International Journal of Mathematical Archive-3(3), 2012, Page: 1029-1031
Available online through www.ijma.info ISSN 2229 – 5046

ON RARELY *gp*-CONTINUOUS MULTIFUNCTIONS

M. Rameshkumar*

Department of Mathematics, P. A. College of Engineering and Technology, Pollachi, Tamil Nadu, India
E-mail: rameshngm@gmail.com

(Received on: 24-02-12; Accepted on: 18-03-12)

ABSTRACT

Popa [13] introduced the notion of rare continuity. The authors [5] introduced and investigated a new class of functions called rarely gp-continuous functions. This paper is devoted to the study of upper (and lower) rarely gp-continuous multifunctions.

Keywords and Phrases: *Rare set, gp -open,rarely gp-continuous multifunctions.*

2000 Mathematics Subject Classification: *54C60, 54C08; Secondary 54D05.*

1. INTRODUCTION

In 1979, Popa [13] introduced the notion of rare continuity as a generalization of weak continuity [8] which has been further investigated by Long and Herrington [10] and Jafari [6] and [7]. Levine [9] introduced the concept of generalized closed sets of a topological space. Authors [1] introduced the concept of gp-continuous functions. The authors [5] introduced and investigated rarely gp-continuous in topological spaces. In this paper we study some characterization of rarely gp-continuous multifunctions.

2. PRELIMINARIES

Throughout this paper, X and Y are topological spaces. Recall that a rare set is a set R such that $Int(R) = \phi$. Noiri et al[11]. Introduced the notion of gp-closed sets: A set A in X is called gp-closed if $Cl_p(A) \subset G$ whenever $A \subset G$ and G is open in X. The complement of a gp-closed set is called gp-open [11]. The family of all gp-open (resp. open) sets will be denoted by GPO(X) (resp. O(X)). We set $GPO(X, x) = \{U / x \in U \in GPO(X)\}$, $GO(X, x) = \{U / x \in U \in GO(X)\}$ and $O(X, x) = \{U / x \in U \in O(X)\}$.

Definition 1: A function $f: X \to Y$ is called:

i) weakly continuous [7] (resp. weakly-g-continuous [4] and weakly-gp- continuous[5]) if for each $x \in X$ and each open set G containing $f(x)$, there exists $U \in O(X, x)$ (resp. $U \in GO(X, x)$ and $U \in GPO(X, x)$) such that $f(U) \subset Cl(G)$,

ii) rarely continuous [13] (resp. rarely-g-continuous [2] and rarely-gp- continuous[5]) if for each $x \in X$ and each $G \in O(Y, f(x))$, there exists a rare set R_G with $G \cap Cl(R_G) = \phi$ and $U \in O(X, x)$ (resp. $U \in GO(X, x)$ and $U \in GPO(X, x)$) such that $f(U) \subset G \cup R_G$,

iii) gp-continuous [1] if the inverse image of every closed set in Y is gp-closed in X.

3. UPPER (LOWER) RARELY gp-CONTINUOUS MULTIFUNCTIONS

We provide the following definitions which will be used in the sequel. Let $F: X \to Y$ be a multifunction. The upper and lower inverses of a set $V \subset Y$are denoted by $F^+(V)$ and $F^-(V)$ respectively, that is,

$$F^+(V) = \{x \in X / F(x) \subset V\} \text{ and } F^-(V) = \{x \in X / F(x) \cap V = \phi\}.$$

Definition 2: A multifunction $F: X \to Y$ is said to be

i) upper rarely gp-continuous (briefly u.r.g.p.c) at $x \in X$ if for each $V \in O(Y, F(x))$, there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GPO(X, x)$ such that $F(U) \subset V \cup R_V$,

***Corresponding author: M. Rameshkumar*, *E-mail: rameshngm@gmail.com**

ii) lower rarely g-continuous (briefly l.r.g.p.c) at $x \in X$ if for each $V \in O(Y)$ with $F(x) \cap V = \phi$ there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GPO(X, x)$ such that $F(u) \cap (V \cup R_V) \neq \phi$ for every $u \in U$,

iii) upper/ lower rarely gp-continuous if it is upper/lower rarely gp-continuous at each point of X.

Definition 3: A multifunction $F: X \rightarrow Y$ is said to be

i) upper weakly gp-continuous at $x \in X$ if for each $V \in O(Y, F(x))$,there exist $U \in GPO(X, x)$ such that $F(U) \subset Cl(V)$,

ii) lower weakly gp-continuous at $x \in X$ if for each $V \in O(Y)$ with $F(x) \cap V \neq \phi$ there exists $U \in GPO(X, x)$ such that $F(u) \cap Cl(V) \neq \phi$ for every $u \in U$,

iii) upper/ lower weakly gp-continuous if it is upper/lower weakly gp-continuous at each point of X.

Theorem 1: The following statements are equivalent for a multifunction $F : X \rightarrow Y$:

i) F is u.r.g.p.c at $x \in X$,

ii) For each $V \in O(Y, F(x))$, there exists $U \in GPO(X, x)$ such that $Int[F(U) \cap (Y - V)] = \phi$

iii) For each $V \in O(Y, F(x))$, there exists $U \in GPO(X, x)$ such that $Int[F(U)] \subset Cl(V)$.

Proof: (i) $\Rightarrow$ (ii): Let $V \in O(Y, F(x))$. By $F(x) \subset V \subset Int(Cl(V))$ and the fact that $Int(Cl(V)) \in O(Y)$, there exist a rare set R_V with $Int(Cl(V)) \cap Cl(R_V) = \phi$ and a gp-open set $U \subset X$ containing x such that $F(U) \subset Int(Cl(V)) \cup R_V$. We have $Int\,[F(U) \cap (Y - V)] = Int(F(U)) \cap Int(Y - V) \subset Int(Cl(V) \cup R_V) \cap (Y - Cl(V)) \subset (Cl(V) \cup Int(R_V)) \cap (Y - Cl(V)) = \phi$.

(ii) $\Rightarrow$ (iii) : Obvious.

(iii) $\Rightarrow$ (i) : Let $V \in O(Y, F(x))$. Then, by (iii) there exists $U \in GPO(X, x)$ such that $Int[F(U)] \subset Cl(V)$. Thus $F(U) = [F(U) - Int(F(U))] \cup Int[F(U)] \subset [F(U) - Int(F(U))] \cup Cl(V) = [F(U) - Int(F(U))] \cup V \cup (Cl(V) - V) = [(F(U) - Int(F(U)) \cap (Y - V)] \cup V \cup (Cl(V) - V)$. Put $P = (F(U) - Int(F(U))) \cap (Y - V)$ and $G = Cl(V) - V$, then P and G are rare sets. Moreover, $R_V = P \cup G$ is a rare set such that $Cl(R_V) \cap V = \phi$ and $F(U) \subset V \cup R_V$. Hence F is u.r.g.p.c.

Theorem 2: The following are equivalent for a multifunction $F: X \rightarrow Y$:

i) F is l.r.g.p.c at $x \in X$,

ii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gp}(F^{-}(V \cup R_V))$,

iii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $Cl(V) \cap R_V = \phi$ such that $x \in Int_{gp}(F^{-}(Cl(V) \cup R_V))$,

iv) For each $V \in RO(Y)$ such that $F(x) \cap V \neq \phi$ there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gp}(F^{-}(V \cup R_V))$.

Proof: (i) $\Rightarrow$(ii): Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (i), there exist a rare set R_V with $V \cap Cl(R_V) = \phi$ and $U \in GPO(X, x)$ such that $F(x) \cap (V \cup R_V) \neq \phi$ for each $u \in U$. Therefore, $u \in F^{-}(V \cup R_V)$ for each $u \in U$ and hence $U \subset F^{-}(V \cup R_V)$. Since $U \in GPO(X, x)$, we obtain $x \in U \subset Int_{gp}(F^{-}(V \cup R_V))$.

(ii) $\Rightarrow$ (iii): Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (ii), there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gp}(F^{-}(V \cup R_V))$. We have $R_V \subset Y - V = (Y - Cl(V)) \cup (Cl(V) - V)$ and hence $R_V \subset [R_V \cap (Y - Cl(V))] \cup (Cl(V) - V)$. Now, put $P = R_V \cap (Y - Cl(V))$. Then P is a rare set and$P \cap Cl(V) = \phi$. Moreover, we have $x \in Int_{gp}(F^{-}(V \cup R_V)) \subset Int_{gp}(F^{-}(P \cup Cl(V)))$.

(iii) $\Rightarrow$ (iv): Let V be any regular open set of Y such that $F(x) \cap V \neq \phi$. By (iii), there exists a rare set R_V with $Cl(V) \cap R_V = \phi$ such that $x \in Int_{gp}(F^{-}(Cl(V) \cup R_V))$. Put $P = R_V \cup (Cl(V) - V)$, then P is a rare set and $V \cap Cl(P) = \phi$. Moreover, we have $x \in Int_{gp}(F^{-}(Cl(V) \cup R_V)) = Int_{gp}(F^{-}(R \cup (Cl(V) - V) \cup V)) = Int_{gp}(F^{-}(P \cup V))$.

(iv) $\Rightarrow$ (i) : Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. Then $F(x) \cap Int(Cl(V)) \neq \phi$ and $Int(Cl(V))$ is regular open in Y . By (iv), there exists a rare set R_V with $V \cap Cl(R_V) = \phi$ such that $x \in Int_{gp}(F^{-}(V \cup R_V))$. Therefore, there exists $U \in$

GPO(X, x) such that $U \subset F^{-}(V \cup R_V)$; hence $F(u) \cap (V \cup R_V) \neq \phi$; for each $u \in U$. This shows that F is lower rarely g-continuous at x.

Corollary 1: ([[2], Theorem 2]) The following statements are equivalent for a function $f : X \rightarrow Y$:

i) f is rarely gp-continuous at $x \in X$,

ii) For $V \in O(Y, f(x))$, there exists $U \in GPO(X, x)$ such that $Int[f(U) \cap (Y - V)] = \phi$,

iii) For each $V \in O(Y, f(x))$, there exists $U \in GPO(X, x)$ such that $Int[f(U)] \subset Cl(V)$.

REFERENCES

[1] I. Arokiarani, K. Balachandran and J. Dontchev, Some characterizations of gp-irresolute and gp-continuous maps between topological spaces, Topology Atlas, Preprint #246.

[2] M. Caldas and S. Jafari, On rarely g-continuous functions, Glas. Mat. Ser. III (2005), 317-322.

[3] M. Caldas, S. Jafari and T. Noiri, Characterizations of rarely g-continuous multifunctions, Sarajevo Journal of Mathematics, Vol.1 (13) (2005), 129-133.

[4] M. Caldas, S. Jafari and T. Noiri, Characterizations of weakly g-continuous multifunctions (under preparation).

[5] M. Caldas, S. Jafari, G. Navalagi and G. Sajjanshettar, On rarely gp-continuous, International Journal of Mathematics and Computing Applications, Vol. 1, Number 1-2 (2009), 47-51

[6] S. Jafari, A note on rarely continuous functions, Stud. Cercet. Stiint., Ser. Mat., Univ. Bacau, 5 (1995), 29-34.

[7] S. Jafari, On some properties of rarely continuous functions, Stud. Cercet. Stiint., Ser. Mat., Univ. Bacau, 7 (1997), 65-73.

[8] N. Levine, A decomposition of continuity in topological spaces, Amer. Math. Monthly, (60) (1961), 44-46.

[9] N. Levine, Generalized closed sets in topological, Rend. Circ. Mat. Palermo, 19 (2) (1970), 89-96.

[10] P. E. Long and L. L. Herrington, Properties of rarely continuous functions, Glas. Mat. Ser. III 17 (37) (1982), 147-153.

[11] T. Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. (Math) 19 (1998) , 13-20.

[12] V. Popa, Weakly continuous multifunctions, Boll. Unione Mat. Ital., (5), 15(A) (1978), 379-388.

[13] V. Popa, Some properties of rarely continuous multifunctions, Conf. Nat. Geom. Topologie, Univ. Al. I. Cuza Iasi, (1989), 269-274.

[14] V. Popa, Sur certain decomposition de la continuit´e dans les espaces topologiques, Glas. Mat. Ser. III, 14 (34) (1979), 359-362.

****

