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LEFT JORDAN AND LEFT DERIVATIONS ON PRIME RINGS

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ABSTRACT

In this paper first we studied some results of Jordan left derivation. Using these first we prove that for D:R→R be a left Jordan derivation 2r [a ,b] a^b=0 for all a, b ,r ЄR .And also we prove that in a prime ring R with characteristic ≠ 2, if D:R→R is a left Jordan derivation then D is a left derivation.

Key words: *Derivation of a Ring, Left Derivation of A Ring, Jordan Derivation of a ring, Left Jordan Derivation of a ring, Characteristic of a ring, Center.*

INTRODUCTION:

Throughout this paper R (≠0) will represent an associative ring with centre Z and X a non zero left R-module. An additive mapping D: R→R will be called a derivation if D (ab) =D (a) b+aD (b) holds for all pairs x, y є R. An additive mapping D: R→R will be called a left derivation if D (ab) =aD (b) +bD (a) holds for all pairs x, yЄR. Following [1], X is called prime if aRx=0 for a ЄR and x Є implies that either x=0 or aX=0.

As is well known, R is prime ring if and only if there exists a non zero faithful prime left R-module. Following (2), an additive mapping D: R→R is called a Jordan left derivation if D (a^2) =2aD (a) for all a ЄR.

I. N. Hersein [1] was shows that for a rather wide class of rings, namely prime rings of characteristic different from 2 a Jordan derivation of A is automatically an ordinary derivation of A. M. Bresar and J. Vukman [2] was present a brief proof of the well known result of Herstein which states that any Jordan derivation on a prime ring with characteristic not two is a derivation. We shall extend the results of M. Bresar and J. Vukman[2] results for left Joran and left derivations on prime rings.

MAIN RESULTS:

Theorem 1: Let R be a prime ring with characteristic not two and let D: R→R be a left Jordan derivation. Then D is a left derivation. For the proof of the theorem1 we need several steps. First we have

Lemma 1: Let R be a ring of characteristic 2.If D: R→R is a Jordan left derivation, then for all a, b, c ЄR, there holds the following:

(1) D (ab +ba)=2aD(b)+2bD(a).

(2) D (aba) =a^2D (b) +3abD (a)-baD (a).

(3) D(abc+cba)=(ab+ca)D(b)+3abD(c)+ 3cbD(a)-baD(c)-bcD(a).

(4) (ab-ba)aD(a)=a(ab-ba)D(a)

(5) (ab-ba)(D(ba)-bD(b)-bD(d))=0.

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Proof: From the Jordan derivation

$D(a^2) = 2aD(a)$ (1)

Substituting a+b for a in (1) we get

$D((a+b)^2) = 2(a+b)D((a+b)^2)$

$D(a^2+b^2+ab+ba) = 2(a+b)D(a+b)$

$D(a^2+b^2+ab+ba) = (2a+2b)D(a+b)$

$D(a^2)+D(b^2)+D(ab+ba) = 2aD(a)+2aD(b)+2bD(a)+2bD(b)$

Which implies

$D(ab+ba) = 2aD(b)+2bD(a)$

Hence (1) is proved.

Let us prove (2) from (1) it follows that

$D(a(ab+ba)+(ab+ba)a) = 2aD(ab+ba)+2(ab+ba)D(a)$

$= 2a(2aD(b)+2bD(a))+2(ab+ba)D(a)$

$= 4a^2D(b)+6abD(a)+2baD(a)$

On the other hand we have

$D(a(ab+ba)+(ab+ba)a) = D(a^2b+ba^2)+2D(aba)$

$= 2a^2D(b)+2bD(a^2)+2D(aba)$

$= 2a^2D(b)+4baD(a)+2D(aba).$

In comparison we obtain

$2D(aba) = 2(a2D(b)+3abD(a)-baD(a)$

Which proves (2).

Since X is2-torsion free by the assumption.

The linearization of (2) gives (3).

Now we are able to prove (4).

Let us denote D (ab(ab) +(ab)ba) by A

Then using (3) we obtain

$A=(a(ab)+(ab)a)+3abD(ab)+3ab^2D(a)-baD(ab)-babD(a).$

On the other hand, since $A=D((ab)^2+ab^2a)$ and using (1) and (2) we obtain

$A=2abD(ab)+a^2D(b^2)+3ab^2D(a)-$

$b^2aD(a) = 2abD(ab)\ 2a2bD(b)+3ab^2D(a)-b^2aD(a).$

By comparing the two expressions obtained from A we have

$(ab-ba)D(ab) = a(ab-ba)D(b)+b(ab-ba)D(a).$

Replacing a+b for b in (2), we have

$(ab-ba)D(ab) + (ab-ba)D(a^2) = a(ab-ba)D(a) + a(ab-ba)D(b) + b(ab-ba)D(a) + a (ab-ba)D(a)$

And according to (1) and (2) we obtain (4).

Let us write a+b for a in (4), using (4) we obtain

$(ab-ba)aD(b)$ $(ab-ba)bD(a) = a(ab-ba)D(b)+ b(ab-ba)D(a)$.

Combining this relation with (2) we prove (5).

The proof of the lemma is complete.

For any Jordan left derivation D we shall write a^b for $D(ab)-bD(a)-aD(b)$.

Now from (1) in lemma1 we see that

$D (ab +ba) =2aD (b) +2bD (a)$

$D(ab) + D(ba) = aD(b) + aD(b) + bD(a) + bD(a)$

Which implies $a^b = - b^a$ (2)

holds for all a,b∈R.

And $a^{b+c} =D (a (b+c)) - (b+c) D(a) - aD(b+c)$

$=D (ab+ac) - bD(a) - cD(a) - aD(b) - aD(c)$

$a^{b+c} = a^b + b^a$ (3)

holds for all a,b∈R.

Theorem 2: Let R be a ring of characteristic not two, and let D: R→R be a left Jordan derivation .In for all a, b, r ∈R we have

$$2r [a, b] a^b=0.$$

Proof: Let us write W for abrba+barba. Then by (2) of lemma (1) we obtain

$D(W) =D(a(brb)a)+b(ara)b)$

$= a^2D(brb)+3abrbD(a)brbaD(a)+b^2D(ara)+3baraD9b) - arabD(b)$

$= a^2[b^2D(r)+3br D(b)-rbD(b)]+3abrbD(a)-brbaD(a)+b^2[a^2D(r)+3arD(a)-raD(a)]+3baraD(b)-arabD(b)$

$= a^2b^2D(r)+3 a^2br D(b)- a^2rbD(b)+3abrbD(a)-brbaD(a)+ b^2a^2D(r)+3 b^2arD(a)- b^2raD(a)+3baraD(b)-rabD(b)$ (4)

On the other hand we obtain using (3) of lemma1.

$D (W) =D ((ab) r (ba) + (ba) rD (ab))$

$= ((ab)(ba)+(ba)(ab)D(r)+3abrD(ba)+3barD(ab)-rabD(ba)-rbaD(ab)$

$= ((ab)ba)+(ba)(ab))D(r)+3abr[bD(a)+aD(b)]+3bar[bD(a)+aD(b] - rba[bD(a)+aD(b)]$ (5)

Comparing (4) and (5) we have

$3abr\, b^a +3bar\, a^b - rab\, b^a - rba\, a^b = 0$

Which implies

$-3ab\, a^b +3ba\, a^b +rab\, a^b - rba\, a^b = 0$

$-3[a, b]r\, a^b + [a, b]r\, a^b = 0$

Which gives

$2[a, b]r\, a^b = 0.$

Hence theorem is proved.

The proof theorem1: Let a and b be fixed elements from R. If $ab \neq ba$ then from above theorem obtains immediately that $a^b = 0$.

If a and b are both in Z(R) then by (1) of lemma 1 implies

D (2ab) =2aD (a) +2bD (b)

2D (ab) = 2(aD(a)+bD(b))

Which implies

D (ab) = aD(b)+bD(a)

Which gives $a^b=0$.

It remains to prove that $a^b=0$ also in the case when a does not lie in Z(R) and $b \in Z(R)$ there exists $c \in R$ such that $ac \neq ca$.

Since $ac \neq ca$ and $a (b+c) \neq (b+c) a$ we have $a^c=0$ and $a^{b+c}=0$. Then we obtain using (B)

$0= a^{b+c}= a^b+ a^c= a^b.$

Therefore the proof of the theorem.

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