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FOLDING OF THE CHAOTIC TREE WITH KNOTS

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ABSTRACT

In this paper, we will define the chaotic tree with knots together with its adjacent and incidence matrices. The limit of foldings on it is deduced. The corresponding changes in the adjacent and incidence matrices under these transformations are achieved.

Keywords: *Tree with knots, folding.*

2010 Mathematics subject classification: *5 C 5, 68 R 10.*

DEFINITIONS AND BACKGROUNDS:

1- Knots [3]:

A knot is a subset of 3- space homeomorphic to the unit circle, while the link is a union of finitely many disjoint knots. The individual knots that make up a link are called its components (so a knot is a link with just one component, i.e. a connected link). A singular knot is a knot with self-intersection

Fig. (1) shows left handed trefoil, right handed trefoil, Hopf link and singular terfoil knot.

Left hand trefoil   Right hand trefoil   Hopf link   Singular trefoil knot

Fig. (1)

2- Tree with knots [5]:

It is a connected graph that contains number of knots, see Fig. (2)

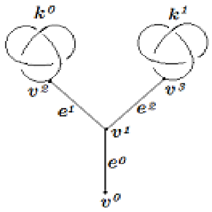

Fig. (2)

Where K^0, K^1 indicates the knots in the graph .Both the adjacency $A(T^K)$and incidence$I(T^K)$ matrices representing the tree with knots takes the form

$$A(T^k)=\begin{bmatrix}0&1&0&0\\1&0&1&1\\0&1&1^{1k}&0\\0&1&0&1^{1k}\end{bmatrix}, I(T^k)=\begin{bmatrix}1&0&0\\1&1&1\\0&1&0\\0&0&1\end{bmatrix}$$

Such that the upper suffix1 in the adjacent matrix refers to the number of knots exist in the graph.

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3 -Folding [2]:

Let f: G→ G′ be a map between any two graphs G and G′ (not necessary to be simple) such that if (u, v) ∈G, (f(u), f(v))∈G′. Then f is called a "topological folding" of G into G′ provided that d(f(u),f(v)) ≤ d(u, v) (or the number of vertices and edges are decreased).

MAIN RESULTS:

1- Chaotic Knot:

It is a knot carries many physical characters, Fig. (3).

Chaotic left hand Chaotic right hand Chaotic hopf link Chaotic singular trefoil
trefoiltrefoil knot

Fig. (3)

2- Chaotic tree with knots:

It is a tree with knots carries many physical characters, Fig. (4).

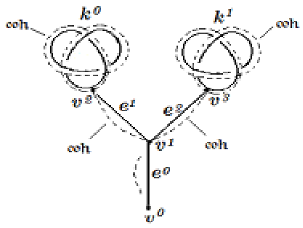

Fig. (4)

Such that itsadjacent $A(T_h^k)$and incidence $I(T_h^k)$matrices are as follows'

$$A(T_h^k)=\begin{bmatrix} 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 1_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 1^{1k_{012...\infty h}}_{012...\infty h} & 0_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 1^{1k_{012...\infty h}}_{012...\infty h} \end{bmatrix}, I(T_h^k)=\begin{bmatrix} 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 1_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} \\ 0_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} \end{bmatrix}$$

Folding of the chaotic tree with knots:

*Folding of the chaotics:

Here, the folding acts on the chaotic vertices, chaotic edges, and chaotic knots. The change in the chaotic tree with knots are deduced together with the change in its matrices.

First: Folding of the chaotic vertices:

Case (1): Identity case:

In this case, the folding gives the original chaotic tree with knots without any change.It's adjacent matrix $A(T_h^k)$and incidence matrix $I(T_h^k)$will remain as they are, see the following Figure.

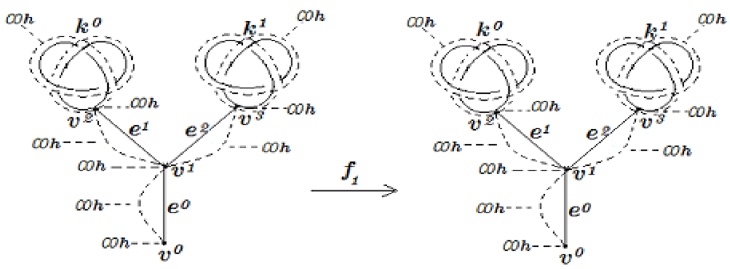

Fig. (5)

Case (2):

In this case, the folding acts on the chaotics of the vertices v^0,v^1,v^2,v^3, we fold only one physical character to another one and the result is the same system but with decreasing of the number of physical character by one, such that $f_{2m}\left(v^i_{jh}\right) = v^i_{(j-1)h}$, where m=1,2...n , i=0,1,2,3 , j=1,2...∞.

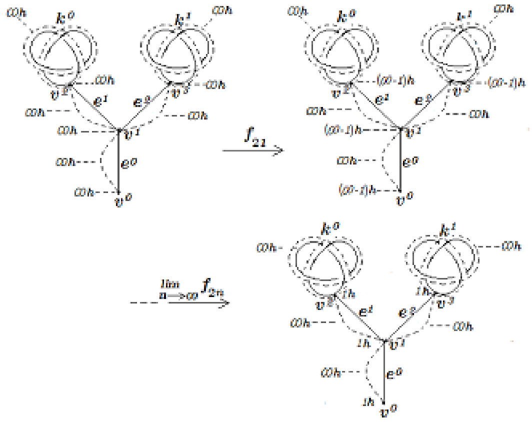

Fig. (6)

where the chaotic incidence matrix $I(T_h^k)$will not change, but the chaotic adjacent matrix $A(T_h^k)$ will be changed as follows:

$$\mathrm{A}(\mathrm{T}_\mathrm{h}^\mathrm{k}) = \begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} \end{bmatrix} \xrightarrow{f_{21}} \begin{bmatrix} 0_{012\ldots(\infty-1)h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots(\infty-1)h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots(\infty-1)h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots(\infty-1)h} \end{bmatrix}$$

$$\ldots \xrightarrow{lim.f_{21}} \begin{bmatrix} 0_{01h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{01h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{01h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{01h} \end{bmatrix}$$

Second: Folding of the chaotic edges:

Here, the folding acts on the chaotics of the edges e^0,e^1,e^2., such that $f_{3m}\left(e^i_{jh}\right) = e^i_{(j-1)h}$, where m=1,2...n , i=0,1,2 , j=1,2...∞,See the following Fig.

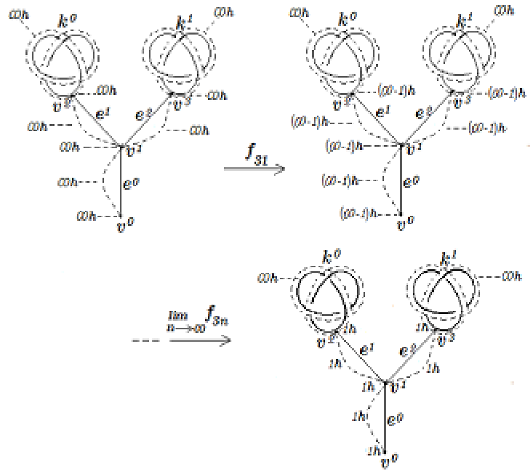

Fig. (7)

Such that

$$A(T_h^k)=\begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} \end{bmatrix} \xrightarrow{f_{31}} \begin{bmatrix} 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} \\ 1_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} \\ 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 1^{1k_{012\ldots\infty h}}_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} \\ 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} & 1^{1k_{012\ldots\infty h}}_{012\ldots(\infty-1)h} \end{bmatrix}$$

$$\ldots \xrightarrow{\lim . f_{31}} \begin{bmatrix} 0_{01h} & 1_{01h} & 0_{01h} & 0_{01h} \\ 1_{01h} & 0_{01h} & 1_{01h} & 1_{01h} \\ 0_{01h} & 1_{01h} & 1^{1k_{012\ldots\infty h}}_{01h} & 0_{01h} \\ 0_{01h} & 1_{01h} & 0_{01h} & 1^{1k_{012\ldots\infty h}}_{01h} \end{bmatrix}.$$

$$I(T_h^k)=\begin{bmatrix} 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{bmatrix} \xrightarrow{f_{31}} \begin{bmatrix} 1_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} \\ 1_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} \\ 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} \\ 0_{012\ldots(\infty-1)h} & 0_{012\ldots(\infty-1)h} & 1_{012\ldots(\infty-1)h} \end{bmatrix}$$

$$\ldots \xrightarrow{\lim . f_{31}} \begin{bmatrix} 1_{01h} & 0_{01h} & 0_{01h} \\ 1_{01h} & 1_{01h} & 1_{01h} \\ 0_{01h} & 1_{01h} & 0_{01h} \\ 0_{01h} & 0_{01h} & 1_{01h} \end{bmatrix}.$$

Third: Folding of the chaotic knots:

As shown in the next Figure, the folding acts on the chaotics of the knots k^0,k^1, such that $f_{4m}\left(k^i_{jh}\right)=k^i_{(j-1)h}$, where m=1,2...n , i=0,1 , j=1,2...∞.The final step of folding gives a 1- chaotic tree with knots(fuzzy tree with knots).

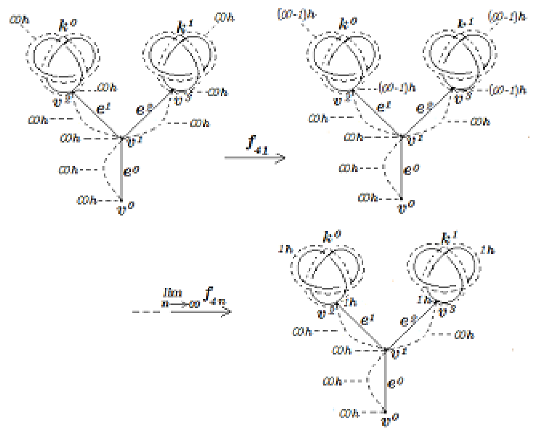

Fig. (8)

$$A(T_h^k)=\begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} \end{bmatrix} \xrightarrow{f_{41}} \begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{012\ldots(\infty-1)h}}_{012\ldots(\infty-1)h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots(\infty-1)h}}_{012\ldots(\infty-1)h} \end{bmatrix} \ldots \xrightarrow{\lim . f_{41}}$$

$$\begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{1k_{01h}}_{01h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{01h}}_{01h} \end{bmatrix}.$$

It's incidence matrix $I(T_h^k)$ will not change.

***FOLDING OF THE GEOMETRY:**

Folding of the geometric tree with knots which requires to fold the corresponding chaotic tree with knots to each other, and the result is a new chaotic tree with knots.

First: Folding of the geometric vertices:

Case (1): In this case, the folding acts on the geometric vertex v^0, which fold it on the vertex v^1 (which requires folding of e^0) forming a knot k^2 such that $f_5(v^0)=v^1, f_5(e^0)=k^2$.

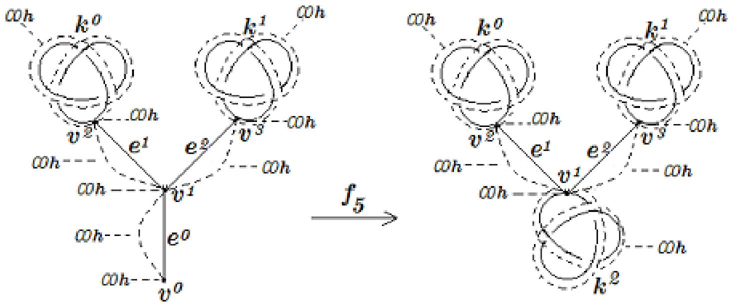

Fig. (9)

The chaotic adjacent and incident matrices resulting from the folding take the following form

$$A(f_5(T_h^k)) = \begin{bmatrix} 1_{012\ldots\infty h}^{1k_{012\ldots\infty h}} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h}^{1k_{012\ldots\infty h}} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h}^{1k_{012\ldots\infty h}} \end{bmatrix},\ I(f_5(T_h^k)) = \begin{bmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{bmatrix}.$$

Case (2):
Let f_6: $T_h^k \to G$, such that $f_6(v^3)=v^2$.The result of folding is not a chaotic tree with knots, but a chaotic graph with multiple edge.

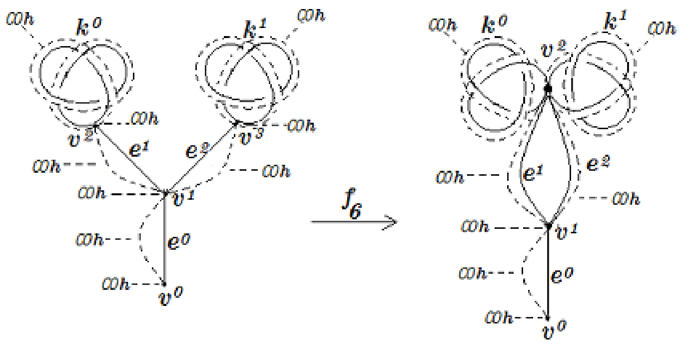

Fig. (10)

Its matrices will be changed.

$$A(f_6(T_h^k)) = \begin{bmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 2_{012\ldots\infty h} & 1_{012\ldots\infty h}^{2k_{012\ldots\infty h}} \end{bmatrix},\ I(f_6(T_h^k)) = \begin{bmatrix} 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{bmatrix}.$$

As shown in the next Figure, the folding acts on the chaotics of the knots k^0, k^1, such that where the upper suffix 2k refers to the existence of two knots connected to the edge, and $2_{012\ldots\infty h}$ refers to the multiple edge.

Second: Folding of the geometric edges:

Case (3): In this case, the folding acts on the edge e^0, such that f_7: $T_h^k \to T_h^k$, $f_7(e^0)=e^2$. The result of folding is a chaotic tree with knots, see the following Fig.

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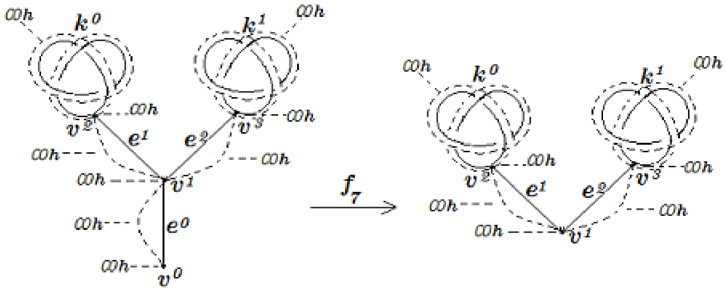

Fig. (11)

$$A(f_6(T_h^k))=\begin{bmatrix}0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} & 0_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h}\end{bmatrix} \quad A(f_7(T_h^k))=\begin{bmatrix}0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h} & 0_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1^{1k_{012\ldots\infty h}}_{012\ldots\infty h}\end{bmatrix},$$

$$I(f_7(T_h^k))=\begin{bmatrix}1_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h}\\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h}\end{bmatrix}.$$

Case (4): Here, we fold the edge e^2 on the edge e^1, which consequently leads to folding of the vertex v^3 on the vertex v^2, where f_8: $T_h^k \rightarrow T_h^k$, $f_8(e^2)=e^1$.

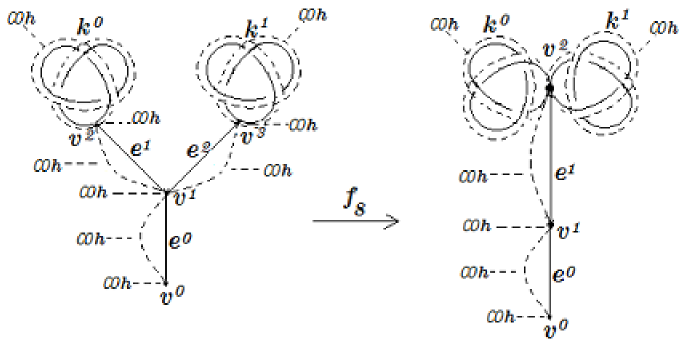

Fig. (12)

The change in it's matrices will be in the following form

$$A(f_8(T_h^k))=\begin{bmatrix}0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1^{2k_{012\ldots\infty h}}_{012\ldots\infty h}\end{bmatrix}, \quad I(f_8(T_h^k))=\begin{bmatrix}1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h}\\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h}\\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h}\end{bmatrix}.$$

Third: Folding of the geometric knots:

Case (5):

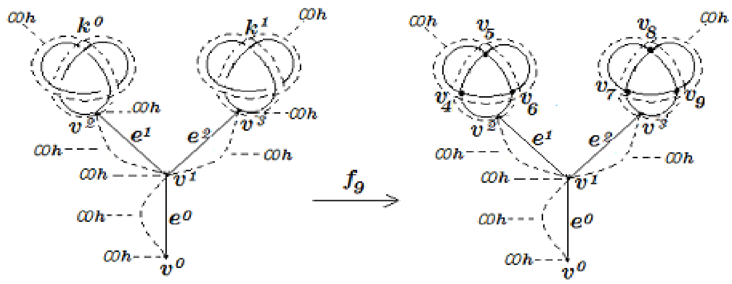

Fig. (13)

$$A(f_9(T_h^k)) = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0\\ 1&0&1&1&0&0&0&0&0&0\\ 0&1&0&0&1&0&1&0&0&0\\ 0&1&0&0&0&0&0&1&0&1\\ 0&0&1&0&0&2&1&0&0&0\\ 0&0&0&0&2&0&2&0&0&0\\ 0&0&1&0&1&2&0&0&0&0\\ 0&0&0&1&0&0&0&0&2&1\\ 0&0&0&0&0&0&0&2&0&2\\ 0&0&0&1&0&0&0&1&2&0 \end{bmatrix}_{012\ldots\infty h},$$

$$I(f_9(T_h^k)) = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0\\ 0&0&0&1&1&1&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&1&1&1&1 \end{bmatrix}_{012\ldots\infty h}$$

Such that the lower suffix which exist in the matrices $012\ldots\infty h$ means that all the vertices and all the edges carries the same physical characters.

Case (6): Here, the folding acts on the length of the knots k^0, k^1, until it reaches the null knot. The result of the folding is the usual chaotic tree .See Fig. (14).

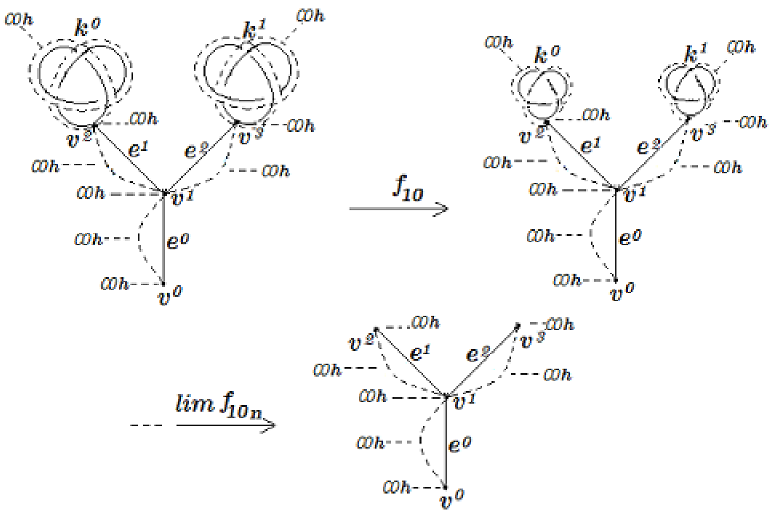

Fig. (14)

$A(T_h^k) = A(f_{10}(T_h^k))$, $I(T_h^k) = I(f_{10}(T_h^k))$, but $A(\lim. f_{10}(T_h^k))$ and $I(\lim. f_{10}(T_h^k))$ will be changed

$$A(\lim. f_{10}(T_h^k)) = \begin{bmatrix} 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 1_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} \end{bmatrix},$$

$$I(\lim. f_{10}(T_h^k)) = \begin{bmatrix} 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 1_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} \\ 0_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} \end{bmatrix}.$$

Theorem (1): The chaotic tree with knots changed into the usual tree by using some types of folding.

Proof: The proof is clear from the above discussion.

Lemma: The matrices of the chaotic tree with knots changed also into the matrices of the usual tree under the folding transformation.

Theorem (2): The chaotic tree with knots changes into the chaotic tree by using some geometric transformations.

Proof: The proof comes directly from the above discussion.

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