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THE NUMERICAL COMPUTATIONS FOR ANTISYMMETRIC MODES OF VIBRATION OF A TRANSVERSELY ISOTROPIC GENERALIZED THERMOELASTIC PLATE

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ABSTRACT

The objective of this paper is to investigate the propagation of plane harmonic waves in a homogenous transversely isotropic plate of finite width. A generalized theory of thermoelasticity effects are taken into consideration in detail with three different relaxation time theories. The frequency equation for the plate in closed form and suitable mathematical conditions for antisymmetric wave mode propagation is derived. Numerical computations for the frequency equation for three various theories of generalized thermoelasticity is carried out for zinc crystal. The real and imaginary parts of the frequency equation as a function of phase velocity for different values of thermal relaxation times are illustrated graphically. It is found that the frequency equation of the antisymmetric motion can be oscillating with respect to the medial of the plate. Moreover, it gets modified due to the thermal relaxation times and anisotropic effects. Finally, the results for the coupled thermoelasticity can be obtained as particular cases of the results by setting thermal relaxation times equal to zero.

Keywords: *Thermoelasticity, Frequency equation, Thermal relaxation times, Harmonic wave propagation*

1. INTRODUCTION

The frequency equation in anisotropic plates find use in many engineering structures and other areas of practical interest, such as slabs on columns, printed circuit boards or solar panels supported at a few points. With their potential applications of antisymmetric modes of vibration of plates for considering the theories of generalized thermoelasticity has received considerable attention from researchers.

The theory of thermoelasticity has aroused intense attention in our attempt to understand the nature of the interaction between temperature and strain fields because of its application in most heavy industries where various structural elements are often subjected to mechanical loads at an elevated temperature. Engineering materials such as fiber reinforced composite, graphite, zinc, ceramics, and aluminum-epoxy, where high strength-to- weight and stiffness-to-weight ratios are required. These materials are crucial for structural applications, and have resulted in considerable research activities on their behavior. Consequently studies of the propagation of elastic waves in the layered media [1], [2], [11] and [12] which are anisotropic in nature become very important and have long been of interest to researchers in the fields of geophysics, acoustics and nondestructive evaluation.

The heat conduction equations for classical uncoupled and coupled theories of thermoelasticity (here called conventional dynamics or CD theory) are of the diffusion type and predict an infinite speed of propagation of the heat wave which is physically inadmissible. To eliminate this paradox of the classical approach, theories of generalized thermoelasticity were developed. At present, there are various generalized approaches but the theories proposed by Lord and Shulman [9] and Green and Lindsay [7] (here called L-S and G-L theories respectively) are most popular. These theories have been developed by introducing one or two relaxation times in the thermoelastic process, with an aim to eliminate the paradox of an infinite speed for the propagation of thermal signals. The L-S model is based on a modified Fourier's law, but the G-L model even allows second sound without violating the classical Fourier's law. The two theories are structurally different, and one cannot be obtained as a particular case of the other. The two theories both ensure finite speeds of propagation for thermal wave. Dhaliwal and Sherief [6] extended the theory of Lord and Shulman to an anisotropic media. Various problems characterizing these two theories have been investigated and have revealed some interesting phenomena. Chandrasekharaiah [4, 5] has reported brief reviews of this topic. Recently, Li et al. [8] discussed the vibration of thermally post buckled orthotropic circular plate. Nayfeh [10] illustrated the

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propagation of horizontally polarized shear waves in multilayered anisotropic media. Several others authors have studied thermoelastic waves in a plate. For example, the theory of micropolar generalized thermoelastic continua has been employed by Sharma and Kumar [14] to study the propagation of plane waves in micropolar thermoelastic plates bordered with inviscid liquid layers (or half-spaces) with varying temperature on both sides. Also, Partap and Kumar [13] have studied the free vibration analysis of micropolar thermoelastic cylindrical curved plate in circumferential direction. Moreover, Shaw and Mukhopadhyay [21] have investigated the thermoelastic waves with thermal relaxation in isotropic micropolar plate. Furthermore, Son and Kang [22] have illustrated the effect of initial stress on the propagation behavior of SH waves in piezoelectric coupled plates. Verma et al. [15], [16], [17], [18], [19] and [20] have studied wave propagation in anisotropic media in the context of generalized thermoelasticity with different hypotheses.

In this paper, analysis for the propagation of thermoelastic waves in a thin homogenous transversely isotropic plate is carried out in the framework of the generalized theory of thermoelasticity. Commencing with a formal analysis of waves in a heat-conducting layered plate of a transversely isotropic media, the frequency equation as function of the phase velocity of thermoelastic waves is obtained by invoking continuity at the interface and boundary of conditions on the surfaces of layered plate. Numerical solution of the frequency equations for a zinc material is carried out for different values of relaxation times and illustrated graphically. Finally, when the two thermal relaxation times are neglected, one may get the results as in [11] and [12].

2. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

The basic governing equations of motion for homogeneous anisotropic generalized thermoelasticity in the absence of body forces and heat sources are given by:

$$\frac{\partial^2 \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{1}$$

The equation of heat conduction

$$K_{ij} \frac{\partial^2 T}{\partial x_i \partial x_j} - \rho C_e \left(\frac{\partial T}{\partial t} + \tau_o \frac{\partial^2 T}{\partial t^2}\right) = T_o \beta_{ij} \left(\frac{\partial e_{kk}}{\partial t} + \tau_o \frac{\partial^2 e_{kk}}{\partial t^2}\right), \tag{2}$$

Where the constitutive relations and equations governing linear generalized thermoelastic interaction in a homogenous anisotropic solid are as follows:

$$\sigma_{ij} = c_{ijkl} e_{kl} - \beta_{ij} \left(T + \delta \tau_1 \frac{\partial T}{\partial t}\right), \tag{3}$$

and

$$\beta_{ij} = c_{ijkl} \alpha_{kl}, \quad i, j, k, l = 1,2,3, \tag{4}$$

The strain–displacement relations

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

where ρ is the density, t is the time, u_i is the displacement in the x_i direction, K_{ij} are the thermal conductivities C_e is the specific heat at constant strain, τ_o, τ_1 are thermal relaxation times, σ_{ij} are the components of stress tensor e_{ij} are the components of strain tensor, β_{ij} is thermal moduli, α_{kl} is the coefficients of linear thermal expansion tensor, *T* is the temperature, T_o is the reference temperature and the fourth-order tensor of the elasticity c_{ijkl} satisfies the symmetry conditions. The parameters in equations (3) and (4) are assumed to satisfy the following conditions [20]:

(i) The thermal conductivity K_{ij} is symmetric and positive-definite

(ii) The thermoelastic coupling tensor β_{ij} is non-singular

(iii) The specific heat C_e at constant strain is positive

(iv) The isothermal linear elasticity is positive-definite in the sense that $c_{ijkl} e_{ij} e_{kl} > 0.$

The use of symbol δ makes the above equations possible for three of generalized thermoelasticity materials. For The L-S (Lord and Shulman) theory $\tau_1 = 0, \quad \tau_o > 0$ and $\delta = 1$. For G-L (Green and Lindsay) theory the thermal relaxation times τ_o and τ_1 satisfy the inequality $\tau_1 \geq \tau_o > 0$ with $\delta = 0$. While for the C-D (Classical Dynamical Coupled) theory, the thermal relaxation times satisfy $\tau_1 = \tau_o = 0, \quad \delta = 0.$

This matrix notation consists of replacing the indices ij or kl by p or q, where $i, j, k, l = 1,2,3$ and $p, q = 1,2,3,4,5,6.$ We may write:

$$c_{ijkl} = c_{pq}, \qquad e_{ikl} = e_{ip}, \qquad \tau_{ij} = \tau_p, \tag{6}$$

where

Table 1. Indices for contracted notation

ij or pq		11	22	33	32=23	31=13	12=21
↓	↓	↓	↓	↓	↓	↓	↓
p	q	1	2	3	4	5	6

3. FORMULATION OF THE PROBLEM AND ITS SOLUTION

We assume an infinite, homogeneous, transversely isotropic, thermally conducting elastic plate of thickness $2d$ initially at uniform temperature T_o. We consider the faces of the plate to be the planes $z = \pm d$ referred to as a rectangular set of Cartesian axes $Oxyz$. We suppose that the x-axis to be in the direction of the propagation of waves so that all particles on a line parallel to y-axis are equally displaced. Therefore, all the field quantities will be independent of y-coordinate. The motion is assumed to take place in the dimensions (x, y). Here, u, w are the displacements of a point in the x, z directions, respectively. The basic governing equations for homogeneous anisotropic generalized thermoelasticity in the absence of body forces and heat sources, are given by

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{44}\frac{\partial^2 u}{\partial z^2} + (c_{13} + \tfrac{1}{2}c_{44})\frac{\partial^2 w}{\partial x \partial z} - \beta_1 \frac{\partial}{\partial x}(T + \tau_1 \frac{\partial T}{\partial t}) = \rho \frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$(c_{13} + \tfrac{1}{2}c_{44})\frac{\partial^2 u}{\partial x \partial z} + \tfrac{1}{2}c_{44}\frac{\partial^2 w}{\partial x^2} + c_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z}(T + \tau_1 \frac{\partial T}{\partial t}) = \rho \frac{\partial^2 w}{\partial t^2}, \tag{8}$$

$$K_1 \frac{\partial^2 T}{\partial x^2} + K_3 \frac{\partial^2 T}{\partial z^2} - \rho C_e (\frac{\partial T}{\partial t} + \tau_o \frac{\partial^2 T}{\partial t^2}) = T_o[\beta_1(\frac{\partial^2 u}{\partial x \partial t} + \delta\tau_o \frac{\partial^3 u}{\partial x \partial t^2}) + \beta_3(\frac{\partial^2 w}{\partial z \partial t} + \delta\tau_o \frac{\partial^3 w}{\partial z \partial t^2})] \tag{9}$$

Now, we introduce the following dimensionless quantities:

$$x^* = \frac{v_1}{k_1}x, \quad z^* = \frac{v_1}{k_1}z, \quad t^* = \frac{v_1^2}{k_1}t, \quad u^* = \frac{v_1^3 \rho}{k_1 \beta_1 T_o}u,$$

$$w^* = \frac{v_1^3 \rho}{k_1 \beta_1 T_o}w, \quad T^* = \frac{T}{T_o}, \quad \tau_o^* = \frac{v_1^2}{k_1}\tau_o, \quad \tau_1^* = \frac{v_1^2}{k_1}\tau_1,$$

$$c_1 = \frac{c_{33}}{c_{11}}, \quad c_2 = \frac{c_{44}}{2c_{11}}, \quad c_3 = \frac{c_{13} + 0.5c_{44}}{c_{11}}, \quad \overline{K} = \frac{K_3}{K_1}, \tag{10}$$

$$\overline{\beta} = \frac{\beta_3}{\beta_1}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho^2 C_e v_1^2}, \quad \omega_1^* = \frac{v_1^2}{k_1}.$$

where $v_1 = (c_{11}/\rho)^{1/2}$ is the velocity of longitudinal waves, $k_1 = K_1/(\rho C_e)$ is the thermal diffusivity in the x-direction, ε_1 is the thermoelastic coupling constant, ω_1^* is the characteristic frequency of the medium and τ^*, τ_1^* are the dimensionless thermal relaxation constants.

Introducing quantities (10) in Equations (7)-(9), after excluding the asterisk (*) for convenience, one may get

$$\frac{\partial^2 u}{\partial x^2} + c_2 \frac{\partial^2 u}{\partial z^2} + c_3 \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(T + \tau_1 \frac{\partial T}{\partial t}), \tag{11}$$

$$c_3 \frac{\partial^2 u}{\partial x \partial z} + c_2 \frac{\partial^2 w}{\partial x^2} + c_1 \frac{\partial^2 w}{\partial z^2} - \frac{\partial^2 w}{\partial t^2} = \overline{\beta} \frac{\partial}{\partial z}(T + \tau_1 \frac{\partial T}{\partial t}), \tag{12}$$

$$\frac{\partial^2 T}{\partial x^2} + \overline{K} \frac{\partial^2 T}{\partial z^2} - (\frac{\partial T}{\partial t} + \delta \tau_o \frac{\partial^2 T}{\partial t^2}) = \varepsilon_1 [(\frac{\partial^2 u}{\partial x \partial t} + \delta \tau_o \frac{\partial^3 u}{\partial x \partial t^2} + \overline{\beta}(\frac{\partial^2 w}{\partial z \partial t} + \delta \tau_o \frac{\partial^3 w}{\partial z \partial t^2})]. \tag{13}$$

The stresses and temperature gradient relevant to our problem in the plate are:

$$\tau_{zz} = (c_3 - c_1)\frac{\partial u}{\partial x} + c_1 \frac{\partial w}{\partial z} - \overline{\beta}(T + \tau_1 \frac{\partial T}{\partial t}), \tag{14}$$

$$\tau_{zx} = \overline{\beta} T_o c_2 (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}), \tag{15}$$

$$T_{,z} = \frac{\partial T}{\partial z} \tag{16}$$

As we considering plane harmonic wave traveling in the x-direction therefore we may take the solutions for u, w and T of Eqs. (11), (12) and (13) is follows:

$$\{u, w, T\} = \{f(z), g(z), h(z)\} exp[i\zeta(x - ct)] \tag{17}$$

where $(c = \omega/\zeta)$ is the phase velocity, ζ and ω are the wave number, circular frequency, respectively and $i = \sqrt{-1}$.

Now using solutions (17) into equations (11), (12) and (13) we get

$$(c_2 D^2 - \zeta^2 + \zeta^2 c^2) f + i c_3 \zeta D g - \tau_a \zeta h = 0, \tag{18}$$

$$i c_3 \zeta D f + (c_1 D^2 - c_2 \zeta^2 + \zeta^2 c^2) g + i \overline{\beta} \tau_a D h = 0, \tag{19}$$

$$i \varepsilon_1 \tau c \zeta^2 f + \varepsilon_1 \overline{\beta} \tau c \zeta D g + (\overline{K} D^2 - \zeta^2 + \zeta^2 c^2 \tau) h = 0 \tag{20}$$

where

$$D = \frac{\partial}{\partial z}, \qquad \tau = i + \delta \tau_o \zeta c, \qquad \tau_a = i + \tau_1 \zeta c. \tag{21}$$

The solutions of equations (18)-(20) can be written in the form:

$$f(z) = \sum_{j=1}^{3} [P_j exp(-\zeta M_j z) - Q_j exp(\zeta M_j z)], \tag{22}$$

$$g(z) = \sum_{j=1}^{3} m_j [P_j exp(-\zeta M_j z) - Q_j exp(\zeta M_j z)], \tag{23}$$

$$h(z)=\zeta \sum_{j=1}^{3} l_j\,[P_j\,exp(-\zeta M_j z)-Q_j\,exp(\zeta M_j z)] \tag{24}$$

where

$$m_j=\frac{[\bar{\beta}(c_2M_j^2+c^2-1)+c_3]M_j}{i[M_j^2(\bar{\beta}c_3-c_1)+c_2-c^2]}, \tag{25}$$

$$l_j=[c_2M_j^2+c^2-1)-ic_3M_jm_j]/\tau_a. \tag{26}$$

where, $P_j, Q_j\ (j=1,2,3)$ are arbitrary constants, and M_1, M_2 and M_3 are the roots of the following equation

$$M^6+B_1M^4+B_2M^2+B_3=0 \tag{27}$$

where

$$B_1=-[\bar{k}c_2(c_2-c^2)+c_1c_2(1-\tau c/\zeta)+\bar{K}c_1(1-c^2)-\bar{K}c_3^2+ic_2\varepsilon_1\beta^{-2}\tau_a\tau c/\zeta]/\bar{K}c_1c_2,$$

$$B_2=[\{\bar{K}(c_2-c^2)+c_1(1-\tau c/\zeta)+i\varepsilon_1\beta^{-2}\tau_a\tau c/\zeta\}(1-c^2)-c_3^2(1-\tau c^2)+c_2(c_2-c^2)$$
$$\times(1-\tau c/\zeta)-i\varepsilon_1(2\bar{\beta}c_3-c_1)\tau_a\tau c/\zeta]/\bar{K}c_1c_2,$$

$$B_3=-[(1-c^2)(c_2-c^2)(1-\tau c/\zeta)+i\varepsilon_1(c_2-c^2)\tau_a\tau c/\zeta]/\bar{K}c_1c_2.$$

The displacement components and temperature of the plate become:

$$u=\sum_{j=1}^{3}[P_j\,exp(-\zeta M_j z)+Q_j\,exp(\zeta M_j z)]\,exp[i\zeta(x-ct], \tag{28}$$

$$w=\sum_{j=1}^{3}m_j[P_j\,exp(-\zeta M_j z)-Q_j\,exp(\zeta M_j z)]\,exp[i\zeta(x-ct], \tag{29}$$

$$T=\zeta\sum_{j=1}^{3}l_j[P_j\,exp(-\zeta M_j z)+Q_j\,exp(\zeta M_j z)]\,exp[i\zeta(x-ct]. \tag{30}$$

4. BOUNDARY CONDITIONS

The non-dimensional boundary conditions at the surfaces $z=\pm d$ of the plate are given by:
(i)Mechanical conditions (stress-free surfaces)

$$\tau_{zz}=0, \qquad \tau_{xz}=0. \tag{31}$$

(ii) Thermal condition (thermally insulated)

$$\frac{\partial T}{\partial z}=0. \tag{32}$$

The use of equations (28), (29) and (30) in equations (14), (15) and (16) leads to a system of the following coupled equations for the arbitrary unknown coefficients P_1, P_2, P_3, Q_1, Q_2 and Q_3:

$$\sum_{j=1}^{3}(iF-c_1m_jM_j+i\bar{\beta}\tau_al_j)[P_j\,exp(-\zeta M_j d)+Q_j\,exp(\zeta M_j d)]\,exp[i\zeta(x-ct]=0,$$

$$\sum_{j=1}^{3}(im_j-M_j)[P_j\,exp(-\zeta M_j d)-Q_j\,exp(\zeta M_j d)]\,exp[i\zeta(x-ct]=0,$$

$$\sum_{j=1}^{3}(-l_j M_j)[P_j \exp(-\zeta M_j d)+Q_j \exp(\zeta M_j d)] \exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(iF - c_1 m_j M_j + i\bar{\beta}\tau_a l_j)[P_j \exp(\zeta M_j z)+Q_j \exp(-\zeta M_j d)] \exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(im_j - M_j)[P_j \exp(\zeta M_j d)-Q_j \exp(-\zeta M_j d)] \exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(-l_j M_j)[P_j \exp(\zeta M_j d)+Q_j \exp(-\zeta M_j d)] \exp[i\zeta(x-ct] = 0. \quad (31\text{-}33)$$

where $F = c_3 - c_2$. We notice that the above six equations which are coming from applying the boundary conditions (31) must be satisfied simultaneously.

5. FREQUENCY EQUATION

The system of equation (33) has a nontrivial solution if and only if the determinant of the coefficients amplitudes P_i and Q_i, where ($i = 1,2,3$) vanishes. After applying algebraic reductions and manipulations this leads to the frequency equation (also called dispersion equation or secular equation) for thermally (insulated) of the plate oscillations. The frequency equation which corresponds to the antisymmetric motion of the plate with respect to the medial plane $z = 0$ may be written as:

$$A_1\Delta_1 - A_2\Delta_2 - A_3\Delta_3 = 0 \quad (34)$$

where we have used:

$$A_j = iF - c_1 m_j M_j + i\bar{\beta}\tau_a l_j, \quad (35)$$

$$\Delta_1 = U_2W_3 - U_3W_2, \qquad \Delta_2 = U_1W_3 - U_3W_1, \qquad \Delta_3 = U_1W_2 - U_2W_1, \quad (36)$$

$$U_j = im_j - M_j, \qquad W_j = -l_j M_j, \qquad j = 1,2,3 \quad (37)$$

with m_j and l_j are given in Eqs. (25) and (26).

6. NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating the theoretical results obtained in the preceding sections and comparing these in the context of various theories of thermoelasticity we now present some numerical results. The material chosen for detailed computation is single crystal of zinc (Zn), of hexagonal symmetry, which is transversely isotropic material, the physical data of which is listed in

Table 1:

Table (1): The physical constants of Zn [19]		
Quantity	**Units**	**Zinc**
ρ	7.14×10^3	Kgm^{-3}
c_{11}	1.628×10^{11}	Nm^{-2}
c_{12}	0.362×10^{11}	Nm^{-2}
c_{13}	0.508×10^{11}	Nm^{-2}
c_{33}	0.627×10^{11}	Nm^{-2}
c_{44}	0.385×10^{11}	Nm^{-2}

β_1	5.75×10^6	$Nm^{-2}\,deg^{-1}$
β_3	5.15×10^6	$Nm^{-2}\,deg^{-1}$
C_e	3.9×10^2	$Jkg^{-1}\,deg^{-1}$
K_1	1.24×10^2	$Wm^{-1}\,deg^{-1}$
K_2	1.24×10^2	$Wm^{-1}\,deg^{-1}$
T_o	298	deg
ω_1^*	1.0×10^5	s^{-1}
ε_1	2.21×10^{-2}	......

We restrict our attention to make the dimensionless phase velocity η and the dimensionless wave number ξ to be: $\eta=\sqrt{\rho c^2 / c_{11}}$ and $\xi=\zeta d/2$ respectively.

The complex roots of characteristics equation (27) have been computed with the help of Cardan's procedure, which are then employed to solve frequency equation (FE) (34). Then, the real and imaginary parts of the (FE) (34) are obtained for the phase velocity η for different values of thermal relaxation times by utilizing iteration method and illustrated graphically in Figs. (1)- (7).

The real and imaginary parts of the frequency equations multiplied by 10^{-16} profiles are plotted in Figs. (1) and (2) versus the phase velocity η for (G-L) model

(i.e., $\tau_1 = 5\tau_o$, $\tau_o = 0.1, 0.2, 0.3$). It is noticed that Re(FE) and Im(FE) start from zero as $\eta = 0$ and vary linearly until $\eta = 0.9$. After that, in the period $\eta = 0.9 - 2.0$, one may find that Re(FE) decrease nonlinearly as η increases, when $(\tau_o = 0.2, 0.3)$. But for $(\tau_o = 0.1)$ Re(FE) decreases slowly and attains a minimum value, then rises again.

Also, in the period $\eta = 0.9 - 2.0$, all the curves for Im(FE) decrease with increasing η, see Fig. (2), (3) and (4) represent variations of the real and imaginary parts of (FE) multiplied by 10^{-25} with respect to the phase velocity η in case of (L-S) model for various values of the first thermal relaxation time τ_o (i.e. $\tau_1 = 0, \delta = 0, \tau_o = 0.1, 0.2, 0.3$). From Figs (3), it is noted that the behavior and trend of the variations of Re(FE) are almost similar as in case of Im(FE) for (G-L) in Fig. (2) for (G-L) model. From Fig. 4, it is observed that Im(FE) starts from zero as $\eta = 0$ and vary linearly until $\eta = 0.9$. After that, in the period $\eta = 0.9 - 2.0$, the curves increase nonlinearly as η increases and increasing of the first relaxation time τ_o.

Fig. (5) exhibits changes of Re(FE) and Im(FE) multiplied by 10^{-20} versus η in case of (C-D) model. The trend and behavior of these profiles are similar to that of Figs, (2)-(4), while in this case, both of Re(FE) and Im(FE) are identical. This means that the values of (FE) are real only.

Fig. (6) shows a comparison between Re(FE) for (G-L) model multiplied by 10^{-19} and Re(FE) for (L-S) model multiplied by 10^{-25} against to η. It is noticed that there are no variations in this case with respect to η in the range ($\eta = 0.0 - 0.9$). Moreover, it is clear that Re(FE) and Im(FE) in (L-S) model are higher than that of (G-L) model, whereas Re(FE) for (G-L) and (L-S) models in the range ($\eta = 0.9 - 2$) are just the opposite.

In the similar way we observe that Fig. (7), displays a comparison between Im(FE) for (G-L) multiplied by 10^{-21} with respect to η. It is seen that the profiles of the curves in this case behave similar to the earlier cases.

CONCLUSION: In this paper, the boundary value problems concerning the propagation of plane harmonic thermoelastic waves in flat infinite homogeneous transversely isotropic plate of finite thickness in the generalized theory of thermoelasticity with two thermal relaxation times is studied. The frequency equation against the phase velocity for a heat conducting thermoelastic plate corresponding to flexural (antisymmetric) thermoelastic modes of vibration is obtained and discussed. A numerical solution to the frequency equations for zinc plate (transversely isotropic) is given, and the dispersion relations are presented graphically. It is found that the phase velocity of the waves is modified due to the thermal and anisotropic effects and is also influenced by the thermal. It is interesting to note that the important clues to the wide-ranging utility of the frequency equation came from its used of the form of integrals relating the real and imaginary parts of a property, called the complex refractive, of any medium in which waves travel. The real part of this index describes how waves of different frequency refract (change speed and hence bend or disperse) on entering the medium, while the imaginary part of the index describes how the wave is absorbed in the medium. The frequency equation of the waves gets modified due to the thermal and anisotropic effects and is also influenced by the thermal relaxation times. The increasing ratios of thermal relaxation times tend to increase the values of the frequency equation of different modes. Within the framework of the generalized theory of thermoelasticity, dispersion curves are similar to those of the elastic waves. When the phase velocity is small, it is seen that there is no change for Re(FE) and Im(FE) among the three various models of generalized thermoelasticity.

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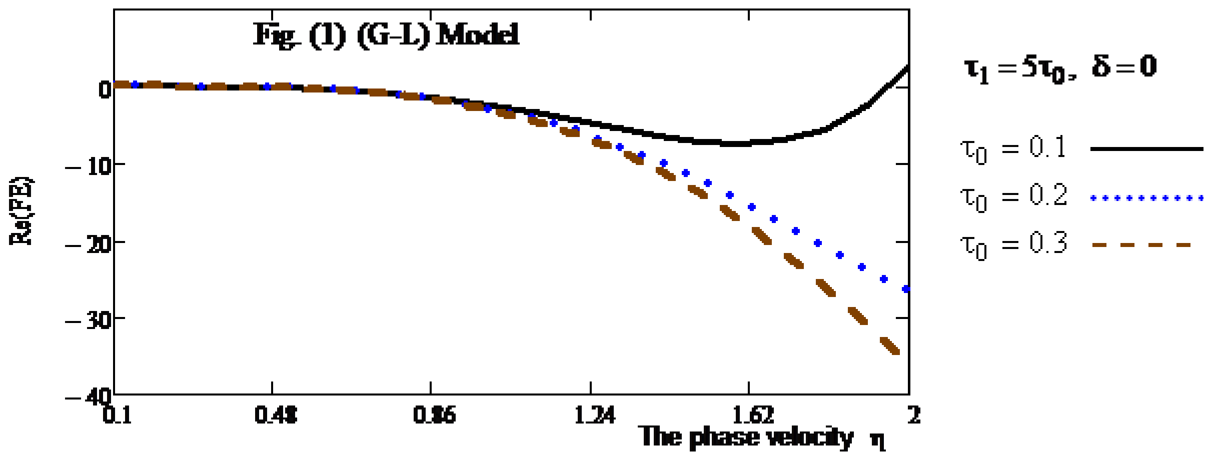
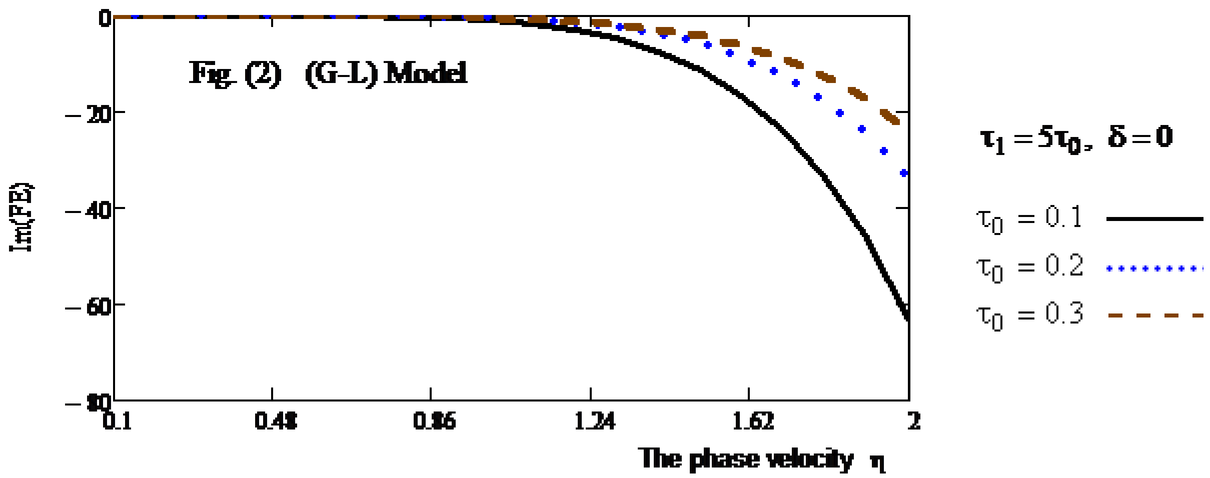
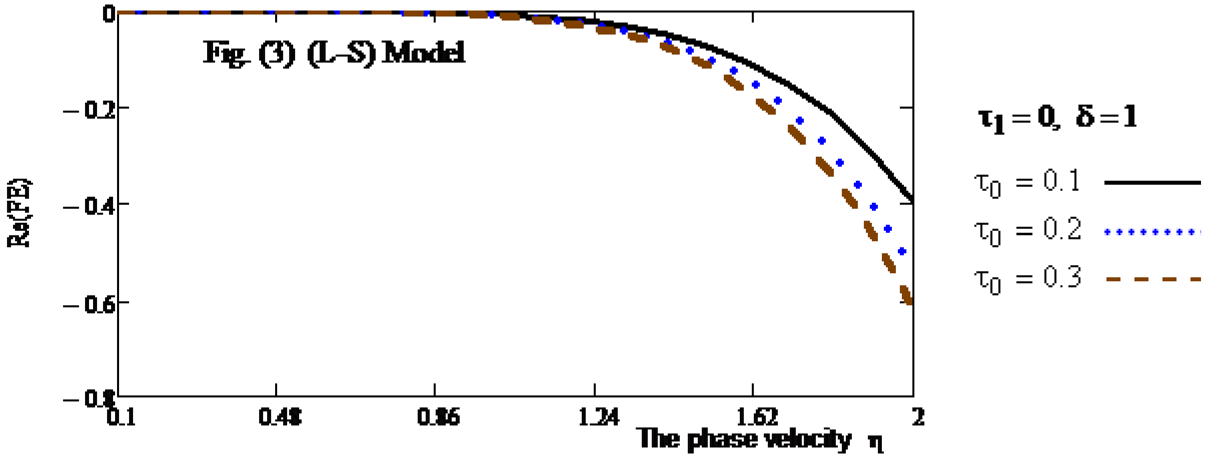
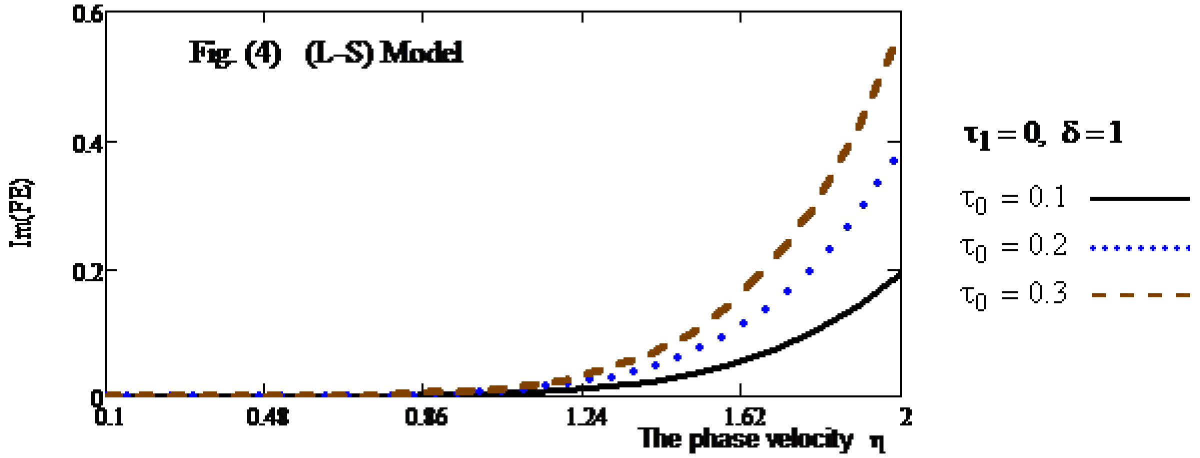




Figs. (1) and (2): The real and imaginary parts of the frequency equation multiplied by 10^{-16} versus the phase velocity for (G-L) model for different values τ_1.





Figs. (3) and (4): The real and imaginary parts of the frequency equation multiplied by 10^{-25} versus the phase velocity for (L-S) model for different values τ_0.

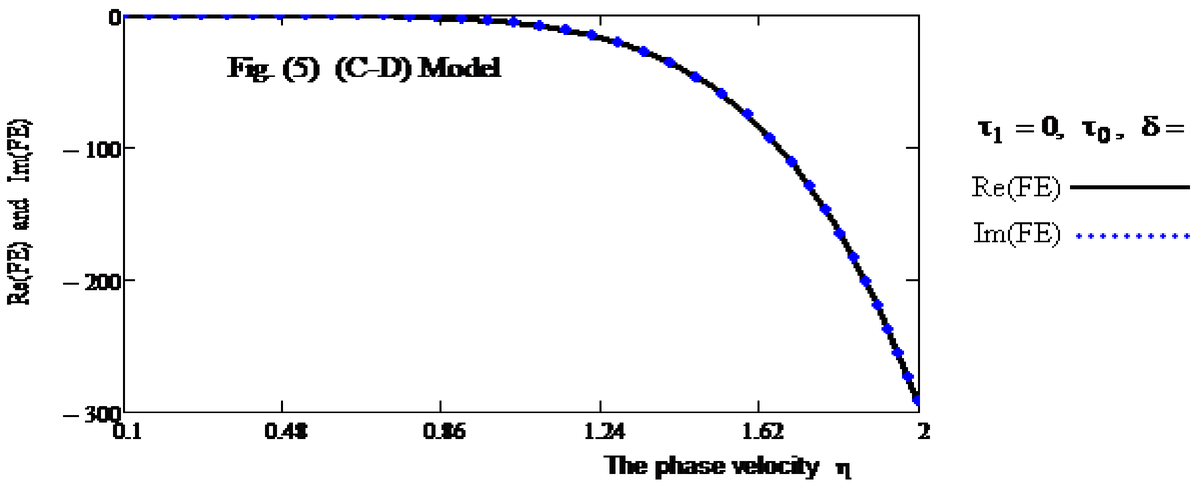


Fig. (5): The real and imaginary parts of the frequency equation multiplied by 10^{-20} versus the phase velocity for (C-D) model.

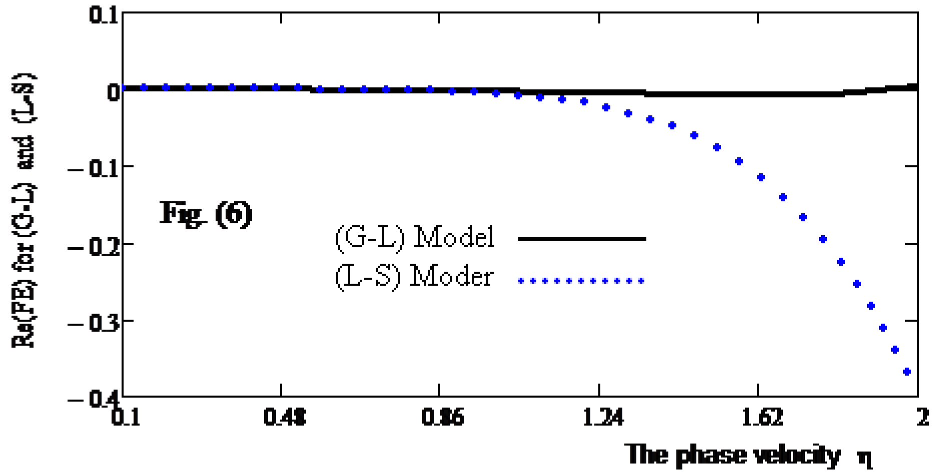


Fig. (6): The real parts of the frequency equation for (G-L) model multiplied by 10^{-19} and (L-S) model multiplied by 10^{-25} versus the phase velocity.

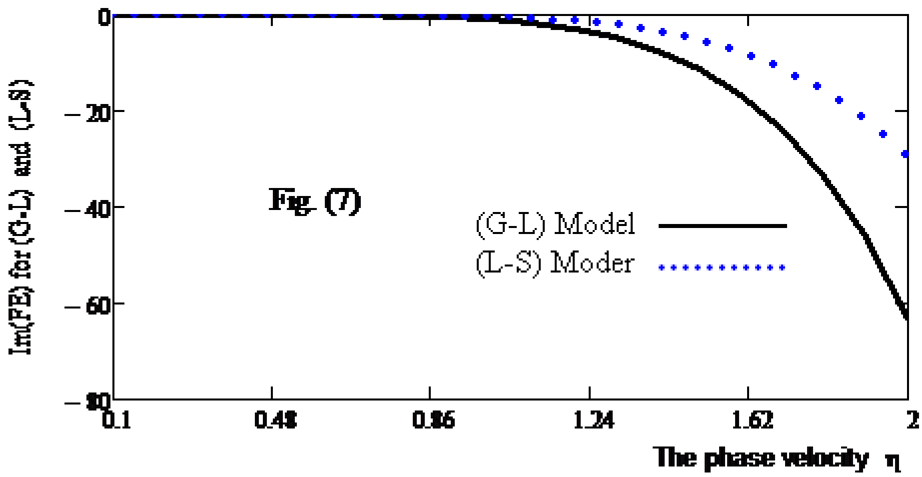


Fig. (7): The imaginary parts of the frequency equation for (G-L) model multiplied by 10^{-16} and (L-S) model multiplied by 10^{-21} versus the phase velocity.

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