International Journal of Mathematical Archive-3(3), 2012, Page: 1063-1068
Available online through www.ijma.info ISSN 2229 – 5046

β*-CLOSED SETS IN TOPOLOGICAL SPACES

J. ANTONY REX RODGIO, JESSIE THEODORE* AND A. HANA SELVI JANSI

Department of mathematics, V. O. Chidambaram College, Tuticorin-628 008, Tamil Nadu, INDIA

Ph.D. Scholar, Department of mathematics, Sarah Tucker College, Tirunelveli-7, India

Ph.D. Scholar, Department of mathematics, Sarah Tucker College, Tirunelveli-7, India
E-mail: lwellingtondaniel7@yahoo.in

(Received on: 06-03-12; Accepted on: 26-03-12)

ABSTRACT

New classes of sets called β^*-closed sets are introduced and studied some of their properties.

Keywords:**g-closed,*g-open,$\tilde{g}$-closed, $\tilde{g}$ open, ρ-closed,,$\hat{\eta}*$ -closed, β*-closed*

Mathematics subject classification: *54A05, 54A10, 54D10.*

---

1. INTRODUCTION

The study of generalized closed sets in topological space was initiated by Levin [4]. In 1986 Andrijevic [2] defined semi pre open sets and it is also known under the name β-closed sets. In 1996, Julian Dontchev [3] introduced the notion of generalized semi-pre closed (*briefly gsp-closed sets)* via the concept of semi pre open sets. Generalised closed sets namely g-closed sets, gs closed sets, r-g closed sets, s-g closed sets, α-closed sets,α-g closed sets were introduced and studied by various authors. The class of gsp- closed sets contains properly the classes of all the above mentioned generalied closed sets except r-g closed sets. The class of ω-closed set was introduced by M. Shiek John[11]in 2002.In this paper we introduce a new classes of sets called β^*Closed sets.This class lies between the class of open and semi pre closed sets and the class of $\hat{\eta}*$-closed sets [9].

2. PRELIMINARIES

Throught this paper (X, τ), (Y, σ) and (Z, η) will always denote topological spaces, on which no separation axioms are assumed unless otherwise mentioned. When A is a subset of (X, τ), Cl (A), Int(A) and D[A] denote the closure, the interior and the derived set of A, respectively.

We recall some known definitions needed.

Definitions 2.1: Let (X,τ) be topological space. A subset A of X is said to be
1.Preopen [7] if A ⊆ Int(cl(A)) and preclosed if Cl(Int(A))⊆A.
2.Semi open[6] if A⊆Cl(Int(A)) and semi closed if Int(Cl(A))⊆A.
3.Semi pre open[1] if A⊆Cl(Int(Cl(A))) and semi pre closed if Int(Cl(Int(A))) ⊆A.

Definition 2.2: Let (X,τ) be a topological space. A subset A of X is said to be
1.generalised closed (briefly g-closed) [5] if Cl(A)⊆U whenever A⊆U and U is open in X.
2. generalized pre closed (briefly gp-closed) [8] if Pcl(A)⊆U whenever A⊆U and U is open in X.
3. generalized semi pre closed (briefly gsp closed) [3]if Spcl(A)⊆U whenever A⊆U and U is open in X.
4.ω-closed if [11]if Cl(A)⊆U whenever A⊆U and U is semi open in X.
5.*g-closed if [12]cl(A)⊆U whenever A⊆U and U is ω-open in X.
6. #gs-closed [13]if Scl(A) ⊆U whenever A⊆U and U is *g-open in X.
7.$\tilde{g}$-closed [4] if cl(A)⊆ U whenever A⊆U and U is #gs-open.
8.ρ-closed [10] if Pcl(A)⊆Int(U) whenever A⊆U and U is $\tilde{g}$ open in X.
9.$\hat{\eta}*$ -closed [9] if Spcl(A) ⊆Int(cl(U)) whenever A⊆U and U is ω-open in X.

*Corresponding author: Wellington Daniel, *E-mail: lwellingtondaniel7@yahoo.in

The compliments of above mentioned sets are called their respective closed sets.

Basic Properties Of β*Closed Sets

We introduce the following **Definition**

Definition 3.1: A subset A of a space (X,τ) is said to be β*- closed in (X, τ) if spcl (A) ⊆ Int (U) whenever A⊆U and U is ω -open in (X, $\boldsymbol{\tau}$)

Theorem 3.2: Every open and semi preclosed subset of (X,τ) is β*- closed but not conversely.

Proof: Let A be an open and semi preclosed subset of (X,τ)

Let A⊆U and U be ω -open in X

Then spcl (A) = A = Int (A)⊆ Int (U)

Hence A is β*closed

The converse of the above **Theorem** need not be true as seen from the following example

Example 3.3: Let X = {a, b, c, d} and τ = {ϕ, {a}, X}

Then the set A = {a, c} is β* closed but neither open nor preclosed

Theorem 3.4: Every β*- closed set is gsp – closed but not conversely.

Proof: Let A be any β* - closed set in X

Let A⊆U and U be open in X

Since every open set is ω -open and A is β*- closed Spcl (A) ⊆ Int (U) = U

Hence A is gsp – closed

Converse of the above **Theorem** need not be true as seen from the following

Example 3.5: Let X = {a, b, c} and τ = {ϕ,{a},{b, c},X}

Then the set A = {a, b} is gsp closed but not β*- closed in X

Theorem 3.7: Every open and preclosed subset of(X,τ) is β*- closed

Proof: Let A be an open and preclosed subset of (X, τ)

Let A ⊆ U and U be ω -open in X

Then spcl (A) ⊆ pcl (A) = A = Int (A) ⊆ Int (U)

Hence A is β*- closed

The converse of the above **Theorem** need not be true. It is seen from the following example.

Example 3.8: Let X = {a, b, c, d} and τ = {ϕ,{a},X}

Then the set {a, b} is β*- closed but neither open are preclosed.

Remark 3.9: β*- closedness and preclosedness are independent. It is shown by the following examples.

Example 3.10: Let X = {a, b, c} and τ = {ϕ,{a},{b},{a, b},X}

Then the set A = {a} is β*- closed but not preclosed.

Example 3.11: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}}

Then the set A = {a, b} is preclosed but not β*- closed

Remark 3.12: β*- closedness and α-closedness are independent. It is shown by the following examples.

Example 3.13: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}

Then the set A = {a} is α-closed but not β*- closed

Example 3.14: Let X = {a, b, c, d} and τ = {ϕ, {a}, X}

Then the set {a, b} is β*- closed but not α-closed.

Remark 3.15: β*- closed sets are independent of semi closed sets and semi preclosed sets. It is shown by the following examples.

Example 3.16: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}

Then the set A = {a, b} is both semi preclosed and semi closed but not β*- closed.

Example 3.17: Let X = {a, b, c} and τ = {ϕ, {a}, {a, c}, X}

Then the set A = {a, b} is β*- closed but neither semi preclosed nor semi closed.

Remark 3.18: β*- closedness and pre semi closedness are independent. It is shown by the following examples.

Example 3.19: Let X = {a, b, c} and τ = {ϕ, {a, b}, X}

Then the set A = {a} is pre semi closed but not β*- closed.

Example 3.20: Let X = {a, b, c, d} and τ = {ϕ, {a}, X}

Then the set A = {a, b} is β*- closed but not pre semi closed.

Remark 3.21: β*- closedness and g closedness are independent. It is shown by the following examples.

Example 3.22: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}

Then the set A = {a, c} is g closed but not β*- closed.

Example 3.23: Let X = {a, b, c} and τ = {ϕ,{a},{b},{a, b},X}

Then the set A = {b} is β*- closed but not g closed.

Theorem 3.24: β*- closed set is $\widehat{\eta *}$ closed set but not conversely.

Proof: Let A be any β*- closed set in X.

Let A⊆U and U be ω -open in X.

Then spcl (A) ⊆ Int (U) ⊆ U

Hence A is $\widehat{\eta *}$ closed

Converse of the above **Theorem** need not be true. It is seen from the following example

Example 3.25: Let X = {a, b, c, d, e} and τ = {ϕ, {a, b}, {a, b, d}, {a, b, c, d}, {a, b, d, e}, X}

Then the set A = {a} is $\widehat{\eta *}$ closed but not β*- closed

Example 3.26: β*- closedness and ρ closedness are independent. It is shown by the following examples.

Example 3.27: Let X = {a, b, c, d, e} and τ = {ϕ, {a, b}, {a, b, d}, {a, b, c, d}, {a, b, d, e}, X}

Then the set A = {c} is β* closed but not ρ - closed

Then the set B = {b, d} is ρ closed but not β*- closed.

Definition 3.28: A subset A of a space (X, τ) is said to be β*s- closed in (X, τ) if spcl (A) ⫅ Int (cl(U)) whenever A⊆U and U is ω -open in (X, τ).

Theorem 3.29: Every β*- closed set is βs*- closed but not conversely.

Proof: Let A be any β*- closed set.

Let A⊆U and U be ω -open

A is β*- closed, spcl (A) ⊆ Int (U) ⊆ Int (cl(U))

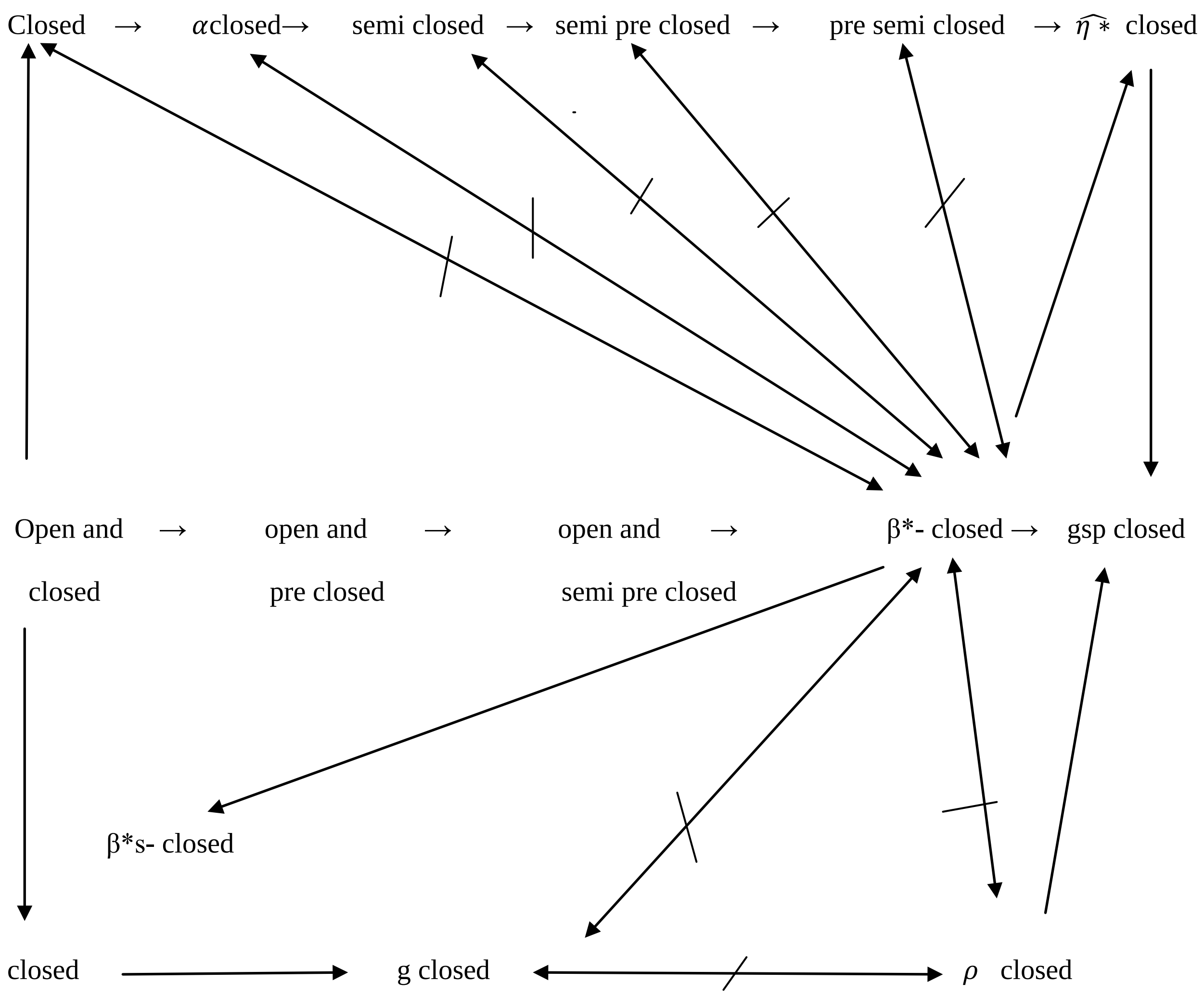
Hence A is βs*- closed.

The converse of the above **Theorem** need not be true. It is seen from the following example.

Example 3.30: Let X = {a, b, c, d, e} and τ = {ϕ, {a, b},{a, b, d},{a, b, c, d},{a, b, d, e},X}

Then the set A = {b, d} is βs*- closed but not β*- closed

Remark 3.31: From the above discussion and known results we have the following implications. A→B represents A implies B but not conversely and A ↔ B represents A and B are independent of each other



Properties Of β*Closed Sets

Remark 3.32: The union and intersection of two β*- closed sets need not be β*- closed. It is shown in the following examples.

Example 3.33: Let X = {a, b, c, d} and τ = {ϕ, {a}, X}

Then set A ={a, b} and B = {a, c} are β*- closed but A∩B = {a} is not β*- closed

2. Let X = {a, b, c} and τ = {ϕ, {a},{b},{a, b},X}

Then set A ={a} and B ={b} are β*- closed but AUB = {a, b} is not β*- closed

Theorem 3.34: A set A is β*- closed Then spcl (A) – A contains no non empty closed set.

Proof: Suppose H ⊆ spcl(A)-A is a non empty closed set

Then H ⊆ spcl(A) and A ⊆ X-H. Since X-H is ω - open and A is β*- closed, we have spcl(A) ⊆ Int (X-H) = X – cl(A).

Hence cl(H) ⊆ X-spcl(A). which is a contradiction.

Hence spcl(A)-(A)contains no non empty closed set. Converse of the above

Theorem need not be true as seen from the following example.

Example 3.35: Let X = {a, b, c} and τ = {ϕ, {a},{b, c},X}

Let A = {a, c} Then spcl(A) – (A) = {a, c}-{a, c} = ϕ contains no non empty closed set but A is not β*- closed

Theorem 3.36: A set A is β*- closed Then spcl(A) – (A) contains no non empty ω -closed sets.

Proof: Let F be a non empty ω -closed set of F ⊆ spcl (A) –(A). Then F ⊆ spcl (A) and A ⊆ X-F. Since A is β*- closed and X- F is ω –open. We have spcl(A) ⊆ Int (X-F) = X – cl(F). Hence cl (F) ⊆ X – spcl (A) and so F ⊆ X – spcl(A).

Already F ⊆ spcl (A) hence we get a contradiction. Hence spcl (A) – (A) contains no non empty ω – closed set.

Theorem 3.37: If A is β*- closed and A ⊆ B ⊆spcl (A) Then B is β*- closed

Proof: Let U be a ω – open set of X such that B ⊆U. Since A ⊆ U and U is ω – open we have spcl (A) ⊆ Int (U) .We have B ⊆ spcl (A) and so spcl (B) ⊆ spcl (spcl(A))=spcl (A). Hence spcl (A) ⊆ Int (U) and so B is β*- closed

Theorem 3.38: If a subset A of (X, τ) is ω – open and β*- closed Then A is semipreclosed in (X, τ).

Proof: If A is ω – open and β*- closed, since A ⊆ A, we have spcl (A) ⊆ Int (A) ⊆A. but A ⊆ spcl (A). Hence A = spcl (A).So A is semipreclosed.

Lemma 3.39: If A is open and gsp closed .Then A is semi pre closed.

Proof: Let A ⊆ U and U be open in (X, τ), since A is open and A ⊆ A, we have spcl (A) ⊆ A ⊆ U, hence spcl (A) ⊆ U and so A is semi pre closed.

Theorem 3.40: A open set of (X,τ) is gsp closed if and only if A is β*- closed.

Proof: Let A be a open set of (X,τ) and A is gsp closed.

Let A ⊆ U and U be ω – open in (X, τ), since A is open and gsp closed by the lemma 3.39; A is semi pre closed. Hence spcl (A) = A = Int (A) ⊆ Int (U), therefore spcl (A) ⊆ Int (U) and so A is β*- closed. Conversely let A be a β*- closed set. Then by Theorem 3.4, A is gsp closed.

Theorem 3.41: Let A be β*- closed in (X, τ) Then A is semi pre closed if and only if spcl (A) – (A) is ω closed.

Proof: Let A be semi pre closed. Then spcl (A) = (A) and so spcl (A) – (A) = ϕ which is ω closed. Conversely Let spcl (A) – (A) is ω closed, Since A is β*- closed by **Theorem** 3.36 spcl (A) – A contains no non empty ω – closed set.

Hence spcl (A) – (A) = ϕ which implies spcl (A) = (A) and so A is semi pre closed

Definition 3.42: Let (X, τ) be a topological space and A ⊆ X and x ∈ X, Then x is said to be a semi pre limit point of A if every semi pre open set containing x contains a point of A different from x.

Definition 3.43: Let (X,τ) be a topological space and A ⊆ X, the set of all semi pre limit point of A is said to be semi pre derived set of A and is denoted by D_{sp} [A]

Theorem 3.44: If D [A] ⊆ D_{sp} [A] for each subset A of a space (X , τ), Then the union of two β*- closed set is β*- closed.

Proof: Let A and B be β*- closed subsets of X and U be ω – open set with A U B ⊆ U, Then spcl (A) ⊆ Int(U) and spcl (B) ⊆ Int (U). Since for each subset A of X, we have D_{sp} [A] ⊆ D[A], we get cl(A) = spcl (A) and cl(B) = spcl (B), therefore cl (AUB) = cl (A) U cl(B) = spcl (A) U spcl (B) ⊆ Int(U), but spcl (AUB) ⊆ cl (AUB). So spcl (AUB) ⊆ cl(AUB) ⊆ Int (U), hence AUB is β*- closed

REFERENCE:

[1] M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud, β-open sets and β-continuous mappings. Bull. Fac. Sci. Univ. (1983),77-90.

[2] D. Andrijevic, semi-pre open sets. Mat. Vesnic. (1986), 24-32.

[3] J. Dontchev, On generalizing semi-pre open sets.Mem.fac. Sci. Kochi Uni. Ser. A Math., (1995), 35-48.

[4] S. Jafari, T. Noiri, N. Rajesh, M. L. Thivagar. Another generalization of closed sets, Kochi J. Math, S (2008), 25-38

[5] N. Levin. Generalised closed sets in topology, Rend. Circ. Math., Palermo, (1970), 89-96.

[6] N. Levin, semi open sets, semi continuity in topological spaces, Amer Math, Monthly,70(1963), 36-41

[7] A. S. Mashour, M.E.ABD EL-Monsef and S. N. El-Deep, ON pre continuous and weak pre continuous mapping, Proc, Math, Phys, Soc. Egypt, 53(1982),47-53

[8] Noiri H.Maki and J. UMEHARA, generalized pre closed functions, Mem. Fac. Sci. Kochi Univ. Ser. A. Maths., 19(1998), 13-20

[9] N.Palaniappan ,J.Antony Rex Rodgio and S.P.Misser,On .$\hat{\eta}*$ closed sets in topological spaces,International journal of general topology Vol. 1, No.1 (2008), 77-88.

[10] S.Pious miser and C. Devamanoha, On ρ-closed sets, ON Contra ρ continuous functions and strongly ρ closed spaces, Demonstratio Mathematica vol.XLV NO 12012

[11] M. Shiek John,A study of generalization of ω closed sets and continuous maps in topological and by topological spaces, Ph. D Thesis, Bharathiar Univerrsity, Coimbathore (2002),44-55

[12] M.K.R.S Veera Kumar, Between g* closed and g closed sets, Antarctica J. Maths.

[13] M. K. R. S. Veerarakumar , #gs closed in topological spaces, Antartica J. maths 2(2)(2005),201-202

****