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EULER CHARACTERISTIC AND CELLULAR FOLDING

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ABSTRACT

In this paper we investigate the action of Euler characteristic on CW-complexes under some known operations such as, Cartesian product, Join product, Suspension, Wedge sum, Quotient, Smash product. Also we investigate the action of Euler characteristic on finite graphs under some known operations such as, Cartesian product, Tensor product, Join product, Composition product and Normal product. Finally, we obtained the relation between the regular CW-complex and its image under a cellular folding in terms of Euler characteristic.

Keywords: *Euler characteristic, Join product, Suspension, Quotient, Wedge sum, Smash product, Cellular folding.*

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1. INTRODUCTION

A Cellular folding is a folding defined on regular *CW*-complexes first defined by E-El-Kholy and H. Al-Khurasani, [1], and various properties of this type of folding are also studied by them. By a cellular folding of regular *CW*-complexes, it is meant a cellular map $f : K \to L$ which maps *i*-cells of *K* to *i*-cells of *L* and such that $f \setminus e^i$, for each *i*-cell *e*, is a homeomorphism onto its image.

The set of regular *CW*-complexes together with cellular foldings form a category denoted by $C(K, L)$. If $f \in C(K, L)$; then $x \in K$ is said to be a singularity of f iff f is not a local homeomorphism at *x*. The set of all singularities of f is denoted by $\sum f$. This set corresponds to the "folds" of the map. It is noticed that for a cellular folding f, the set $\sum f$ of singularities of f is a proper subset of the union of cells of dimension $\leq n-1$. Thus when we consider any $f \in C(K, L)$, where *K* and *L* are connected regular *CW*-complexes of dimension 2, the set $\sum f$ will consists of 0-cells, and 1-cells, each 0-cell (vertex) has an even valency, [2, 3,13], of course $\sum f$ need not be connected.

From now we mean by a complex a regular *CW*-complex.

2. DEFINITIONS

(i) If (X, Y) is a *CW*-pair consisting of a cell complex *X* and a subcomplex *Y*, then the quotient space X/Y inherits a natural cell complex structure from *X*. The cells of X/Y are the cells of $X - Y$ plus one new 0-cell, the image of *Y* in X/Y. For a cell e_α^n of $X - Y$ attached by $\phi_\alpha : S^{n-1} \to X^{n-1}$, the attaching map for the corresponding cell in X/Y is the composition $S^{n-1} \to X^{n-1} \to X^{n-1}/Y^{n-1}$ [4]. For example, if we give S^{n-1} any cell structure and build D^n from S^{n-1} by attaching *n*-cell, the quotient, D^n / S^{n-1} is S^n with its usual cell structure.

(ii) For a space *X*, the suspension SX is the quotient of $X \times I$ obtained by collapsing $X \times \{0\}$ to one point and $X \times \{1\}$ to another point. In other words the suspension SX is the union of all line segments joining points of *X* to two external vertices, [4]. The motivating example is $X = S^n$, where $SX = S^{n+1}$ with two suspension points at the

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north and south poles of S^{n+1}, the points $(0,\, 0,\, ...,\, 0,\, \pm 1)$. One can regard $S\,X$ as a double cone on *X*, the union of two copies of the cone $C\,X = (X \times I)/(X \times \{0\})$. If *X* is a *CW*-complex, so $S\,X$ and $C\,X$ as quotient of $X \times I$ with its cell structure, *I* being given the standard cell structure of two 0-cells joined by a 1-cell.

(iii) Given two spaces *X* and *Y*. This is the join $X * Y$, the quotient space of $X \times Y \times I$ under identification $(x,\, y_1,\, 0) \sim (x,\, y_2,\, 0)$ and $(x_1,\, y,\, 1) \sim (x_2,\, y,\, 1)$. Thus we are collapsing the subspace $X \times Y \times \{0\}$ to *X* and $X \times Y \times \{1\}$ to *Y*. The join product $X * Y$ is a cell complex if *X* and *Y* are cell complexes, [4].

For example if *X* and *Y* are both closed intervals, then we are collapsing two opposite faces of a cube onto line segments so that the cube becomes a tetrahedron.

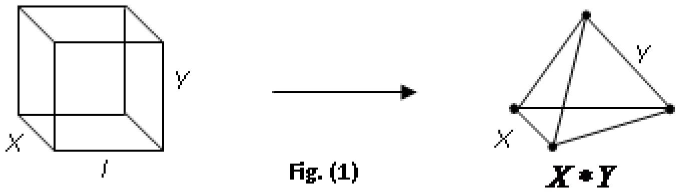

Fig. (1)

(iv) Let *X* and *Y* be connected cell complexes with $X \bigcap Y = \{\,p\}$ for a vertex *p*, the space $X \vee Y$, so formed is called the wedge sum or one-point union., [5].

(v) Inside a product space $X \times Y$ there are copies of *X* and *Y* namely $X \times \{y_0\}$ and $\{x_0\} \times Y$ for points $x_0 \in X$ and $y_0 \in Y$. These two copies of *X* and *Y* in $X \times Y$ intersect only at the point $(x_0,\, y_0)$, so their union can be identified with the wedge sum $X \vee Y$. The smash product $X \wedge Y$ is then defined to be the quotient $X \times Y \,/\, X \vee Y$, [4]. The smash product $X \wedge Y$ is a cell complex if *X* and *Y* are cell complexes with x_0 and y_0 0-cells, assuming that we give $X \times Y$ the cell complex topology rather than the product topology in cases when these two topologies differ.

For example, $S^m \wedge S^n$ has a cell structure with just two cells, of dimension 0 and $m+n$, hence $S^m \wedge S^n = S^{m+n}$. In particular, when $m = n = 1$ we see that collapsing longitude and meridian circles of a torus to a point produces a 2-sphere.

Let $G_1 = (V_1,\, E_1)$ and $G_2 = (V_2,\, E_2)$ be finite simple graphs, then:

(vi) The cartesian product, $G_1 \times G_2$, is the simple graph with vertex set $V(G_1 \times G_2) = V_1 \times V_2$ and edge set $E(G_1 \times G_2) = [(E_1 \times V_2) \bigcup (V_1 \times E_2)]$ such that two vertices $(u_1,\, u_2)$ and $(v_1,\, v_2)$ are adjacent in $G_1 \times G_2$ iff either:

(1) $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2, or

(2) u_1 is adjacent to v_1 in G_1 and $u_2 = v_2$, [6].

(vii) If G_1 and G_2 are vertex-disjoint graphs. Then the join, $G_1 \vee G_2$, is the super graph of $G_1 + G_2$, in which each vertex of G_1 is adjacent to every vertex of G_2, [6].

(viii) The composition, or lexicographic product, $G_1[G_2]$, is the simple graph with $V_1 \times V_2$ as the vertex set in which the vertices $u = (u_1,\, u_2),\ v = (v_1,\, v_2)$ are adjacent if either u_1 is a adjacent to v_1 or $u_1 = v_1$ and u_2 is adjacent to v_2.

The graph $G_1[G_2]$ need not to be isomorphic to $G_2[G_1]$, [6].

(ix) The normal product, or the strong product, $G_1 \circ G_2$, is the simple graph with $V(G_1 \circ G_2) = V_1 \times V_2$ where $(u_1,\, u_2)$ and $(v_1,\, v_2)$ are adjacent in $G_1 \circ G_2$ iff either:

(1) $u_1 = v_1$ and u_2 is adjacent to v_2, or
(2) u_1 is adjacent to v_1 and $u_2 = v_2$, or
(3) u_1 is adjacent to v_1 and u_2 is adjacent to v_2, [6].

(x) The tensor product, or Kronecher product, $G_1 \otimes G_2$, is a simple graph with $V(G_1 \otimes G_2) = V_1 \times V_2$ where (u_1, u_2) and (v_1, v_2) are adjacent in $G_1 \otimes G_2$ iff u_1 is adjacent to v_1 in G_1 and u_2 is adjacent to v_2 in G_2.

Note that $G_1 \circ G_2 = (G_1 \times G_2) \bigcup (G_1 \otimes G_2)$, [6].

3. MAIN RESULTS

Lemma 1: $\chi(nT^2) = (n-1)\chi(2T^2)$.

Lemma 2: $\chi(mP^2) = (2-m)\chi(P^2)$.

Lemma 3: $\sum_{i=0}^{n} (-1)^i [\#i-\text{cells} - \beta_i] = 0$.

Lemma 4: If K is orientable surface, then χ is even but the converse is not true.

Lemma 5: If χ is odd, then the surface is non-orientable but the converse is not true.

Lemma 6: For orientable 2-complex without boundary, $\chi + \beta_1 = 2$.

Lemma 7: For nonorientable 2-complexes without boundary, $\chi + \beta_1 = 1$.

Lemma 8: $g = \begin{cases} \frac{1}{2}\beta_1 & \text{in case of orientable complexes} \\ \beta_1 + 1 & \text{in case of nonorientable complexes.} \end{cases}$

Lemma 9: Two complexes M_1, M_2 are topologically equivalent iff $\beta_1(M_1) = \beta_1(M_2)$ and both are orientable or both are nonorientable.

Theorem 1: Let *M* and *N* be two complexes of dimensions *m*, *n* respectively. Then

$$\chi(|M| \times |N|) = \chi(|M|) \cdot \chi(|N|).$$

Proof:

$$\begin{aligned} \chi(|M|)\chi(|N|) = [\#0-\text{cells of }|M| - \#1-\text{cells of }|M|] + \#2-\text{cells of }|M| \\ + \ldots + (-1)^m \#m-\text{cells of }|M|] \cdot [\#0-\text{cells of }|N| - \#1-\text{cells of }|N| \\ + \#2-\text{cells of }|N| - \cdots + (-1)^n \#n-\text{cells of }|N|] \end{aligned}$$

$$\begin{aligned} = [(\#0-\text{cells of }|M|)(\#0-\text{cells of }|N|) - (\#0-\text{cells of }|M|)(\#1-\text{cells of }|N|) \\ + (\#0-\text{cells of }|M|)(\#2-\text{cells of }|N|) - \cdots + (-1)^n (\#0-\text{cells of }|M|)(\#n-\text{cells of }|N|) \\ - (\#1-\text{cells of }|M|)(\#0-\text{cells of }|N|) + (\#1-\text{cells of }|M|)(\#1-\text{cells of }|N|) \\ - (\#1-\text{cells of }|M|)(\#2-\text{cells of }|N|) + \cdots + (-1)^{n+1} (\#1-\text{cells of }|M|)(\#n-\text{cells of }|N|) \\ + (\#2-\text{cells of }|M|)(\#0-\text{cells of }|N|) - (\#2-\text{cells of }|M|)(\#1-\text{cells of }|N|) + \cdots \\ + (-1)^{n+2} (\#2-\text{cells of }|M|)(\#n-\text{cells of }|N|) - \cdots + (-1)^m (\#m-\text{cells of }|M|)(\#0-\text{cells of }|N|) \\ - \cdots + (-1)^{m+n} (\#m-\text{cells of }|M|)(\#n-\text{cells of }|N|)]. \end{aligned}$$

$$= [\,(\#0-\text{cells of }|M|)(\#0-\text{cells of }|N|)\,] - [\,(\#0-\text{cells of }|M|)(\#1-\text{cells of }|N|) + (\#1-\text{cells of }|M|)(\#0-\text{cells of }|N|)\,] + [\,(\#0-\text{cells of }|M|)(\#2-\text{cells of }|N|) + (\#1-\text{cells of }|M|)(\#1-\text{cells of }|N|) + (\#2-\text{cells of }|M|)(\#0-\text{cells of }|N|)\,] - \cdots + [\,(-1)^{m+n}(\#m-\text{cells of }|M|)(\#n-\text{cells of }|N|)\,]$$

$$= [\,(\#0-\text{cells of }(|M|\times|N|)] - [\,(\#1-\text{cells of }(|M|\times|N|)] + [\,(\#2-\text{cells of }(|M|\times|N|)] - \cdots + (-1)^{m+n}\#(m+n)-\text{cells of }(|M|\times|N|)$$

$$= \chi(|M|\times|N|).$$

The above theorem can be generalized for a finite number of complexes as follows:

Corollary 1: Let $M_1, M_2, ..., M_n$ be complexes, then

$$\chi(|M_1|)\times(|M_2|)\times\cdots\times(|M_n|) = \chi(|M_1|)\chi(|M_2|)...\chi(|M_n|)\,.$$

Example 1: Let *M*, *N* be complexes such that $|M|=|N|=S^1$. Then $|M|\times|N|=T^2$ (tours), see Fig. (2)

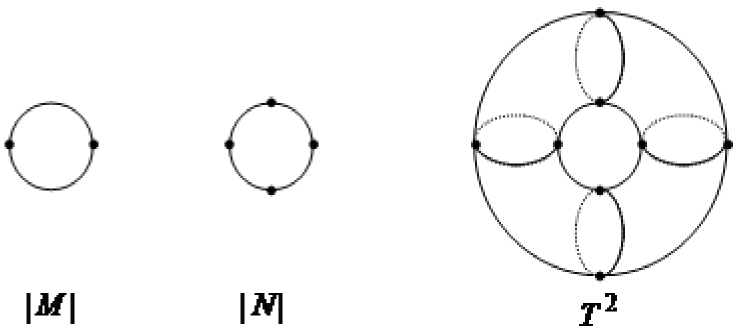

Fig. (2)

It is easy to check that $\chi(|M|\times|N|) = \chi(|M|)\cdot\chi(|N|)$

Theorem 2: Let *M* and *N* be two complexes of dimensions *m* and *n* respectively, then

$\chi(|M|*|N|) = \chi(|M|) + \chi(|N|) - \chi(M|\times|N|)$.

Proof: From the definition of join product spaces we have:

$\#0-\text{cells of }(|M|*|N|) = \#0-\text{cells of }|M| + \#0-\text{cells of }|N|$,

$\#1-\text{cells of }(|M|*|N|) = \#1-\text{cells of }|M| + \#1-\text{cells of }|N| + \#0-\text{cells of }(|\mathrm{M}|\times|N|)$,

$\#2-\text{cells of }(|M|*|N|) = \#2-\text{cells of }|M| + \#2-\text{cells of }|N| + \#1-\text{cells of }(|\mathrm{M}|\times|N|)$,

$\#3-\text{cells of }(|M|*|N|) = \#3-\text{cells of }|M| + \#3-\text{cells of }|N| + \#2-\text{cells of }(|\mathrm{M}|\times|N|)$,

$\#k-\text{cells of }(|M|*|N|) = \#k-\text{cells of }|M| + \#k-\text{cells of }|N| + \#(k-1)-\text{cells of }(|\mathrm{M}|\times|N|)$.

Thus we have

$$\chi(|M|*|N|) = \chi(|M| + x(|N|) - \chi(|M|\times|N|)\,.$$

Example 2: Let *M* and *N* be two complexes such that $|M|=|N|=I$ then $|M|*|N|$ is a tetrahedron

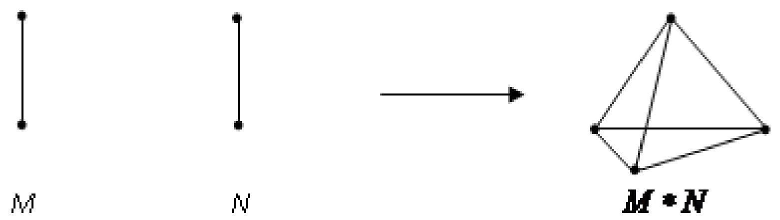

Fig. (3)

It is easy to check that $\chi(|M|*|N|)=\chi(|M|)+\chi(|N|)-\chi(M|\times|N|)$.

The above theorem can be generalized for a finite number of complexes as follows:

Corollary 2: Let $M_1,\ M_2,\ ...,\ M_n$ be complexes, then
$\chi(|M_1|*|M_2|*\cdots*|M_n|)=\chi(|M_1|)+\chi(|M_2|)+\cdots+\chi(|M_n|)-\chi(|M_1|\times|M_2|\times\cdots\times|M_n|)$.

Theorem 3: Let *M* and *N* be two complexes of dimensions *m* and *n* respectively, then

$\chi(|M|\vee|N|)=\chi(|M|)+\chi(|N|)-1$.

Proof: From the definition of the wedge sum, we have

$$\#0-\text{cells of }(|M|\vee|N|)=\#0-\text{cells of }|M|+\#0-\text{cells of }|N|-1,$$

$$\#1-\text{cells of }(|M|\vee|N|)=\#1-\text{cells of }|M|+\#1-\text{cells of }|N|,$$

$$\#2-\text{cells of }(|M|\vee|N|)=\#2-\text{cells of }|M|+\#2-\text{cells of }|N|,$$

$$\#k-\text{cells of }(|M|\vee|N|)=\#k-\text{cells of }|M|+\#k-\text{cells of }|N|,$$

and so on.

Thus we have,

$$\chi(|M|\vee|N|)=\chi(|M|)+\chi(|N|)-1.$$

The above theorem can be generalized for a finite number of complexes as follows:

Corollary 3: Let $M_1,\ M_2,\ ...,\ M_n$ be complexes, then

$$\chi(|M_1|\vee|M_2|\vee\cdots\vee|M_n|)=\chi(|M_1|)+\chi(|M_2|)+\cdots+\chi(|M_n|)-(n-1).$$

Theorem 4: Let *M* be a complex of dimension *n*, then

$$\chi(|SM|)=\begin{cases}\chi(|M|)+2 & \text{if } n \text{ is odd}\\ \chi(|M|)+2\text{-}2(\#0\text{ - cells of }|M|), & \text{if } n \text{ even.}\end{cases}$$

Proof: From the definition of suspension we have:

$$\#0-\text{cells of }(|SM|)=\#0-\text{cells of }|M|+2$$

$$\#1-\text{cells of }(|SM|)=\#1-\text{cells of }|M|+2(\#0-\text{cells of }|M|)$$

$$\#2-\text{cells of }(|SM|)=\#2-\text{cells of }|M|+2(\#0-\text{cells of }|M|)$$

$$\# k - \text{cells of } (|S\,M|) = \# k - \text{cells of } |M| + 2(\# 0 - \text{cells of } |M|)$$

and so on.

Thus

$$\chi(|S\,M|) = \begin{cases} \chi(|M|) + 2 & \text{if } n \text{ is odd} \\ \chi(|M|) + 2 - 2(\#0 - \text{cells of } |M|), & \text{if } n \text{ is even.} \end{cases}$$

Example 3: Let *M* be a complex such that $|M| = S^1$, then $|S\,M| = S^2$, see Fig. (4).

$\chi(|M|) = 0$ $\chi(|SM|) = 2$

Fig. (4)

Theorem 5: Let *M* and *N* be two complexes, then $\chi\left(|M|/|N|\right) = \chi|M| - \chi(|N|) + 1$.

Proof: From the definition of quotient spaces, we have:

$$\#0 - \text{cells of } |M|/|N| = \#0 - \text{cells of } |M| - \#0 - \text{cells of } |N| + 1$$

$$\#1 - \text{cells of } |M|/|N| = \#1 - \text{cells of } |M| - \#1 - \text{cells of } |N|$$

$$\#2 - \text{cells of } |M|/|N| = \#2 - \text{cells of } |M| - \#2 - \text{cells of } |N|$$

$$\#k - \text{cells of } |M|/|N| = \#k - \text{cells of } |M| - \#k - \text{cells of } |N|.$$

Thus we have

$$\chi(|M|/|N|) = \chi(|M|) - \chi(|N|) + 1.$$

Example 4: Let *M* and *N* be two complexes such that $|M| = \text{disc}$, $N = \partial M$, $|N| = S^1$, see Fig. (5).

M $\boldsymbol{M/N}$

Fig. (5)

It is easy to check that the conditions of Theorem (5) is satisfied.

Theorem 6: Let *M*, *N* be complexes, then

$$\chi(|M| \wedge |N|) = \chi(|M|)\,\chi(|N|) - \chi(|M|) - \chi(|N|) + 2.$$

Proof: Since $|M| \wedge |N| = |M| \times |N| / |M| \vee |N|$

Then
$$\begin{aligned} \chi(|M| \vee |N|) &= \chi(|M| \times |N| / |M| \vee |N|) \\ &= \chi(|M| \times |N|) - \chi(|M| \vee |N|) + 1 \\ &= \chi(|M|)\,\chi(|N|) - \chi(|M| - \chi(|N|) + 2. \end{aligned}$$

Example 5: Let *M* and *N* be two complexes such that $|M|=|N|=I$, then $|M|\wedge|N|$ is as shown in Fig. (6).

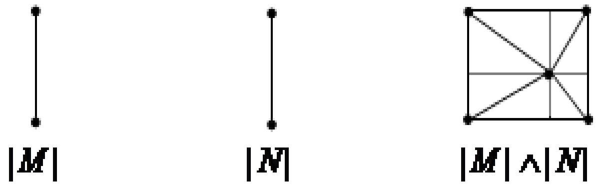

Fig. (6)

It is easy to check that the condition of Theorem (6) is satisfied.

Theorem 7: Let *M*, *N* be two complexes, then

$$\chi(|M|\bigcup|N|)=\chi(|M|)+\chi(|N|)-\chi(|M|\bigcap|N|).$$

Corollary 4: Let *M*, *N* be two disjoint complexes, then

$$\chi(|M|\bigcap|N|)=\chi(|M|)+\chi(|N|).$$

Corollary 5: Let $M_1, M_2, ..., M_n$ be complexes, then

$$\begin{aligned}\chi(\bigcap_{i=1}^{n}|M_i|)=\sum_{i=1}^{n}\chi(|M_i|)-\chi(|M_1|\bigcap|M_2|)-\chi(|M_1|\bigcap|M_3|)-\cdots\\-\chi(|M_1|\bigcap|M_n|)-\chi(|M_2|\bigcap|M_3|)-\cdots-\chi(|M_2|\bigcap|M_n|)\\-\chi(|M_3|\bigcap|M_4|)-\cdots-\chi(|M_1|\bigcap|M_2|\bigcap\cdots\bigcap|M_n|).\end{aligned}$$

Theorem 8: Let *M*, *N* be two complexes of the same dimension 2, then

$$\chi(|M|\#|N|)=\chi(|M|)+\chi(|N|)-2.$$

Proof: From the definition of connected sum we have:

$$\#0-\text{cells of }|M|\#|N|=\#0-\text{cells of }|M|+\#0-\text{cells of }|N|-2$$

$$\#1-\text{cells of }|M|\#|N|=\#1-\text{cells of }|M|+\#1-\text{cells of }|N|-4$$

$$\#2-\text{cells of }|M|\#|N|=\#2-\text{cells of }|M|+\#2-\text{cells of }|N|-4$$

Thus $\chi(|M|\#|N|)=\chi(|M|)+\chi(|N|)-2$.

The above theorem can be generalized for a finite number of 2-complexes as follows:

Corollary 6: Let $M_1, M_2, ..., M_n$ be 2-complexes, then

$$\chi(|M_1|\#|M_2|\#\cdots\#|M_n|)=\chi(|M_1|)+\chi(|M_2|)+\cdots+\chi(|M_n|)-2(n-1)$$

The Euler characteristic of a finite graph *G* denoted by $\chi(G)$ is defined to be the number of vertices of *G* minus the number of edges. It is easy to see $\chi(G)\le 1$ and if *G* is a finite tree, then $\chi(G)=1$. Also if two finite connected graphs G_1, G_2 have the same homotopy type, then $\chi(G_1)=\chi(G_2)$, [7].

In the following theorem we investigate the action of Euler characteristic under some known operations of graphs such as union, join product, cartesian product, tensor product, composition product and normal product.

Theorem 9: Let G_1, G_2 be two finite connected graphs with number of vertices and edges are n_1, n_2 and m_1, m_2 respectively, then

(i) $\chi(G_1 \cup G_2) = \chi(G_1) + \chi(G_2) - \chi(G_1 \cap G_2)$.

(ii) $\chi(G_1 \vee G_2) = \chi(G_1) + \chi(G_2) - m_1 m_2$.

(iii) $\chi(G_1 \times G_2) = \chi(G_1)\chi(G_2) - m_1 m_2$.

(iv) $\chi(G_1 \otimes G_2) = \chi(G_1)\chi(G_2) + n_1 m_2 + n_2 m_1 - 3m_1 m_2$.

(v) $\chi(G_1 \circ G_2) = \chi(G_1)\chi(G_2) - 3m_1 m_2$.

(iv) $\chi(G_1[G_2]) = \chi(G_1)\chi(G_2) + n_2 m_1(1 - n_2) - m_1 m_2$.

The following theorem gives the relation between the regular *CW*-complex and its image under a cellular folding.

Theorem 10: Let *M* and *N* be complexes of the same dimension *n* and $f : M \to N$ be a cellular folding. Then $\chi(|M|) = \chi(|N|) + \sum_{i=0}^{n} (-1)^i (\#i\text{-cells of } |M| - |N|)$.

Corollary 7: Let *M* and *N* be complexes of the same dimension 2 and $f : M \to N$ be a cellular folding. Then

(i) $g(|M|) = g(|N|) - \frac{1}{2}\sum_{i=0}^{2} (-1)^i (\#i\text{-cell of } |M| - |N|)$, *M, N* are orientable.

(ii) $g(|M|) = g(|N|) - \sum_{i=0}^{2} (-1)^i \#(i\text{-cell of } |M| - |N|)$, *M, N* nonorientable.

(iii) $g(|M|) = \frac{1}{2}[g(|N|) - \sum_{i=0}^{2} (-1)^i \#(i\text{-cell of } |M| - |N|)$,

in case of *M* is orientable and *N* is nonorientable 2-complexes.

(iv) $\beta_1(|M|) = \beta_1(|N|) - \sum_{i=0}^{2} (-1)^i \#(i\text{-cell of } |M| - |N|)$,

in case of *M*, *N* are orientable or nonorientable 2-complexes.

(v) $\beta_1(|M|) = \beta_1(|N|) + 1 - \sum_{i=0}^{2} (-1)^i \#(i\text{-cell of } |M| - |N|)$,

in case of *M* is orientable and *N* is nonorientable 2-complexes.

Corollary 8: Let *M*, *N* and *L* be complexes of the same dimension *n* and $f : M \to N$, $g : N \to L$ be cellular folding. Then

$$\chi(|M|) = \chi(|L|) + \sum_{i=0}^{n} (-1)^i \#(i\text{-cell of } |M| - |L|).$$

Example 6:

(a) Consider the cellular folding *f* of a complex *M* such that $|M|$ is a sphere into itself with cellular subdivision shown in Fig. (7). The image of this map is a complex *N* such that $|L|$ is a disc.

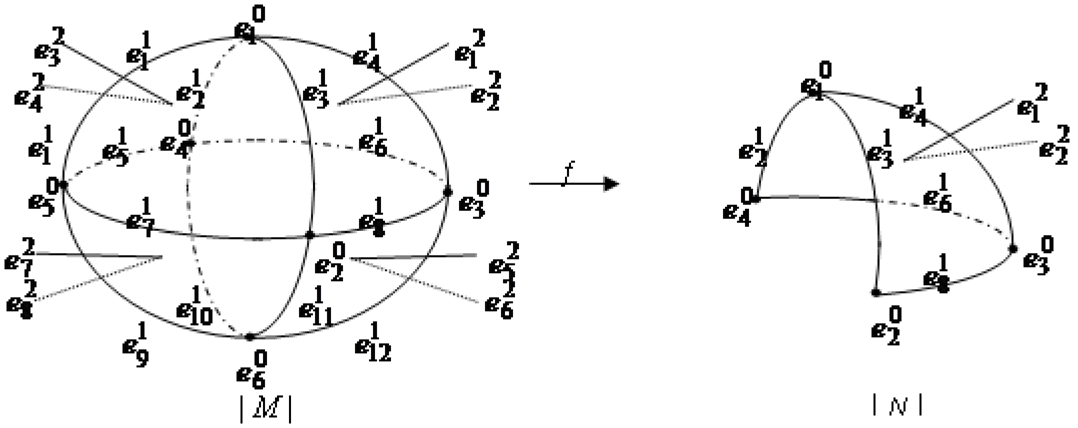

Fig. (7)

It is easy to see that the conditions of theorem (10) and corollary (7) are satisfied.

(b) Consider the complexes *M*, *N* and *L* such that $|M|$, $|N|$ and $|L|$ are torus, cylinder and disc respectively with cellular subdivision shown in Fig. (8) and let $f : M \to N$, $g : N \to L$ be cellular foldings.

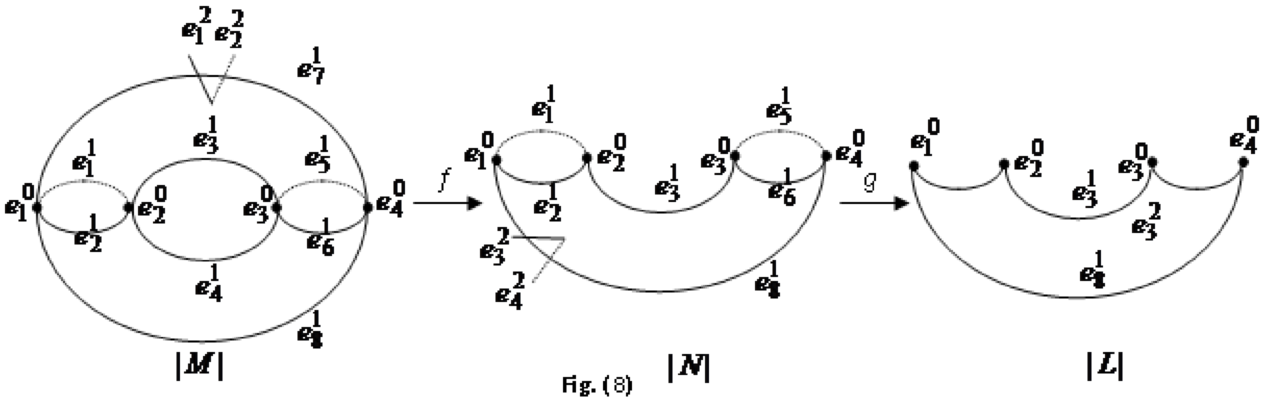

Fig. (8)

It is easy to check that the conditions of corollaries (7) and (8) are satisfies.

CONCLUSION

In the theorems we introduced an attention is paid to the algebraic presentation of 2-manifolds. This is a good way to describe the algebraic and geometric information about orbits of an electron round the nucles throughout its direct disturbance.

The continuous and discontinuous effects of the orbit after and before folding are discussed as discussed in some topological quantum field theories, [8-12].

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