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SLOW STEADY MOTION OF A SECOND ORDER THERMO-VISCOUS FLUID BETWEEN TWO PARALLEL PLATES WHEN THERE IS NO TEMPERATURE GREDIENT

J. Srinivas[1*], N. Ch. Pattabhi Ramacharyulu[2] and P. Nageswararao[3]

[1]Sr. Asst. Prof. of Mathematics, (H&S Department), VNRVJIET, Hyderabad (A.P), India
Email: joshi_seenu@yahoo.com

[2]Prof. of Mathematics, (Retd.) National Institute of Technology, Warangal (A.P), India
Email: pattabhi1933@yahoo.com

[3]Prof. of Mathematics, (H&S Department), VNRVJIET, Hyderabad (A.P), India
Email: nageswararao_p@vnrvjiet.in

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ABSTRACT

The slow steady motion of a second order thermoviscous fluid between two parallel plates is examined when there is no temperature gradient, for following two cases (i) when upper plate is in relative motion and (ii) when upper plate is thermally insulated. It is observed that forces are generated in transverse directions which are special feature of these type of fluids.

Key Words: *Theromoviscous fluids, Strain thermal conductivity, Thermo stress Coefficient, heat flux bivector.*

INTRODUCTION

The non-Newtonian nature of fluids has been a subject of study for a long time. The development of a non-liner theory reflecting the interaction between the thermal and viscous natures of the fluids have been formulated by KOH and ERINGEN [2]. KELLY examined some simple shear flows of thermoviscous fluids. Later NAGESWARA RAO and PATTABHI RAMACHARALU [7] examined some steady flows of these type of fluids.

The present paper deals with slow motion of a second order thermoviscous fluid between two parallel plates in relative motion which are kept at different temperatures and also one of the plate insulated. Assuming that there is no temperature gradient is also considered. In each of these cases, the closed form solutions to sustain the flows are obtained. for velocity and temperature field. The forces generated in transverse direction even in the absence of temperature gradient are obtained.

FORMULATION

Consider the steady flow of a second order incompressible thermoviscous fluid characterized by the constitutive relation

The stress tensor

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db-bd) \qquad (1)$$

and the heat flux bivector

$$h = \beta_1 b + \beta_3 (bd+db) \qquad (2)$$

where 'd' stands for deformation rate tensor given by

$2\, d_{k,m} = v_{k,m} + v_{m,k}$

and 'b' is the thermal bigradient $b_{ij} = \epsilon_{ijk}\, \theta_{,k}\, v_k$ being the k-th component of velocity and θ is the temperature field. The coefficient α's and β's being the polynomials in the invariants of d and b in which the coefficients depends on density and temperature only. These coefficients are independent of temperature field which are taken as constants.

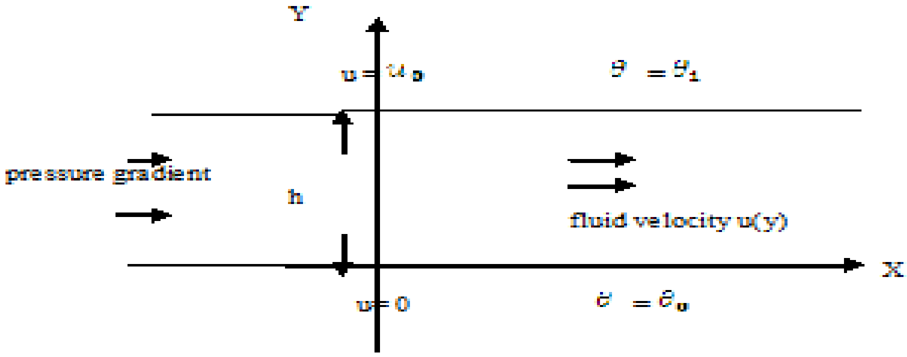
α_1 = -p = pressure, $\alpha_3 = 2\mu$ = kinematics coefficient of viscosity,

$\alpha_5 = 4\mu_c$ = Riner-Rilivin cross viscosity

α_6 = thermo stress coefficient, α_8 = thermo stress viscosity coefficient

β_1 = -k = thermal conductivity, β_3 = strain thermal conductivity

With reference to the Cartesian coordinates system O(x, y, z) with origin on the lower plate, the X-axis in the direction of flow. The flow is characterized by the velocity field [u(y), 0, 0] and temperature by θ (y).



In the absence of any external force in the direction of flow, the equations of motion reduces to

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial^2 \theta}{\partial y^2} \tag{3}$$

$$\mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho F_y = 0 \tag{4}$$

$$\alpha_8 \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y}\right) + \rho F_z = 0 \tag{5}$$

In the absence of any heat source, the energy equation reduces to

$$\rho c\left(u \frac{\partial \theta}{\partial x}\right) = \mu \left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + k \frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x} \frac{\partial^2 u}{\partial y^2} \tag{6}$$

Together with the boundary conditions $u(0) = 0 \quad \theta(0) = \theta_0$ (7)

$u(h) = u_0 \quad \theta(h) = \theta_1$ (8)

Introducing the non-dimensional quantities as,

$$y = hY,\quad u = (\mu/\rho h)\,U,\quad T = \frac{\theta - \theta_0}{\theta_1 - \theta_0},\quad \frac{\partial \theta}{\partial x} = \frac{\theta_1 - \theta_0}{h} c_2,\quad -\frac{\partial p}{\partial x} = \frac{\mu^2}{\rho h^3} c_1 \tag{9}$$

The equation (3) and (6) can be written as

$$0 = c_1 + \frac{d^2 U}{dY^2} - A_6 c_2 \frac{d^2 T}{dY^2} \tag{10}$$

and

$$Uc_2 = A_1 \left[\left(\frac{dU}{dY}\right)^2 - A_6 C_2 \frac{dU}{dY}\frac{dT}{dY}\right] + \frac{1}{p_r}\frac{d^2T}{dY^2} + B_3 C_2 \frac{d^2U}{dY^2} \tag{11}$$

with

$p_r = \frac{c\mu}{k}$ (Prandtle number), $S = \frac{\rho h v_0}{\mu}$, $B_3 = \frac{\beta_3}{\rho h^2 c}$, $A_1 = \frac{\mu^2}{\rho h^2 c(\theta_1 - \theta_0)}$,

$$A_6 = \frac{\alpha_6(\theta_1-\theta_0)^2}{\mu^2} \tag{12}$$

C_1 = constant pressure gradient, C_2 = constant temperature gradient

The boundary conditions U(0) = 0 , U(1) = U_0

T(0) = 0 , T(1) = 1

Considering when there is no temperature gradient .i.e. $C_2 = 0$

The velocity and temperature distribution are obtained by solving equations (10) and (11) and using boundary conditions

$$U(Y) = \frac{C_1}{2} Y(1-Y) + U_0 Y \tag{A.1}$$

$$T(Y) = Y + A_1 P_r \left[\frac{C_1^2}{24}(2Y^2 - 2Y + 1) + \frac{U_0^2}{2} - \frac{U_0 C_1}{6}(2Y-1)\right] Y(1-Y) \tag{A.2}$$

The force generated in transverse directions are

$$\rho F_y = 2\mu_c \frac{\mu^2}{\rho^2 h^4}\left[U_0 + \frac{C_1}{2}(2Y-1)\right] C_1 \tag{A.3}$$

$$\rho F_z = \frac{\alpha_8 \mu(\theta_1 - \theta_0)}{h^4 \rho}\left[\begin{array}{l} C_1 + \frac{A_1 p_r u_0}{4}\{C_1(1-2Y)\}^2 \\ + \frac{A_1 p_r c_1}{2}\left\{\begin{array}{l}\frac{C_1^2}{3}(1-2Y)^3 \\ + \frac{2U_0 C_1}{3}(2 - 9Y + 9Y^2) + 2U_0^2(1-2Y)\end{array}\right\}\end{array}\right] \tag{A.4}$$

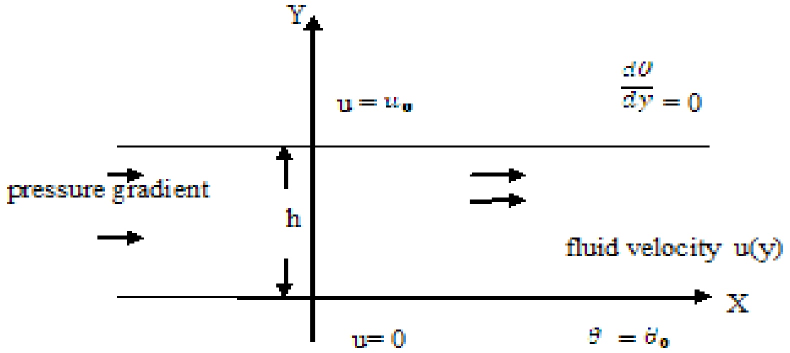
Mean velocity = $\frac{1}{12}(C_1 + 6U_0)$

Which is depend on constant pressure gradient and the relative motion of upper plate.

Nussult number on the upper plate

$$\left(\frac{dT}{dy}\right)_{y=1} = 1 - \frac{A_1 p_r}{24}\left[(C_1 - 2U_0)^2 + 8U_0^2\right]$$

When the upper plate is insulated



The equations of motion and energy reduces to

$$0 = c_1 + \frac{d^2U}{dY^2} - A_6 c_2 \frac{d^2T}{dY^2}$$

$$Uc_2 = A_1 \left[\left(\frac{dU}{dY}\right)^2 - A_6 C_2 \frac{dU}{dY}\frac{dT}{dY}\right] + \frac{1}{p_r}\frac{d^2T}{dY^2} + B_3 C_2 \frac{d^2U}{dY^2}$$

Together with the boundary conditions

$U(0) = 0 \qquad T(0) = 0$

$U(1) = U_0 \qquad \left(\frac{dT}{dY}\right)_{Y=1} = 0$

let there be no temperature gradient . i.e. $c_2 = 0$

The velocity distribution is obtained by solving equation (10) and using above boundary conditions

$$U(Y) = \frac{C_1}{2} Y(1 - Y) + U_0 Y \tag{B.1}$$

The temperature distribution is obtained by solving equation (11) and using above boundary conditions

$$T(Y) = A_1 p_r \left[\frac{C_1^2}{24}(2 - 3Y + 4Y^2 - 2Y^3) - \frac{C_1 U_0}{6}(3 - 2Y)Y + \frac{U_0^2}{2(2-Y)}\right] Y \tag{B.2}$$

$$\rho F_y = 2\mu_c \frac{\mu^2}{\rho^2 h^4}\left[U_0 + \frac{C_1}{2}(2Y - 1)\right] C_1$$

$$\rho F_z = \alpha_8 p_r \mathbf{A}_1 \begin{bmatrix} \frac{C_1^3}{24}(5 - 24Y + 48Y^2 - 32Y^3) \\ + \frac{C_1^2}{12} U_0 (9 - 48Y + 48Y^2) \\ - \frac{U_0^2}{2} C_1 (5 - 8Y) - U_0^3 \end{bmatrix} \tag{B.3}$$

RESULTS AND DISCUSSION

When the upper plate is fixed, with or without thermally insulated of the plate the velocity is parabolic in general. when the upper plate is in relative motion with a given velocity, the velocity of the fluid is steadily increased so as to attain the velocity of the upper plate. fig(1)

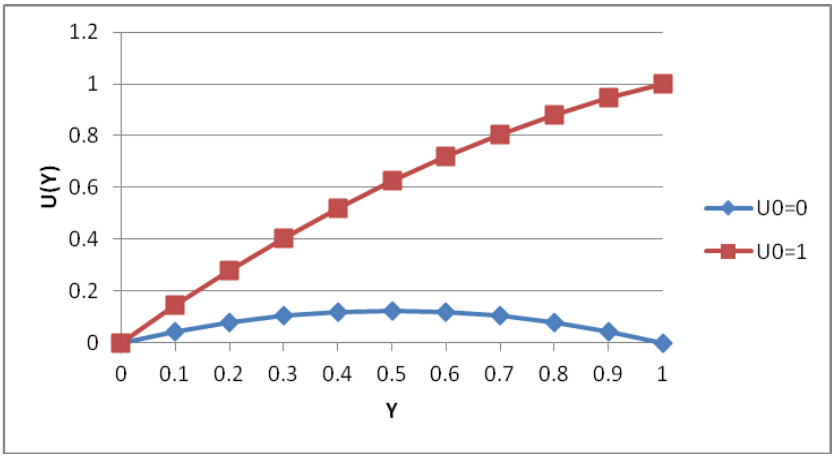
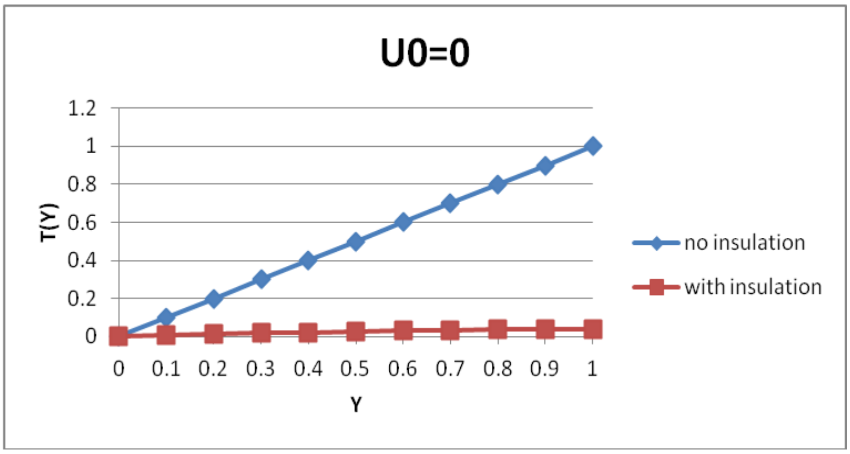


Fig. (1)

When the upper plate is fixed. The temperature distribution is far less when compared to that of the temperature distribution obtained when the upper plate is insulated.



Feg. (2)

When the upper plate is in relative motion, the temperature distribution is same in nature when the plate is with or without thermally insulated. The temperature distribution is steadily increasing from 0 to 1 and the temperature of the fluid when the upper plate is insulated is more compared to that when the upper plate is kept at a constant temperature.

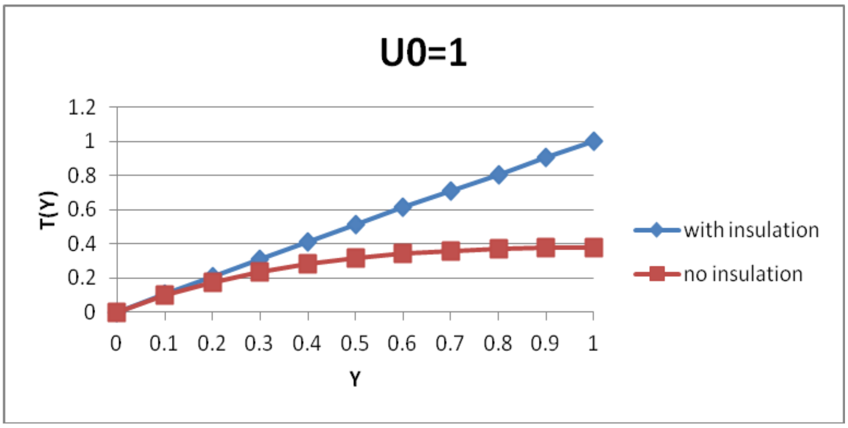


Fig. (3)

Even in the absence of temperature gradient a transverse force is generated with in the flow perpendicular to fluid flow and depends on Reinev-revler cross viscosity μ_c and thermo viscous parameter α_8. The force generated in y-direction is illustrated graphically in fig (4)

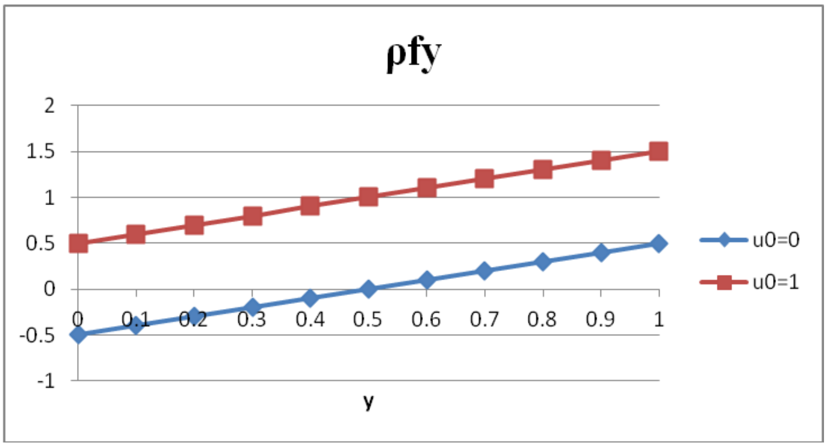


Fig. (4)

It can be observed that the forces generated in the perpendicular direction of the flow are increasing gradually due to the presence of the Reinev-revler cross viscosity μ_c .

The force generated in z-direction is illustrated graphically in fig (5)

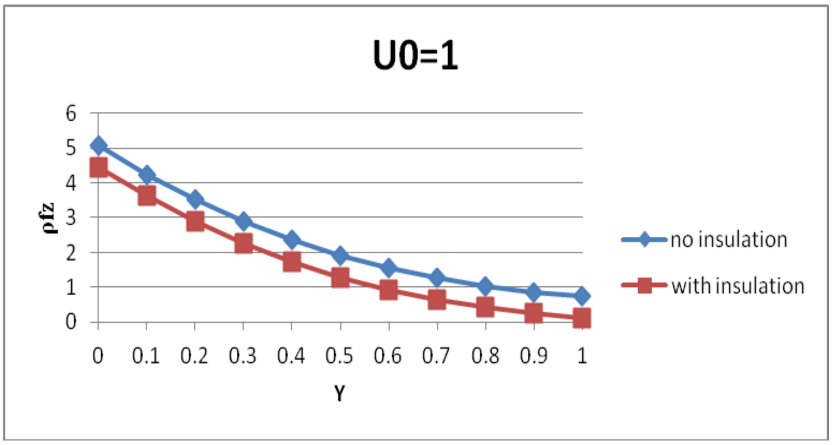


Fig. (5)

The transverse force generated in z direction is maximum near the cooler plate and decreasing gradually towards the end of the hotter plate and this effect is because of the presence of the parameter α_8. The quantity of the force is more when upper plate is not insulated where as it is less when upper plate is insulated.

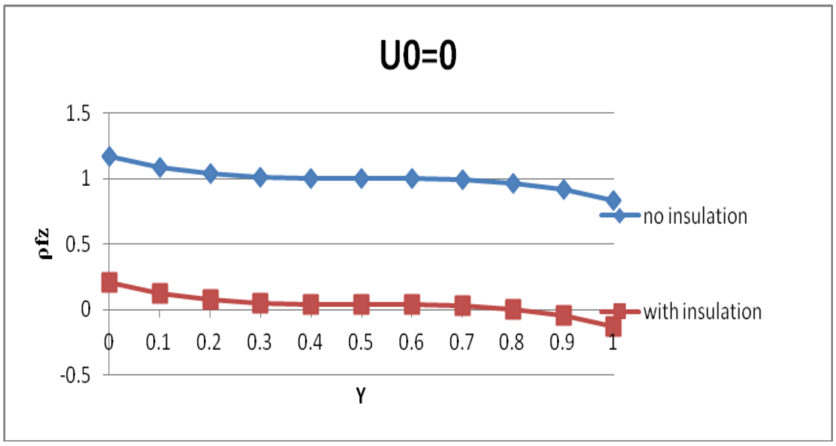


Fig. (6)

When the upper plate is fixed. The transverse force generated in z direction is maximum near the cooler plate and decreasing gradually towards the end of the hotter plate and this effect is because of the presence of the parameter α_8. The quantity of the force is more when upper plate is not insulated where as it is less when upper plate is insulated as shown in fig(6)

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