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# ACTIVE VIBRATION CONTROL OF AN AIRCRAFT TAIL SUBJECTED TO MULTI-HARMONIC AND MULTI-TUNED EXCITATIONS WITH MULTI-SIMULTANEOUS RESONANCE

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***ABSTRACT***

*In this paper, a negative velocity feedback is added to the dynamical system of twin-tail aircraft to suppress the vibration. The system is represented by two coupled second-order nonlinear differential equations having both quadratic and cubic nonlinearities. The system describes the vibration of an aircraft tail subjected to both multi-harmonic and multi-tuned excitations. The method of multiple time scale perturbation is applied to solve the nonlinear differential equations and obtain approximate solutions up to the second order approximations. The stability of the proposed analytic solution near the simultaneous primary, combined and internal resonance is studied and its conditions are determined. The effect of different parameters on the steady state response of the vibrating system is studied and discussed using frequency response equations. Some different resonance cases are investigated numerically.*

***Keywords:*** *Aircraft tail, Stability, Response, Active vibration control, Resonance.*

## 1. INTRODUCTION

Vibration of structures is often an undesirable phenomenon and should be avoided or controlled. There are two techniques to control the vibration of a system, that is, active and passive control techniques. El-Badawy and Nayfeh [1] used two simple control laws based on linear velocity and cubic velocity feedback to suppress the high-amplitude vibrations of the twin-tail assembly of an F-15 fighter in a structural dynamic model when subjected to primary resonance excitations. Hanagud et al. [2] designed a standoff piezoceramic stack based active control system to control the fine vibrations of an F-15 tail. Eissa et al. [3-4] studied the active control of an aircraft tail subjected to multi-harmonic excitation or multi-parametric excitation forces. They added many active controllers for this system. The system has controlled by negative (linear, quadratic) velocity feedback. Best active control of the system has been achieved via negative velocity feedback. Eissa et al. [5, 6] studied the control of both vibration and dynamics chaos of mechanical system having quadratic and cubic nonlinearities, subjected to harmonic excitation forces. Pai and Schulz [7] studied the control of the first mode vibration of a stainless steel beam through a negative velocity feedback. Eissa et al. [8] applied active nonlinear vibration absorber using higher-order internal resonance and saturation phenomena to suppress the steady-state vibrations of a cantilever skew aluminum plate. Eissa et al. [9-11] investigated saturation phenomena in non-linear oscillating systems subject to multi-parametric and/or external excitations. The system represents the vibration of a single-degree-of-freedom cantilever or the wing of an aircraft. They reported the occurrence of saturation phenomena at different parameters values. They applied saturation values of different parameters as optimum working conditions for vibration suppression of the cantilever. Yassen [12] studied the chaos synchronization between two different chaotic systems using active control. Eissa and Amer [13] studied the vibration control of a second-order system simulating the first mode of a cantilever beam subjected to a primary and sub-harmonic resonance. Eissa et al. [14, 15] makes a comparison between active and passive vibration control of a simple pendulum which described by a second-order nonlinear differential equations having both quadratic and cubic nonlinearities. Y. S. Hamed et al. [16-18] studied the USM model subject to external and tuned excitation forces. The model consists of multi-degree-of-freedom system consisting of the tool holder and three absorbers (tools) simulating ultrasonic machining process. The advantages of using multi-tools are to machine different materials and different shapes at the same time. This leads to time saving and higher machining efficiency.

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This work deals with models having two-degree-of-freedom system with quadratic and cubic nonlinearities subjected to multi-harmonic and multi-tuned excitation forces. The method of multiple time scale perturbation [19-20] is used to solve the nonlinear differential equations describing the controlled system up to third order approximations. In this system, we added many active controllers. Best active control of the system has been achieved via negative velocity feedback.

## 2. MATHEMATICAL MODELING

The modified behavior of the aircraft twin-tail can be modeled; in general, by the following two generalized second-order coupled nonlinear differential equations [3, 4]. This nonlinear system can be written as:

$$\ddot{u}_1+\omega_1^2u_1+2\varepsilon\mu_1\dot{u}_1+\varepsilon\alpha_1u_1^3+\varepsilon\alpha_2u_1^2+\varepsilon\mu_3\dot{u}_1\left|\dot{u}_1\right|-\varepsilon\kappa(u_2-u_1)=\varepsilon\sum_{S=1}^{N}F_S\cos(\Omega_St+\tau_1)$$
$$+\varepsilon\sum_{j=1}^{N}P_j\sin(\Omega_{k_j}t+\tau_3)\cos(\Omega_{k_{j+1}}t+\tau_4)+R_1 \tag{1a}$$

$$\ddot{u}_2+\omega_2^2u_2+2\varepsilon\mu_2\dot{u}_2+\varepsilon\alpha_3u_2^3+\varepsilon\alpha_4u_2^2+\varepsilon\mu_4\dot{u}_2\left|\dot{u}_2\right|-\varepsilon\kappa(u_1-u_2)=\varepsilon\sum_{S=1}^{N}F_S\cos(\Omega_St+\tau_2)$$
$$+\varepsilon\sum_{j=1}^{N}P_j\sin(\Omega_{k_j}t+\tau_5)\cos(\Omega_{k_{j+1}}t+\tau_6)+R_2 \tag{1b}$$

where $u_1$, $u_2$ denote the generalized coordinates of the first bending modes of the aircraft twin-tail, $\omega_1$, $\omega_2$ are the lowest linear natural frequencies of the right and left tails respectively, $\mu_1$, $\mu_2$ are the linear damping coefficients, $\alpha_1$, $\alpha_3$ are the coefficients of the cubic nonlinearity, $\alpha_2$, $\alpha_4$ are the coefficients of the quadratic nonlinearity, $\mu_3$, $\mu_4$ are the nonlinear damping coefficients, $F_S$, $P_J$ are the excitation amplitudes, $\Omega_S$, $\Omega_{k_j}$, $\Omega_{k_{j+1}}$ are the excitation frequencies, t is the time, $\tau_1,\dots\tau_6$ are constants, $\varepsilon$ is small a perturbation parameter, $\kappa$ is the coupling coefficient of the aircraft tails , $R_1=-\varepsilon G_1\dot{u}_1$ and $R_2=-\varepsilon G_2\dot{u}_2$ are the controls forces , $G_1$ and $G_2$ are positive constants known as the gains.

### 2.1. Perturbation analysis

Multiple scale perturbation method is conducted to obtain an approximation solution for Eqs. (1). so, assuming the solution in the form:

$$u_1(t;\varepsilon)=u_{10}(T_0,T_1)+\varepsilon u_{11}(T_0,T_1)+O(\varepsilon^2) \tag{2a}$$

$$u_2(t;\varepsilon)=u_{20}(T_0,T_1)+\varepsilon u_{21}(T_0,T_1)+O(\varepsilon^2) \tag{2b}$$

where, $T_n=\varepsilon^nt$ (n = 0, 1) are the fast and slow time scales respectively and the time derivatives became

$$\frac{d}{dt}=D_0+\varepsilon D_1\ ,\qquad \frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0D_1+\varepsilon^2D_1^2 \tag{3}$$

where, $D_n=\dfrac{\partial}{\partial T_n}$ (n = 0, 1).

Substituting Eqs. (2) and (3) into Eqs. (1) and equating the coefficients of same powers of ε, we obtain the following:

$$(D_0^2+\omega_1^2)u_{10}=0 \tag{4a}$$

$$(D_0^2+\omega_2^2)u_{20}=0 \tag{4b}$$

$$(D_0^2+\omega_1^2)u_{11} = -2D_0D_1u_{10} - 2\mu_1(D_0u_{10}) - \alpha_1u_{10}^3 - \alpha_2u_{10}^2 \mp \mu_3(D_0u_{10})^2 + \kappa(u_{20}-u_{10})$$

$$-G_1(D_0u_{10}) + \sum_{S=1}^{N} F_S\cos(\Omega_S t+\tau_1) + \sum_{j=1}^{N} P_j \sin(\Omega_{k_j} t+\tau_3)\cos(\Omega_{k_{j+1}} t+\tau_4) \tag{5a}$$

$$(D_0^2+\omega_2^2)u_{21} = -2D_0D_1u_{20} - 2\mu_2(D_0u_{20}) - \alpha_3u_{20}^3 - \alpha_4u_{20}^2 \mp \mu_4(D_0u_{20})^2 + \kappa(u_{10}-u_{20})$$

$$-G_2(D_0u_{20}) + \sum_{S=1}^{N} F_S\cos(\Omega_S t+\tau_2) + \sum_{j=1}^{N} P_j \sin(\Omega_{k_j} t+\tau_5)\cos(\Omega_{k_{j+1}} t+\tau_6) \tag{5b}$$

The general solution of Eqs. (4) can be expressed in the form

$$u_{10} = A_{10}\exp(i\,\omega_1 T_0) + cc \tag{6a}$$

$$u_{20} = A_{20}\exp(i\,\omega_2 T_0) + cc \tag{6b}$$

where, $A_{10}$ and $A_{20}$ are complex function in $T_1$ and cc denotes complex conjugate terms. Substituting Eqs. (6) into Eqs. (5) and eliminating the secular terms, then the bounded first order approximation yields

$$u_{11} = A_{11}\exp(i\omega_1T_0) + E_1\exp(i\omega_2T_0) + E_2\exp(2i\omega_1T_0) + E_3\exp(3i\omega_1T_0)$$
$$+E_4\sum_{S=1}^{N}\exp(i\,(\Omega_S T_0+\tau_1)) + E_5\sum_{j=1}^{N}\exp(i\,((\Omega_{k_j}+\Omega_{k_{j+1}})T_0+\tau_3+\tau_4))$$
$$+E_6\sum_{j=1}^{N}\exp(i\,((\Omega_{k_j}-\Omega_{k_{j+1}})T_0+\tau_3-\tau_4)) + E_7 + cc \tag{7a}$$

$$u_{21} = A_{21}\exp(i\,\omega_2T_0) + E_8\exp(i\,\omega_1T_0) + E_9\exp(2i\,\omega_2T_0) + E_{10}\exp(3i\,\omega_2T_0)$$
$$+E_{11}\sum_{S=1}^{N}\exp(i\,(\Omega_S T_0+\tau_2)) + E_{12}\sum_{j=1}^{N}\exp(i\,((\Omega_{k_j}+\Omega_{k_{j+1}})T_0+\tau_5+\tau_6))$$
$$+E_{13}\sum_{j=1}^{N}\exp(i\,((\Omega_{k_j}-\Omega_{k_{j+1}})T_0+\tau_5-\tau_6)) + E_{14} + cc \tag{7b}$$

where $A_{11}$, $A_{21}$, $E_1,\ldots, E_{14}$ are complex conjugate function in $T_1$.

From Eqs. (6-7) the resonance cases are

1. Primary resonance: $\Omega_S = \omega_n$ and $(\Omega_{k_j} \pm \Omega_{k_{j+1}}) = \omega_n$, n, s = (1, 2).
2. Sub-harmonic resonance: $\Omega_S = m\,\omega_n$ and $(\Omega_{k_j} \pm \Omega_{k_{j+1}}) = m\,\omega_n$, m = (2, 3).
3. Internal resonance: $\omega_1 = r\,\omega_2$, r = (1, 2, 3, 4,1/2, 1/3).
4. Combined resonance: $\Omega_n \cong \omega_1 \pm \omega_2,\ \Omega_n \cong 2\omega_1 \pm \omega_2,\ \Omega_n \cong \omega_1 \pm 2\omega_2$
   $(\Omega_{k_j} \pm \Omega_{k_{j+1}}) \cong \omega_1 \pm \omega_2$
5. Simultaneous resonance: Any combination of the above resonance cases is considered as a simultaneous resonance.

**2.2 Numerical results**

To study the behavior of the aircraft twin-tail system of Eqs. (1), Runge-Kutta fourth order method was applied to determine the numerical solution of the given system.

Fig.1. illustrates the response and the phase plane for the non-resonant system at some practical values of the equation parameters. It can be seen from the figure that the steady state amplitudes $u_1$ and $u_2$ are about 75% and 40% of the maximum excitation forces amplitude $F_1$ respectively and the phase plane shows multi-limit cycle.

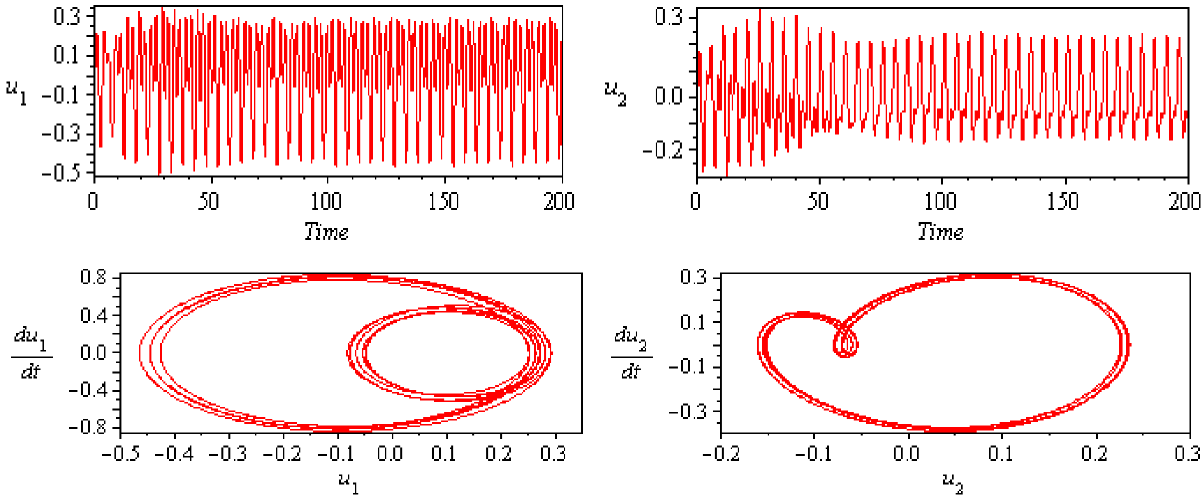
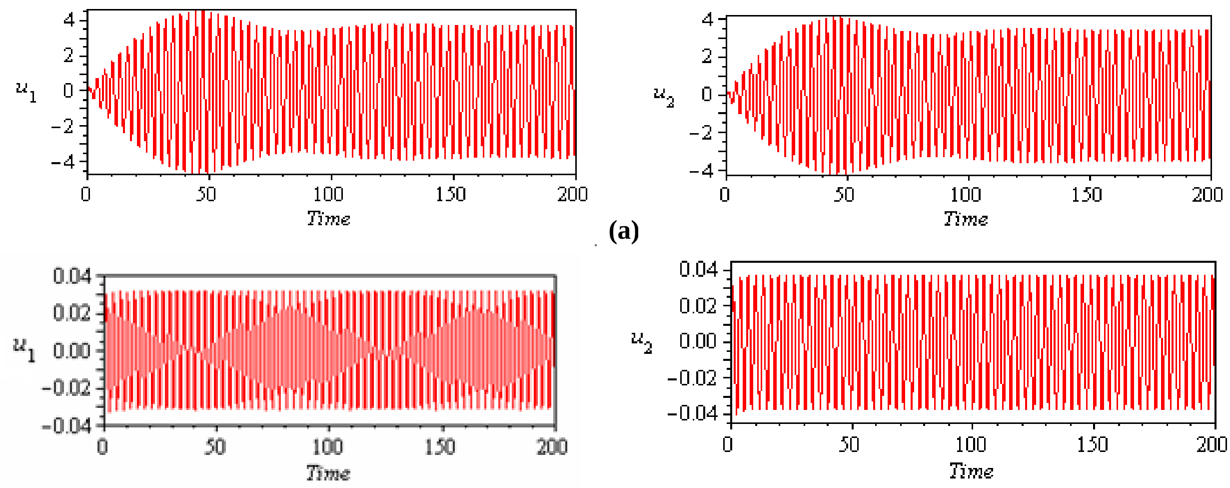



**Fig. 1:** Non-resonant system behavior (basic case)

The worst resonance case is confirmed numerically as shown in Figs. 2. The worst resonance case of the system is the simultaneous primary resonance case, where $\Omega_S \cong \omega_1$, $\Omega_S \cong \omega_2$ and $(\Omega_{k_j} - \Omega_{k_{j+1}}) \cong \omega_1$. The effectiveness of the controller is represented by $E_a$= steady state amplitude of the system before control/steady state amplitude of the system after control, where ($E_a$=125, $E_a$=90), for the simultaneous primary resonance case.




**(b)**

**Fig. 2:** Simultaneous resonance case $\Omega_S \cong \omega_1, \Omega_S \cong \omega_2, (\Omega_{k_j} - \Omega_{k_{j+1}}) \cong \omega_1$ , **a:** without controller, **b:** with controller **$G_1$ =12 and $G_2$=10**

## 3. STABILITY OF THE SYSTEM

For the simultaneous primary, combined and internal resonance case $\Omega_S \cong \omega_1$ , $\Omega_S \cong \omega_2$ and $(\Omega_{k_j} - \Omega_{k_{j+1}}) \cong \omega_1$ (worst case) which was confirmed numerically, we can introduce detuning parameters $\sigma_1$ , $\sigma_2$ and $\sigma_3$ such that:

$$\Omega_S = \omega_1 + \varepsilon\sigma_1, \; \omega_2 \cong \omega_1 + \varepsilon(\sigma_1 - \sigma_2) \; \text{and} \; (\Omega_{k_j} \pm \Omega_{k_{j+1}}) \cong \omega_1 + \varepsilon\sigma_3 \tag{8}$$

Substituting Eq. (8) into Eqs. (5) and putting the coefficients of the secular terms to zero yields the solvability conditions as

$$-2i\,\omega_1 D_1 A_{10} - 2i\,\omega_1\mu_1 A_{10} - 3\alpha_1 A_{10}^2\bar{A}_{10} - i\,\omega_1 G_1 A_{10} + \kappa[A_{20}\exp(i\,(\sigma_1-\sigma_2)T_1) - A_{10}] + \frac{1}{2}\sum_{S=1}^{N} F_S\,\exp(i\,(\sigma_1 T_1+\tau_1)) + \frac{1}{4i}\sum_{j=1}^{N} P_j\,\exp(i\,(\sigma_3 T_1+\tau_3-\tau_4)) = 0 \quad (9a)$$

$$-2i\,\omega_2 D_1 A_{20} - 2i\,\omega_2\mu_2 A_{20} - 3\alpha_3 A_{20}^2\bar{A}_{20} - i\,\omega_2 G_2 A_{20} + \kappa[A_{10}\exp(i\,(\sigma_2-\sigma_1)T_1) - A_{20}] + \frac{1}{2}\sum_{S=1}^{N} F_S\,\exp(i\,(\sigma_2 T_1+\tau_2)) + \frac{1}{4i}\sum_{j=1}^{N} P_j\,\exp(i\,((\sigma_2+\sigma_3-\sigma_1)T_1+\tau_5-\tau_6)) = 0 \quad (9b)$$

Using polar form $A_{n0} = \frac{1}{2}a_n\exp(i\,\beta_n)$, n= (1, 2) (10)

where $a_n$ and $\beta_n$ are the steady-state amplitudes and phases of the motion, respectively. Substituting from Eq. (10) into Eqs. (9) and equating imaginary and real parts, we obtain

$$a_1' + \mu_1 a_1 + \frac{1}{2}a_1 G_1 - \frac{1}{2\omega_1}\kappa a_2\sin\theta_3 - \frac{1}{2\omega_1}F_S\,\sin\theta_1 - \frac{1}{4\omega_1}P_j\,\cos\theta_4 = 0 \quad (11a)$$

$$a_1(\theta_1' - \sigma_1) + \frac{3\alpha_1}{8\omega_1}a_1^3 + \frac{\kappa}{2\omega_1}a_1 - \frac{1}{2\omega_1}\kappa a_2\cos\theta_3 - \frac{1}{2\omega_1}F_S\,\cos\theta_1 - \frac{1}{4\omega_1}P_j\,\sin\theta_4 = 0 \quad (11b)$$

$$a_2' + \mu_2 a_2 + \frac{1}{2}a_2 G_2 + \frac{1}{2\omega_2}\kappa a_1\sin\theta_3 - \frac{1}{2\omega_2}F_S\,\sin\theta_2 + \frac{1}{4\omega_2}P_j\,\cos\theta_5 = 0 \quad (12a)$$

$$a_2(\theta_2' - \sigma_2) + \frac{3\alpha_3}{8\omega_2}a_2^3 + \frac{\kappa}{2\omega_2}a_2 - \frac{1}{2\omega_2}\kappa a_1\cos\theta_3 - \frac{1}{2\omega_2}F_S\,\cos\theta_2 - \frac{1}{4\omega_2}P_j\,\sin\theta_5 = 0 \quad (12b)$$

where, $\theta_1 = \sigma_1 T_1 - \beta_1 + \tau_1$, $\theta_2 = \sigma_2 T_1 - \beta_2 + \tau_2$, $\theta_3 = (\sigma_1-\sigma_2)T_1 + \beta_2 - \beta_1$, $\theta_4 = \sigma_3 T_1 - \beta_1 + \tau_3 - \tau_4$ and $\theta_5 = (\sigma_2+\sigma_3-\sigma_1)T_1 - \beta_2 + \tau_5 - \tau_6$.

For steady state solutions we have $a_n' = \theta_n' = 0$, n=(1, 2) and the periodic solution at the fixed points corresponding to Eqs. (11) and (12) is given by

$$\mu_1 a_1 + \frac{1}{2}a_1 G_1 - \frac{1}{2\omega_1}\kappa a_2\sin\theta_3 - \frac{1}{2\omega_1}F_S\,\sin\theta_1 - \frac{1}{4\omega_1}P_j\,\cos\theta_4 = 0 \quad (13a)$$

$$a_1\sigma_1 - \frac{3\alpha_1}{8\omega_1}a_1^3 - \frac{\kappa}{2\omega_1}a_1 + \frac{1}{2\omega_1}\kappa a_2\cos\theta_3 + \frac{1}{2\omega_1}F_S\,\cos\theta_1 + \frac{1}{4\omega_1}P_j\,\sin\theta_4 = 0 \quad (13b)$$

$$\mu_2 a_2 + \frac{1}{2}a_2 G_2 + \frac{1}{2\omega_2}\kappa a_1\sin\theta_3 - \frac{1}{2\omega_2}F_S\,\sin\theta_2 + \frac{1}{4\omega_2}P_j\,\cos\theta_5 = 0 \quad (14a)$$

$$a_2\sigma_2 - \frac{3\alpha_3}{8\omega_2}a_2^3 - \frac{\kappa}{2\omega_2}a_2 + \frac{1}{2\omega_2}\kappa a_1\cos\theta_3 + \frac{1}{2\omega_2}F_S\,\cos\theta_2 + \frac{1}{4\omega_2}P_j\,\sin\theta_5 = 0 \quad (14b)$$

From equations (13)-(14) we considered only case where, $a_1 \neq 0, a_2 \neq 0$ which is the practical one. Then we get the frequency response equations as:

$$\frac{9\alpha_1^2}{64\omega_1^2}a_1^6 - \left(\frac{3}{4\omega_1}\alpha_1\sigma_1 - \frac{3}{8\omega_1^2}\kappa\alpha_1\right)a_1^4 + \left(\mu_1^2 + \sigma_1^2 + \frac{1}{4}G_1^2 + \mu_1 G_1 + \frac{1}{4\omega_1^2}\kappa^2 - \frac{1}{\omega_1}\kappa\sigma_1\right)a_1^2 = \frac{1}{4\omega_1^2}\left(\kappa^2 a_2^2 + F_S^2 + 2\kappa F_S a_2 + \frac{1}{4}P_j^2\right) \quad (15)$$

$$\frac{9\alpha_3^2}{64\omega_2^2}a_2^6 - \left(\frac{3}{4\omega_2}\alpha_3\sigma_2 - \frac{3}{8\omega_2^2}\kappa\alpha_3\right)a_2^4 + \left(\mu_2^2 + \sigma_2^2 + \frac{1}{4}G_2^2 + \mu_2 G_2 + \frac{1}{4\omega_2^2}\kappa^2 - \frac{1}{\omega_2}\kappa\sigma_2\right)a_2^2 = \frac{1}{4\omega_2^2}\left(\kappa^2 a_1^2 + F_S^2 + 2\kappa F_S a_1 + \frac{1}{4}P_j^2\right) \quad (16)$$

**3.1. Stability of linear solution**

The stability of the obtained fixed points for the simultaneous primary combined and internal resonance case is determined and studied as follows. To determine the stability of the linear solution, one investigates the solution of the linearized form of Eqs. (9) as

$$-2i\,\omega_1 A'_{10} - 2i\,\omega_1\mu_1 A_{10} - i\,\omega_1 G_1 A_{10} + \kappa[A_{20}\exp(i\,(\sigma_1 - \sigma_2)T_1) - A_{10}] + \frac{1}{2}F_S\exp(i\,(\sigma_1 T_1 + \tau_1)) + \frac{1}{4i}P_j\exp(i\,(\sigma_3 T_1 + \tau_3 - \tau_4)) = 0 \quad (17a)$$

$$-2i\,\omega_2 A'_{20} - 2i\,\omega_2\mu_2 A_{20} - i\,\omega_2 G_2 A_{20} + \kappa[A_{10}\exp(i\,(\sigma_2 - \sigma_1)T_1) - A_{20}] + \frac{1}{2}F_S\exp(i\,(\sigma_2 T_1 + \tau_2)) + \frac{1}{4i}P_j\exp(i\,((\sigma_2 + \sigma_3 - \sigma_1)T_1 + \tau_5 - \tau_6)) = 0 \quad (17b)$$

Consider $A_{n0}$ in the form:

$$A_{n0} = \frac{1}{2}[p_n - iq_n]e^{i\sigma_n T_1}, \quad \text{n = (1, 2)} \quad (18)$$

where $p_n$ and $q_n$ are real functions. Substitute Eqs. (18) into Eqs. (17) and separating real and imaginary parts, one obtains

$$p'_1 + \left(\mu_1 + \frac{G_1}{2}\right)p_1 + \left(\sigma_1 - \frac{\kappa}{2\omega_1} + \frac{P_j}{4\omega_1 q_1}\right)q_1 + \frac{\kappa}{2\omega_1}q_2 = 0 \quad (19a)$$

$$q'_1 + \left(\frac{\kappa}{2\omega_1} - \sigma_1 - \frac{F_S}{2\omega_1 p_1}\right)p_1 + \left(\mu_1 + \frac{G_1}{2}\right)q_1 - \frac{\kappa}{2\omega_1}p_2 = 0 \quad (19b)$$

$$p'_2 + \left(\mu_2 + \frac{G_2}{2}\right)p_2 + \left(\sigma_2 - \frac{\kappa}{2\omega_2} + \frac{P_j}{4\omega_2 q_2}\right)q_2 + \frac{\kappa}{2\omega_2}q_1 = 0 \quad (20a)$$

$$q'_2 + \left(\frac{\kappa}{2\omega_2} - \sigma_2 - \frac{F_S}{2\omega_2 p_2}\right)p_2 + \left(\mu_2 + \frac{G_2}{2}\right)q_2 - \frac{\kappa}{2\omega_2}p_1 = 0 \quad (20b)$$

The eigen equation of the above system of equations is obtained from:

$$\begin{vmatrix} \lambda+\mu_1+\frac{G_1}{2} & \sigma_1-\frac{\kappa}{2\omega_1}+\frac{P_j}{4\omega_1 q_1} & 0 & \frac{\kappa}{2\omega_1} \\ \frac{\kappa}{2\omega_1}-\sigma_1-\frac{F_S}{2\omega_1 p_1} & \lambda+\mu_1+\frac{G_1}{2} & -\frac{\kappa}{2\omega_1} & 0 \\ 0 & \frac{\kappa}{2\omega_2} & \lambda+\mu_2+\frac{G_2}{2} & \sigma_2-\frac{\kappa}{2\omega_2}+\frac{P_j}{4\omega_2 q_2} \\ -\frac{\kappa}{2\omega_2} & 0 & \frac{\kappa}{2\omega_2}-\sigma_2-\frac{F_S}{2\omega_2 p_2} & \lambda+\mu_2+\frac{G_2}{2} \end{vmatrix}=0 \qquad (21)$$

The eigenvalues are given by the equation

$$\lambda^4+r_1\lambda^3+r_2\lambda^2+r_3\lambda+r_4=0 \qquad (22)$$

where, $r_1$, $r_2$, $r_3$ and $r_4$ are constants. According to the Routh-Huriwitz criterion, the necessary and sufficient conditions for all the roots of Eq. (22) to possess negative real parts are if and only if:

$$r_1>0,\ r_1r_2-r_3>0,\ r_3(r_1r_2-r_3)-r_1^2r_4>0,\ r_4>0 \qquad (23)$$

## 4. RESULTS AND DISCUSSION

In this section, the results of the steady state response of the given system at various parameters near the simultaneous primary combined and internal resonance case are investigated and studied. The numerical simulation results are achieved using MATLAB 7.0 programs.

### 4.1. Response curves and effects of different parameters

The frequency response equations given by Eqs. (15) and (16) are nonlinear algebraic equations of $a_1$ against $\sigma_1$ and $a_2$ against $\sigma_2$, respectively.

From the obtained figures, we observe that the solid lines stand for the stable solutions and the dashed lines for the unstable ones.

For the practical case, where $a_1 \neq 0, a_2 \neq 0$: Fig. 3a and 4a, show the steady state amplitudes $a_1$ and $a_2$ against the detuning parameter $\sigma_1$ and $\sigma_2$ which have multi-valued solutions where the jump phenomenon exists. Figs. 3 and 4 (c, g , h), show that the steady state amplitudes of the aircraft tail are monotonic increasing function in the excitation amplitudes $F_S$ , $P_j$ and the coupling coefficient κ. Also Figs. 3 and 4 (d, e, f), show that the steady state amplitudes of the aircraft tail are monotonic decreasing function in the natural frequencies $\omega_1$and $\omega_2$ , the linear damping coefficients $\mu_1$ and $\mu_2$ and the gains $G_1$ and $G_2$. For increasing or decreasing nonlinear parameters $\alpha_1$ and $\alpha_3$ the curves are bent to right or left producing either hard or soft spring respectively as shown in Figs. 3b and 4b.

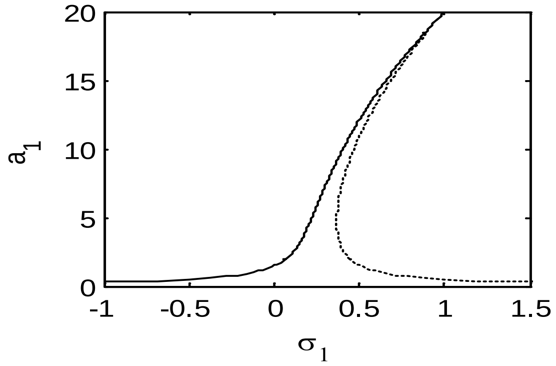



**Fig.3a:** Effects of the detuning parameter $\sigma_1$

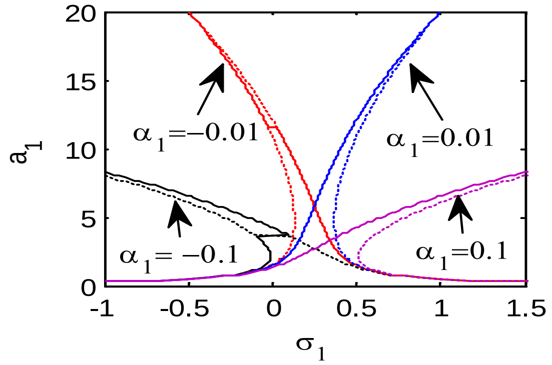



**Fig. 3b:** Effects of the non-linear parameter $\alpha_1$

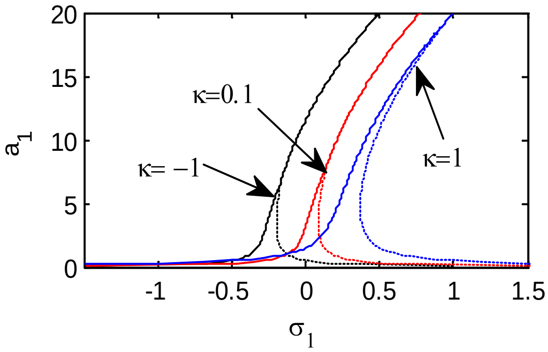



**Fig.3c:** Effects of coupling coefficient $\kappa$

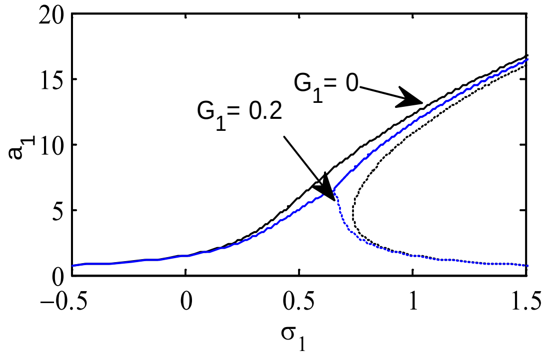



**Fig. 3d:** Effects of the gain $G_1$

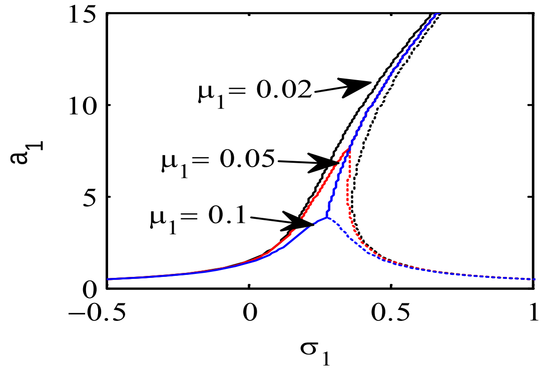



**Fig.3e:** Effects of the damping coefficient $\mu_1$

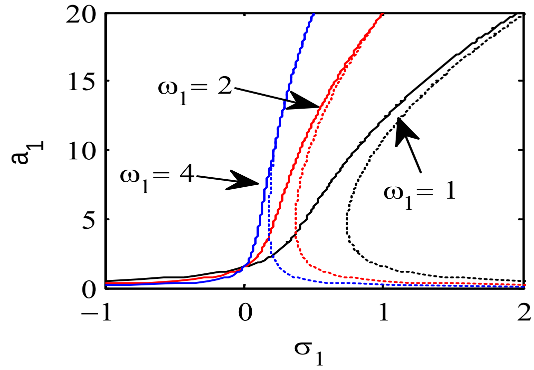



**Fig. 3f:** Effect of the natural frequencies $\omega_1$

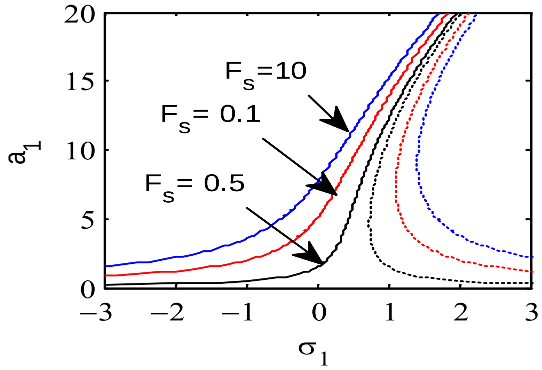



**Fig.3g:** Effects of the excitation amplitudes $F_S$

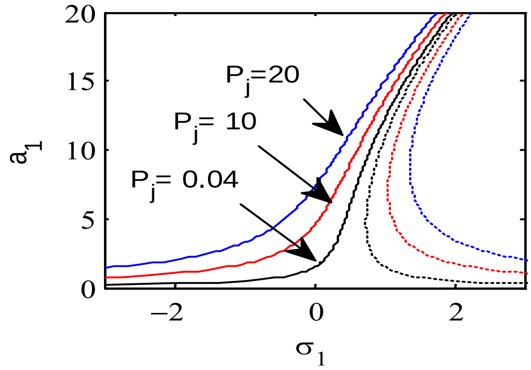



**Fig. 3h:** Effects of the excitation amplitudes $P_j$

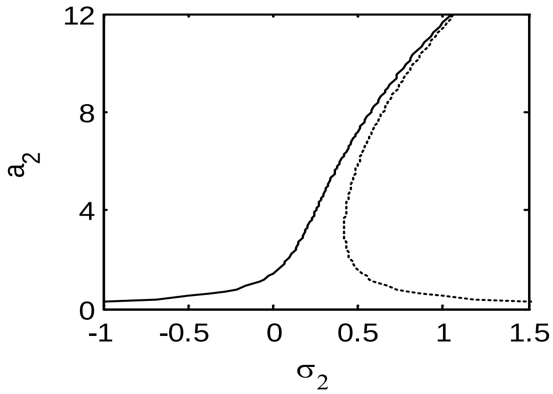



**Fig.4a:** Effects of the detuning parameter $\sigma_2$

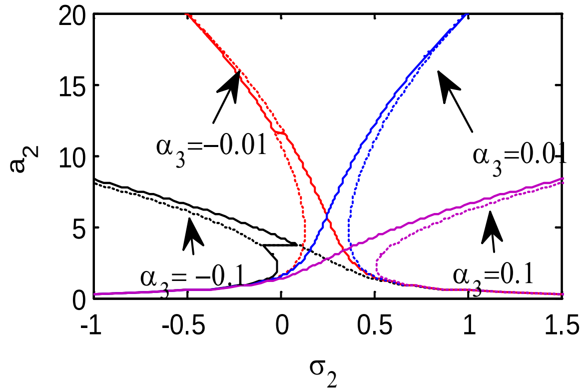



**Fig. 4b:** Effects of the non-linear parameter $\alpha_3$

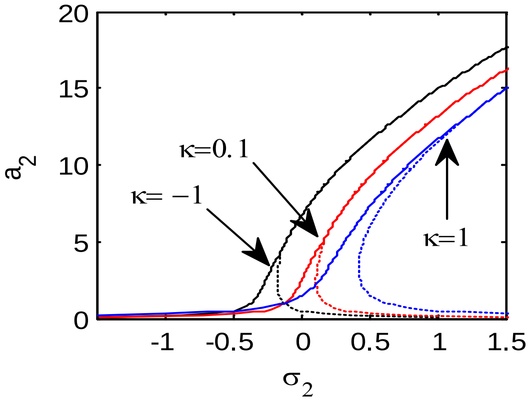



**Fig.4c:** Effects of coupling coefficient $\kappa$

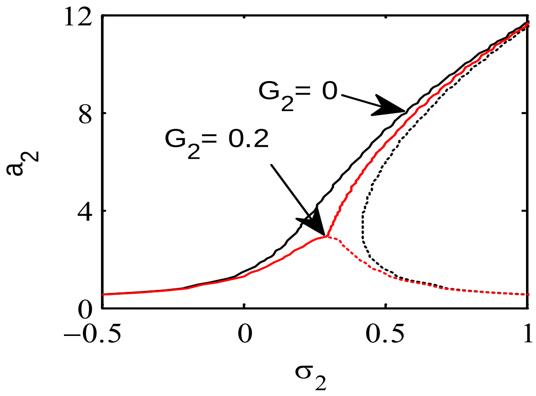



**Fig. 4d:** Effects of the gain $G_2$

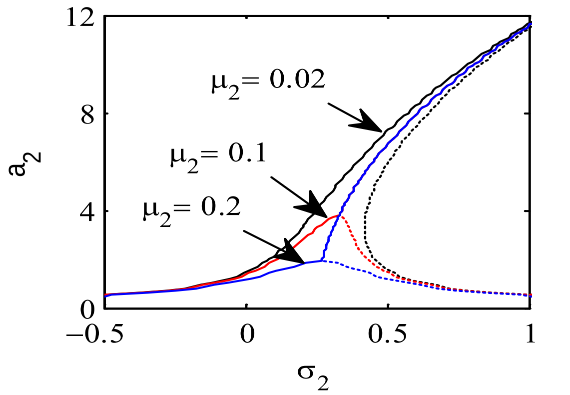



**Fig.4e:** Effects of the damping coefficient $\mu_2$

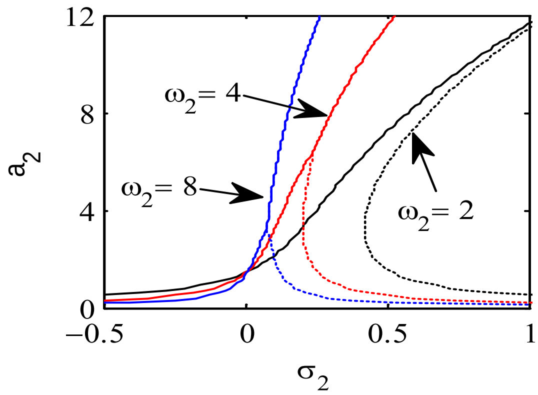



**Fig. 4f:** Effect of the natural frequencies $\omega_2$

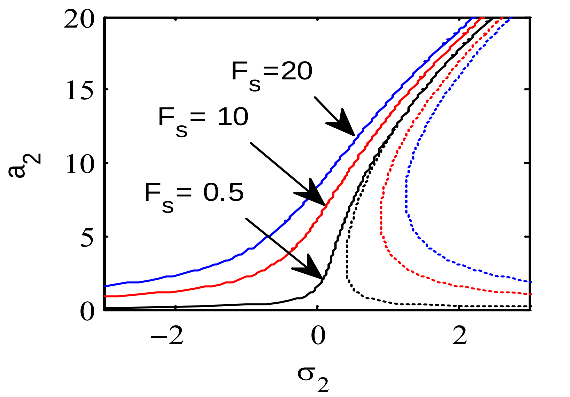



**Fig.4g;** Effects of the excitation amplitudes $F_S$

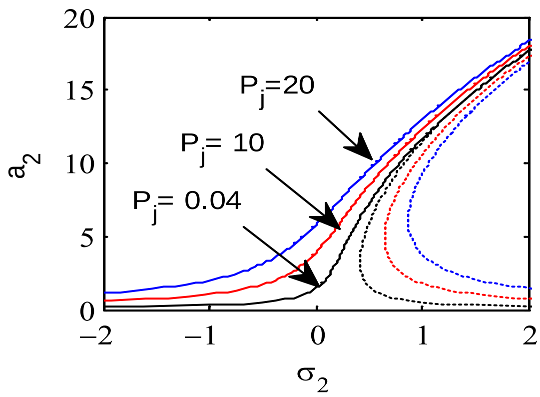



**Fig. 4h:** Effects of the excitation amplitudes $P_j$

## 5. CONCLUSIONS

The vibrations of a second-order nonlinear system having both quadratic and cubic nonlinearities, subjected to multi-harmonic and multi-tuned excitation forces can be controlled via negative linear velocity feedback to the system which are in agreement with Refs [1, 2, 3]. Multiple time scale perturbation method is useful to determine approximate solutions for the coupled differential equations describing the system up to third order approximation. To study the stability of the system, both the frequency response equations and the phase-plane technique are applied. The effect of the different parameters of the system is studied numerically. From the above study the following may be concluded:

1. The worst behavior of the system occurs at simultaneous primary and combined and internal resonance case $(\Omega_S \cong \omega_1,\ \Omega_S \cong \omega_2$ and $(\Omega_{k_j} - \Omega_{k_{j+1}}) \cong \omega_1)$.
2. The effectiveness of the negative velocity feed back controller at the simultaneous primary and combined and internal resonance case $(\Omega_S \cong \omega_1,\ \Omega_S \cong \omega_2$ and $(\Omega_{k_j} - \Omega_{k_{j+1}}) \cong \omega_1)$ is $E_a$ =125 and $E_a$ =90.

3. The steady state amplitudes of the aircraft twin-tail are monotonic increasing function in the excitation amplitudes $F_S$, $P_j$ and the coupling coefficient $\kappa$ which is in agreement with Ref [4].
4. The steady state amplitudes of the aircraft tail are monotonic decreasing function to the natural frequencies $\omega_1$ and $\omega_2$, the linear damping coefficients $\mu_1$ and $\mu_2$ and the gains $G_1$ and $G_2$ which agree with Ref [4].
5. For increasing or decreasing the nonlinear parameters $\alpha_1$ and $\alpha_3$ the curves are bent to right or left producing hard or soft spring respectively and the jump phenomenon occurrence.
6. The reported results are in good agreement with Refs [1, 2, 3] regarding the amplitude reduction and jump phenomenon occurrence.

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