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THE ROLL OF REGULAR INCLINE MATRICES IN AUTOMATA THEORY

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ABSTRACT

In this paper, we have highlighted an application of incline matrices in Automata Theory. We have discussed the structure of DFSA and NDFSA. We obtained that any set can be represented as an incline, which is a DFSA and the conversion of a NDFSA to an equivalent incline, which is a DFSA by way of constructing an incline structure with the set of all states in the given NDFSA. In this way, we exhibited that the corresponding transition matrices are space equivalent.

Key words: *Regular incline, regular matrices, DFSA, NDFSA.*

MS Classification: *18B20, 15B33.*

INTRODUCTION

Inclines are additively idempotent semirings in which products are less than (or) equal to either factor. Further, Incline algebra is a generalization of Fuzzy algebra which is a generalization of Boolean algebra. The notion of inclines and their applications are described comprehensively in Cao, Kim and Roush [1]. Von Neumann [7] has introduced the concept of regular elements in a ring. A ring R is regular if and only if every element of R is regular. Recently in [4], it is proved that an element in an incline is regular if and only if it is idempotent. The equivalence of deterministic finite state automata (DFSA) and the conversion of a non deterministic finite state automata (NDFSA) to an equivalent DFSA play an important role in Automata Theory (For details refer to [3, 6]).We review some basic results of Automata in [2].

In this paper, the roll of regular matrices over an incline in Automata theory is discussed. In section 2, some basic definitions and results are given. In section 3,the structure of DFSA and NDFSA are discussed by way of constructing incline structure.

2. PRELIMINARIES

In this section, the required definition and results are given.

Definition 2.1: A nonempty set £ with two binary operations + and · is called an incline if it satisfy the following conditions (We usually suppress the 'dot' in $x{\cdot}y$ and write as xy)
(i) $(£, +)$ is a semilattice.
(ii) $(£,\cdot)$ is a semigroup.
(iii) $x(y+z) = xy+xz$ for all $x,y,z \in £$.
(iv) $x+xy = x$ and $y + xy = y$ for all $x,y \in £$.

Proposition 2.3 [4]: An element $a \in £$ is regular if and only if a is idempotent if and only if $a^2=a$.

Definition 2.4: A matrix A $\in £_{mn}$ is said to be regular if there exists a matrix X $\in £_{nm}$ such that AXA =A. Then X is called a g-inverse of A and A $\{1\}$ denotes the set of all g-inverses of A.

Theorem 2.5[5]: Let £ be an incline. For $A, B \in £_{mn}$ with $R(A) = R(B)$ (or) $C(A) = C(B)$. Then A is a regular matrix ⇔ B is a regular matrix.

The row (column) space $R(A)$ $(C(A))$ of an $m \times n$ matrix A is the subspace of V^n generated by its rows (columns).

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Definition 2.6: A Deterministic finite state automata is A= (I, S, f, A, q), where
(i) a finite set Q of states
(ii) a finite set X of input symbols
(iii) a next state function f:Q×X→Q
(iv) a subset A of Q of accepting state
(v) an initial state qεQ.

Definition 2.7: A Non Deterministic finite state automata A is A= (Q, X, f, A, s), where
(i) Q is a finite set of states
(ii) X is a finite set of input symbols
(iii) f is a next state function f:Q×X→P(Q)(iv)A is a subset of Q of accepting state
(v) sεQ is the initial state.

3. THE ROLE OF REGULAR INCLINE MATRICES IN AUTOMATA THEORY

Definition 3.1: A six- tuple M= (S, I, O, f, g, s)is called a finite state machine if S,I and O are finite nonempty sets ,f:S×I→O,g:S×X →Y and s∈Q. Where, S - Set of states, I - input symbols, O -Output symbols, f - transition function, g -output function, s - initial state .

Definition 3.2: A finite state machine is said to be Semi - Automata, denoted as $M = \{S, I, O, f, s_0\}$. Where *S*-Finite set of states, *I*-Finite set *I* of input symbols, *O*-Finite set *O* of output symbols, *f*-a transition function f that assigns a new state to every pair of state and input, s_0-an initial state.

Definition 3.3: A finite state machine is said to be Full Automata, denoted as $M = \{S, I, O, f, g, s_0\}$. Where *S* - Finite set of states, *I*-Finite set *I* of input symbols, O-Finite set O of output symbols, *f*-a transition function f that assigns a new state to every pair of state and input, *g*-an output function g that assigns an output to every pair of state and input, s_0 -An initial state.

Lemma 3.4: Given any set *S* we can form an incline $£ = (P(S),\cup,\cap)$, whose elements are subsets of S and the set "$\cup$" and "$\cap$" as an incline operation, which is a Deterministic Full State Automata(DFSA).

Proof: Consider any non deterministic full state automata $M = \{S, Q, \upsilon,\delta,F\}$

Where *S* - Finite set of states
Q - Finite set of inputs
υ -A function υ from $S\times Q \rightarrow S$, referred to as the transition function.
δ - A function δ from $S \times Q \rightarrow F$, referred to as the output function.
F - Final set of state.

The equivalent automata can be written as $M' = \{P(S), Q', \upsilon'\delta',F'\}$

where *P(S)* - the power set of *S*.
Q' -Q
υ - $P(S)\times Q \rightarrow S$
δ - A function δ' from $P(S)$ x $Q \rightarrow F'$
F' - F

Here, M' is a DFSA and M' satisfies the Definition of incline (2.1), hence M' is an incline.

Illustration 3.5: Let us consider the incline $£ = (P(D),\cup,\cap)$, $D = \{a, b, c\}$,where P(D)is the power set of D and set inclusion as the order relation "$\leq$".

From this example it is clear that, any set can be represented as an incline under the incline operation, which is a DFSA.

Definition 3.6: A pair of matrices $A = (a_{ij})$ and $B = (b_{ij})$ ε $£_{mn}$ are said to be space equivalent if and only if $R(A) = R(B)$ and $C(A) = C(B)$.

Example 3.7: First we construct the equivalent incline for the following NDFSA.

$M = < I = \{0, 1\}, Q = \{q_0, q_1,\}\upsilon,\delta, s_0= q_0, F = q_1\} >$

Here υ and δ is given by

υ	0	1
q_0	$\{q_0,q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0,q_1\}$

Table 3.1(a)

δ	0	1
q_0	$\{q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_1\}$

Table 3.1(b)

The state diagram of the given NDFSA

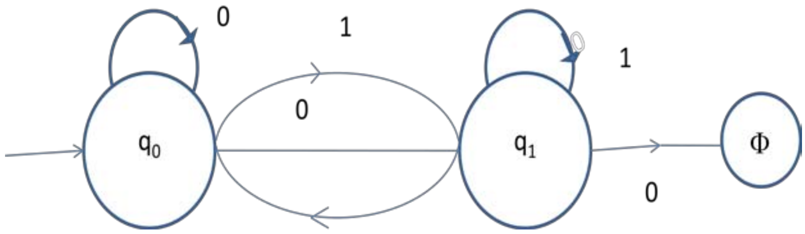

Fig. 3.1(a)

The equivalent DFSA is represented by the equivalent incline

$M' = < I', P(Q), \upsilon', \delta', q_0, F'>$ Where $I' = I = \{0, 1\}$.

$P(Q)$ = all possible subsets of states, that is, $\{\phi, [q_0], [q_1], [q_0, q_1]\}$

$q_0' = [q_0]$ $F' = F = \{q_1\}$

And since $\upsilon'(q_0, 0) = \{q_0, q_1\}$, we have $\upsilon'([q_0],0) = [q_0,q_1]$

Likewise, $\upsilon'([q_1], 0) = [q_0], \upsilon'([q_1],0) = [q_1], \upsilon'([q_1],0) = \phi,$

$\upsilon'([q_1], 1) = [q_0,q_1], \upsilon'(\phi,0) = \upsilon'(\phi,1) = \phi$

Then $\upsilon'([q_0,q_1],0) = [q_0,q_1]$

$\upsilon'(\{q_0,q_1\},0) = \upsilon(q_0,0) \cup \upsilon(q_1,0) = \{q_0,q_1\} \cup \phi = \{q_0,q_1\}$

And $\upsilon'([q_0,q_1],1) = [q_0,q_1]$

Since $\upsilon'(\{q_0,q_1\},1) = \upsilon(q_0,1) \cup \upsilon(q_1,1) = \{q_1\} \cup \{q_0,q_1\} = \{q_0,q_1\}$

Thus υ' and δ' is defined by

υ'	0	1
$[q_0]$	$[q_0,q_1]$	$[q_1]$
$[q_1]$	ϕ	$[q_0,q_1]$
$[q_0,q_1]$	$[q_0,q_1]$	$[q_0,q_1]$

Semi Automata - Incline

Table 3.2(a)

δ'	0	1
$[q_0]$	$[q_1]$	$[q_1]$
$[q_1]$	ϕ	$[q_1]$
$[q_0,q_1]$	$[q_1]$	$[q_1]$

Full Automata – Incline

Table 3.2(b)

The state diagram for an incline given below:

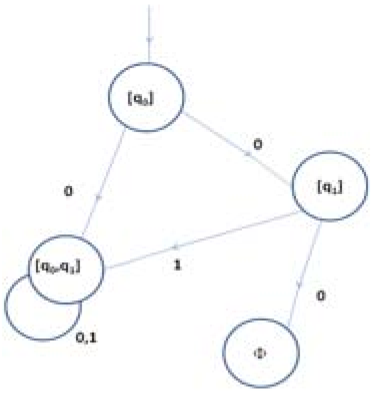

Fig.3.2

Here, for the transition matrix $\upsilon, \upsilon', \delta$ and δ' $R(\upsilon) = R(\upsilon')$ and $R(\delta) = R(\delta')$, υ and δ are regular being idempotent by Proposition (2.3) and by applying Theorem (2.5), υ' and δ' are regular.

Remark 3.8: Any NDFSA is equivalent to an DFSA and the corresponding transition matrices, that is, δ and δ′ are space equivalent, υ and υ' are space equivalent.

Finite state Acceptor 3.9: An acceptor is a machine which can identify the strings of a language.

Example 3.10: Consider the set of non commuting words in *8* generators ordered by length and lexicographically when of equal length is a non commutative incline. Here the ordering is linear and $x+y$ is the greater of the two words that is, $x \geq y$ if the length of the word $x \geq$ the length of the word y.

For this incline, let us construct a finite state acceptor that will accept the set of words multiples of *3*.

Let $M = <I, Q, q_0, \delta>$, Where $I = \{a, b, c, d, e, f, g, h\}$, $Q = \{q_0, q_1, q_2, q_3\}$, $F = \{q_0\}$ and δ is defined by

δ	X	Y
q_0	q_1	q_2
q_1	q_3	q_3
q_2	q_3	q_3
q_3	q_0	q_0

Table 3.3

Where $x = \{a, c, e, g\}$, $y = \{b, d, f, h\}$. Check the word *ad, age, deed, cabbage* is accept or not.

Consider *ad:* $\delta(q_0, ad) = \delta(\delta(q_0, a)\, d) = \delta(q_1, d) = q_3 \notin F$.

Hence *ad* is not accepted by *M.*

Consider *age:* $\delta(q_0, age) = \delta(\delta(q_0, a)ge) = \delta(\delta(q_1, g)e) = \delta(q_3, e) = q_0 \, \varepsilon \, F$.

Hence *age* is accepted by *M.*

Consider *deed:* $\delta(q_0, deed) = \delta(\delta(q_0, d)eed) = \delta(\delta(q_2, e)ed) = \delta(\delta(q_3, e)d) = \delta(q_0, d) = q_2 \notin F$.

Hence *deed* is not accepted by *M.*

Consider *cabbage:* $\delta(q_0, cabbage) = \delta(\delta(q_0, c)abbage) = \delta(\delta(q_1, a)bbage) = \delta(\delta(q_3, b)bage)$
$= \delta(\delta(q_0, b)age) = \delta(\delta(q_3, a)ge) = \delta(\delta(q_0, g)e) = \delta(q_1, e) = q_3 \notin F$.

Hence *cabbage* is not accepted by *M.*

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