e^{2V_0}\right)e^{\frac{V_0 Y}{K}} - 2K\left(1-e^{\frac{V_0}{K}}\right)e^{2V_0 Y}\right] \\ &-\frac{V_0}{2Km^2} e^{\frac{V_0 Y}{K}}\left[(m+2)q_3 + 2m(q_4 - m)\right] + \frac{\left(e^{\frac{V_0}{K}}-1\right)}{m^2}\left[(mY+1)q_3 - mq_4\right] \end{aligned}\right\}$$

$$q_1 = \frac{C_1}{V_0\left(e^{V_0}-1\right)}\left[C_2(1-b_3 V_0^{\,2}) + 2a_1 C_1(1-e^{V_0})\right], \quad q_2 = \frac{a_1 C_1^{\,2}}{(1-e^{V_0})^2},$$

$$q_3 = \frac{C_1 C_2}{V_0}, \quad q_4 = \frac{C_1}{V_0^{\,2}\left(1-e^{V_0}\right)}\left[C_2 V_0 - a_1 C_1(1-e^{V_0})\right]$$

5. REFERENCES

[1] Bear, J., Dynamic of fluids in porous media. American Elsevier Pub. Co. inc. Newyork (1970)

[2] Beaver G.S and Joseph D.D Boundary conditions at a naturally permeable wall. J. Fluid Mechanism, Vol. 30(1967), pp.197-207

[3] Collis, R. E., Flow of fluids through porous materials. Reinhold publishing corporation, Newyork. (1961).

[4] Kelly, P.D., Some viscometric flows of incompressible thermo-viscous fluids, Int.J.Engg.sci. 2, pp.519-537.

[5] Koh S.L. and Eringin A.C., On the foundations of non-linear thermo-viscous elastic, Int. J. Engg. Scil, 1, pp.199 to 229

[6] Langlois,W.E., Steady flow of slightly visco-elastic fluid between rotating cylinders, Q. APP. Math, Vol. XXI , No.1,pp.61-71

[7] Nageswar Rao.P and Srinivas Joshi., steady flow of thermo-viscous fluid between two parallel porous plates in relative motion., IEEMS. vol. (9), pp.58-65, September, 2010

[8] Pattabhi Ramacharyulu.N.Ch. and Nageswar Rao.P., steady flow of thermo-viscous fluid between two non permeable fixed parallel plates

[9] Pattabhi Ramacharyulu.N.Ch. and Anuradha.k., Steady flow of a thermo-viscous fluid between two parallel plates in relative motion.,International journal of Mathematical sciences., vol.5(2006)

[10] Preziosi,L and Farina,A., On Darcy's Law for growing porous media, Int. J. Non- linear mechanics, pp.485-491(2002)

[11] Rivlin, R.S., The solution of problems in second-order elasticity theory, J.Rat.Mech.Anal.2, pp.53-59

[12] Yamamoto, K.V. Yoshida, Z. , Flow through a porous wall with convective accelaration, J. of Physical Society,Japan37, No.3, pp.774-779(1974).

****

