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FIBONACCI MEANS AND ITS APPLICATIONS

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ABSTRACT

Fibonacci sequence has many applications in chemistry, botany, arts, statistics, mathematics, music and among others as it occurs naturally in our surroundings. Arithmetic, geometric and harmonic sequences are commonly tackled in college and high school mathematics but not the Fibonacci sequence. Sequences are generally taught by solving them through their "means."

An equation was already obtained for integer sequences given two initial values to satisfy a Fibonacci or Fibonacci-like sequence. Recently, Natividad [4] derived explicit formulas in solving Fibonacci means based on the number of missing terms. This paper will discuss the usefulness of this formula in computing Fibonacci sequence as well as explaining the possibility of integrating this on different branches of science, preferably in mathematical sequences, by giving examples and discussing its applications.

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Key Words and Phrases: *Fibonacci sequence, Fibonacci means, missing terms.*

INTRODUCTION:

Sequences have been fascinating topic for mathematicians for centuries. They have many applications in nature and to name some of the sequences, these are arithmetic, geometric, Pell, Lucas and one of the famous sequences, the Fibonacci sequence. Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21….) is a succession of numbers that are obtained through adding the two preceding numbers. This is named after Leonardo of Pisa who was known as Fibonacci.

Many applications of Fibonacci sequence have been presented and explored in our nature by different authors due to its relation in golden ratio. Fibonacci numbers could be seen in biology, particularly in spiral phyllotaxis [2, 3], specifically of those sunflowers [8] and of different organisms. History tells us that Johannes Kepler observed the commonness of Fibonacci numbers in plants [2] and two successive Fibonacci numbers can be seen on spiral phyllotaxis going clockwise and counterclockwise in many instances [7].

Fibonacci sequence is important to chemist in describing crystals at the atomic level using diffraction pattern [9]. It can be noted that Dan Schectman has been awarded with Nobel Prize in Chemistry 2011 due to the discovery of quasicrystals which altered the conception of regularity in crystals [9]. This aperiodic crystals cannot be explained without the knowledge of Fibonacci sequence. This sequence can be further seen in periodic crystals and crystallography as a whole. Also, the golden ratio and Fibonacci sequence is evident in music [1], arts of Muslims, arts of Leonardo da Vinci and Dürer, and among others.

Fibonacci means are related to arithmetic, geometric and harmonic means that you will not miss when you are studying sequences. Any high school or college algebra book will discuss on how to find arithmetic mean and geometric mean, a basic concept that has been known for Egyptians and Babylonians thousands of years ago.

Arithmetic means are n – 2 numbers between a_1 and a_n so that the *n* numbers form an arithmetic sequence of *n* terms. In mathematical sentence, if $a_1, a_2, \ldots, a_{n-1}, a_n$ is an arithmetic sequence, then $a_2, \ldots, a_{n-1}$ are called arithmetic means between a_1 and a_n [6]. Harmonic means are terms between any two terms of a harmonic sequence. In this manner, we can define Fibonacci means as terms between any two terms of Fibonacci or Fibonacci-like sequence.

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Solving harmonic means is the same way as calculating arithmetic means just keeping in mind that harmonic sequence is a sequence formed by the reciprocals of the terms of arithmetic sequence. Like the two means, geometric means are terms between any two terms of geometric sequence. Arithmetic, geometric and harmonic means are also known as Pythagorean means.

In this paper, we will present formulas in solving Fibonacci means and discuss this by giving examples.

THE FORMULAS:

All integer sequences that satisfy $g(n+2) = g(n) + g(n+1)$ can be solved given two initial values using the equation

$$g(n) = F(n)g(1) + F(n-1)g(0) \tag{1}$$

where F(n) and F(n-1) are corresponding terms of Fibonacci sequence. A basic formula similar in (1) and derived in [4] with equation based on number of missing terms can be used to solve for Fibonacci mean. This formula is derived to find the first missing term of any Fibonacci-like sequence with *n* missing terms. This can be illustrated as

$$x_1 = \frac{b - F_n a}{F_{n+1}} \tag{2}$$

Since the formula for finding the n[th] term of Fibonacci sequence (also known as Binet's formula) is

$$F_n = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}}, \tag{3}$$

we can substitute (3) to (2) and get the formula which is

$$x_1 = \frac{b - \left[\frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}}\right] a}{\frac{\varphi^{n+1} - (1-\varphi)^{n+1}}{\sqrt{5}}} \tag{4}$$

where x_1 *is the first missing term in Fibonacci like sequence*
a *is the first term given*
b *is the last term given*
n *is the number of missing terms*
φ *is known as golden ratio equal to* $\frac{1+\sqrt{5}}{2}$ *or 1.61803399…*

Simplifying the denominator to cancel $\sqrt{5}$ will give another form of (4) which is

$$x_1 = \frac{b\sqrt{5} - [\varphi^n - (1-\varphi)^n]a}{\varphi^{n+1} - (1-\varphi)^{n+1}}. \tag{5}$$

Amazingly, (4) and (5) are easy to use since it only requires basic knowledge of algebra and method of direct substitution. Also, all the missing terms in a Fibonacci-like sequence could be solved given the first missing term x_1 since it is a succession of numbers that are obtained through adding the two preceding numbers. Since, a general formula was obtained in x_1, the other missing terms will be calculated easily. Hence, inserting Fibonacci means between two numbers are now possible using this formula. This formula is also extended on solving Pell means [5]. In this case, college or even high school students can solve the problems involving Fibonacci mean.

2.1 Specific Examples

To gain insights with the usefulness of the formula, let's study these three examples.

Example 1: *Insert six Fibonacci means between 1 and 21.*
We may write this problem as $a, x_1, x_2, x_3, x_4, x_5, x_6, b$ *…. noting that a = 1, b = 21, n = 6 and* $\varphi = 1.61803399$ *...*
Substituting this to (4) will give

$$x_1 = \frac{21 - \left[\frac{1.618034^6 - (1-1.618034)^6}{\sqrt{5}}\right](1)}{\frac{1.618034^{6+1} - (1-1.68034)^{6+1}}{\sqrt{5}}} \tag{1}$$

$$x_1 = 1.$$

Now that we solve for x_1, *we could easily find* x_2 *which is*

$$a + x_1 = x_2$$

$$1 + x_1 = x_2$$

$$1 + 1 = x_2$$

$$x_2 = 2,$$

and the same method for x_3, x_4, x_5 *up to* x_6. *Therefore the sequence is 1, 1, 2, 3, 5, 8, 13, 21… which is actually the Fibonacci sequence itself.*

Let's deepen our understanding about the formula in the second and third example.

Example 2: *Missy wants to hybrid a sunflower with spirals. She found out that the inner spiral is 7 while the sixth spiral is 61. What are the Fibonacci numbers between them?*

Again substituting a = 7, b = 61, n = 4 in (4) will give

$$x_1 = \frac{61 - \left[\frac{1.618034^4 - (1 - 1.618034)^4}{\sqrt{5}}\right](7)}{\frac{1.618034^{4+1} - (1 - 1.68034)^{4+1}}{\sqrt{5}}}$$

$$x_1 = 8.$$

Solving x_2 *and* x_3 *will bring out the Fibonacci like sequence 7, 8, 15, 23, 38, 61…*

Example 3: *Insert 7 Fibonacci means between 2 and 47.*

Substituting a = 2, b = 47, n = 7 in (4) will give

$$x_1 = \frac{47 - \left[\frac{1.618034^7 - (1 - 1.618034)^7}{\sqrt{5}}\right](2)}{\frac{1.618034^{7+1} - (1 - 1.68034)^{7+1}}{\sqrt{5}}}$$

$$x_1 = 1.$$

Solving for all the missing terms will bring out the Fibonacci-like sequence 2, 1, 3, 4, 7, 11, 18, 29, 47,… Actually this sequence is a variant of Fibonacci sequence which is the Lucas sequence.

CONCLUSION:

Fibonacci means are terms between any two terms of Fibonacci or Fibonacci-like sequence. Unlike arithmetic, geometric and harmonic means, solving Fibonacci means is not popular or known. Formulas based on the number of missing terms derived by Natividad [4] for getting this type of means will be a great help since it is very easy to use that even high school students can calculate it using basic algebra only. The formula has many applications in chemistry, botany, arts, statistics, mathematics, and music.

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