

A NOTE ON INTUITIONISTIC FUZZY \mathfrak{I} -GENERALIZED SEMI IRRESOLUTE MAPPINGS

¹S. Maragathavalli & ²K. Ramesh*

¹Department of Mathematics, Sree Saraswathi thyagaraja College, Pollachi, Tamilnadu, India
 E-mail: smvalli@rediffmail.com

²Department of Mathematics, SVS College of Engineering, Coimbatore, Tamilnadu, India
 E-mail: rameshfuzzy@gmail.com

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ABSTRACT

In this paper a new class of mapping called intuitionistic fuzzy \mathfrak{I} -generalized semi irresolute mapping in intuitionistic fuzzy topological space is introduced and some of its properties are studied.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy \mathfrak{I} -generalized semi closed set, intuitionistic fuzzy \mathfrak{I} -generalized semi open set, intuitionistic fuzzy \mathfrak{I} -generalized semi irresolute mapping, intuitionistic fuzzy $\mathfrak{I}T_{1/2}$ space and intuitionistic fuzzy $\mathfrak{I}T_{1/2}$ space.

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1. INTRODUCTION

After the introduction of Fuzzy set (FS) by Zadeh [12] in 1965 and fuzzy topology by Chang [3] in 1967, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1983 as a generalization of fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy \mathfrak{I} -generalized semi irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy \mathfrak{I} -generalized semi irresolute mapping and established the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

1. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (3) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (4) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (5) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Corresponding author: ²K. Ramesh, *E-mail: rameshfuzzy@gmail.com

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:

- (a) $0_{\sim}, 1_{\sim} \in \tau$,
- (b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
- (c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X . The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then, the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Definition 2.5: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be

- (i) An intuitionistic fuzzy closed (IF closed in short) mapping if $f(A)$ is an IFCS A in Y for each IFCS in X
- (ii) An intuitionistic fuzzy α -closed (IF α closed in short) mapping if $f(A)$ is an IF α CS in Y for every IFCS A in X
- (iii) An intuitionistic fuzzy semiclosed (IFS closed in short) mapping if $f(A)$ is an IFSCS in Y for every IFCS A in X
- (iv) An intuitionistic fuzzy preclosed (IFP closed in short) mapping if $f(A)$ is an IFPCS in Y for every IFCS A in X .

Definition 2.6: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be

- (i) An intuitionistic fuzzy generalized closed (IFG closed in short) mapping if $f(A)$ is an IFGCS in Y for every IFCS A in X
- (ii) An intuitionistic fuzzy pre-regular closed (IFPR closed in short) mapping if $f(A)$ is an IFRCS in Y for every IFRCS A in X .

Definition 2.7:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy almost closed (IFA closed in short) mapping if $f(A)$ is an IFCS in Y for every IFRCS A in X .

Definition 2.8:[8] A subset of A of a space (X, τ) is called:

- (i) regular open if $A = \text{int}(\text{cl}(A))$
- (ii) \mathbb{T} open if A is the union of regular open sets.

Definition 2.9: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is called an

- (a) intuitionistic fuzzy semi closed set [7] (IFSCS) if $\text{int}(\text{cl}(A)) \subseteq A$
- (b) intuitionistic fuzzy α -closed set [7] (IF α CS) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (c) intuitionistic fuzzy pre-closed set [7] (IFPCS) if $\text{cl}(\text{int}(A)) \subseteq A$
- (d) intuitionistic fuzzy regular closed set [7] (IFRCS) if $\text{cl}(\text{int}(A)) = A$
- (e) intuitionistic fuzzy generalized closed set [9] (IFGCS) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
- (f) intuitionistic fuzzy generalized semi closed set [8] (IFGSCS) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
- (g) intuitionistic fuzzy α generalized closed set [8] (IF α GCS) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.
- (h) intuitionistic fuzzy \mathbb{T} -generalized semi closed set [8] (IF \mathbb{T} GSCS) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF \mathbb{T} OS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α -open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy \mathbb{T} -generalized semi open set (IFSOS, IF α OS, IFPOS, IFROS, IFGOS, IFGSOS, IF α GOS and IF \mathbb{T} GSOS) if the complement of A^c is an IFSCS, IF α CS, IFPCS, IFRCS, IFGCS, IFGSCS, IF α GCS and IF \mathbb{T} GSCS respectively.

Result 2.10: [8] Every IFCS, IFSCS, IFGCS, IFRCS, IF α CS, IFGSCS is an IF \mathbb{T} GSCS but the converses may not be true in general. (Every IFOS, IFSOS, IFGOS, IFROS, IF α OS, IFGSOS is an IF \mathbb{T} GSOS but the converses may not be true in general).

Definition 2.11: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.12: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be
 (a) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$
 (b) intuitionistic fuzzy α - continuous (IF α continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$
 (c) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$
 (d) intuitionistic fuzzy completely continuous if $f^{-1}(B) \in \text{IFRO}(X)$ for every $B \in \sigma$.

Definition 2.13: [6] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy γ continuous (IF γ continuous in short) if $f^{-1}(B)$ is an IF γ OS in (X, τ) for every $B \in \sigma$.

Definition 2.14: [12] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFCS B in Y .

Result 2.15: [12] Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

Definition 2.16: [10] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition 2.17: [9] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy \mathbb{T} - generalized continuous (IF \mathbb{T} GS continuous in short) if $f^{-1}(B)$ is an IF \mathbb{T} GSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition 2.18: [12] An IFTS (X, τ) is called an intuitionistic fuzzy $T_{1/2}$ (IFT $_{1/2}$ in short) space if every IFGCS in X is an IFCS in X .

Definition 2.19: [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if $f^{-1}(B) \in \text{IFCS}(X)$ for every IFCS B in Y .

Definition 2.20: [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFGCS B in Y .

Result 2.21: [8] Every IFGSCS is an IF \mathbb{T} GSCS but not conversely.

Definition 2.22:[8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\mathbb{T}_{1/2}$ (IF $\mathbb{T}_{1/2}$ in short) space if every IF \mathbb{T} GSCS in X is an IFCS in X .

Definition 2.23:[8] An IFTS (X, τ) is said to be an intuitionistic fuzzy \mathbb{T} (IF \mathbb{T} in short) space if every IF \mathbb{T} GSCS in X is an IFGCS in X .

3. INTUITIONISTIC FUZZY \mathbb{T} - GENERALIZED SEMI IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy \mathbb{T} - generalized semi irresolute mappings and studied some of their properties.

Definition 3.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy \mathbb{T} - generalized semi irresolute (IF \mathbb{T} GS irresolute) mapping if $f^{-1}(A)$ is an IF \mathbb{T} GSCS in (X, τ) for every IF \mathbb{T} GSCS A of (Y, σ) .

Theorem 3.2: If $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T} GS irresolute mapping, then f is an IF \mathbb{T} GS continuous mapping but not conversely.

Proof: Let A be any IFCS in Y . Since every IFCS is an IF \mathbb{T} GSCS, A is an IF \mathbb{T} GSCS in Y . Since f is an IF \mathbb{T} GS irresolute mapping, $f^{-1}(A)$ is an IF \mathbb{T} GSCS in X . Hence f is an IF \mathbb{T} GS continuous mapping.

Example 3.3: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.1_a, 0.2_b), (0.3_a, 0.3_b) \rangle$, $G_2 = \langle x, (0.1_a, 0.1_b), (0.2_a, 0.3_b) \rangle$, $G_3 = \langle x, (0.1_a, 0.2_b), (0.2_a, 0.3_b) \rangle$, $G_4 = \langle x, (0.1_a, 0.1_b), (0.3_a, 0.3_b) \rangle$, $G_5 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.3_b) \rangle$, $G_6 = \langle y, (0.4_u, 0.2_b_v), (0.6_u, 0.7_v) \rangle$. Then $\tau = \{ 0, G_1, G_2, G_3, G_4, G_5, 1 \}$ and $\sigma = \{ 0, G_6, 1 \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF \mathbb{T} GS continuous mapping. Let $B = \langle y, (0.1_u, 0.1_v) \rangle$,

$(0.3_u, 0.3_v)$ is an IF \mathbb{T}_L GSCS in Y. But $f^{-1}(B) = \langle x, (0.1_a, 0.1_b), (0.3_a, 0.3_b) \rangle$ is not an IF \mathbb{T}_L GSCS in X. Therefore f is not an IF \mathbb{T}_L GS irresolute mapping.

Theorem 3.4: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF \mathbb{T}_L GS irresolute mapping, then f is an IFGS continuous mapping but not conversely.

Proof: Let A be an IFCS in Y. Since every IFCS is an IF \mathbb{T}_L GSCS, A is an IF \mathbb{T}_L GSCS in Y. By hypothesis, $f^{-1}(A)$ is an IF \mathbb{T}_L GSCS in X. This implies $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

Example 3.5: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.2_a, 0.4_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.1_a, 0.3_b), (0.3_a, 0.4_b) \rangle$, $G_3 = \langle x, (0.1_a, 0.3_b), (0.5_a, 0.4_b) \rangle$, $G_4 = \langle x, (0.2_a, 0.4_b), (0.3_a, 0.4_b) \rangle$, $G_5 = \langle x, (0.4_a, 0.4_b), (0.3_a, 0.4_b) \rangle$, $G_6 = \langle y, (0.4_u, 0.2_v), (0.5_u, 0.5_v) \rangle$. Then $\tau = \{ 0, G_1, G_2, G_3, G_4, G_5, 1 \}$ and $\sigma = \{ 0, G_6, 1 \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGS continuous mapping. Let $B = \langle y, (0_u, 0.3_v), (0.5_u, 0.4_v) \rangle$ is an IF \mathbb{T}_L GSCS in Y. But $f^{-1}(B) = \langle x, (0_a, 0.3_b), (0.5_a, 0.4_b) \rangle$ is not an IF \mathbb{T}_L GSCS in X. Therefore f is not an IF \mathbb{T}_L GS irresolute mapping.

Theorem 3.6: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF \mathbb{T}_L GS irresolute mapping, then f is an IF continuous mapping if X is an IF \mathbb{T}_L T_{1/2} space.

Proof: Let A be an IFCS in Y. Then A is an IF \mathbb{T}_L GSCS in Y. Since f is an IF \mathbb{T}_L GS irresolute mapping, $f^{-1}(A)$ is an IF \mathbb{T}_L GSCS in X. Since X is an IF \mathbb{T}_L T_{1/2} space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.7: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two IF \mathbb{T}_L GS irresolute mappings. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IF \mathbb{T}_L GS irresolute mapping.

Proof: Let A be an IF \mathbb{T}_L GSCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IF \mathbb{T}_L GSCS in Y. Since f is an IF \mathbb{T}_L GSCS irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T}_L GSCS in X. That is $(g \circ f)^{-1}(A)$ is an IF \mathbb{T}_L GSCS in X. Hence $g \circ f$ is an IF \mathbb{T}_L GS irresolute mapping.

Theorem 3.8: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T}_L GS irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be an IF \mathbb{T}_L GS continuous mapping. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IF \mathbb{T}_L GS continuous mapping.

Proof: Let A be an IFCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IF \mathbb{T}_L GSCS in Y. Since f is an IF \mathbb{T}_L GS irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T}_L GSCS in X. That is $(g \circ f)^{-1}(A)$ is an IF \mathbb{T}_L GSCS in X. Hence $g \circ f$ is an IF \mathbb{T}_L GS continuous mapping.

Theorem 3.9: If $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T}_L GS irresolute mapping, then f is an IFG irresolute mapping if X is an IF \mathbb{T}_L T_{1/2} space.

Proof: Let A be an IFGCS in Y. Then A is an IF \mathbb{T}_L GSCS in Y. Therefore $f^{-1}(A)$ is an IF \mathbb{T}_L GSCS in X, by hypothesis. Since X is an IF \mathbb{T}_L T_{1/2} space, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG irresolute mapping.

Theorem 3.10: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF \mathbb{T}_L T_{1/2} spaces:

- (i) f is an IF \mathbb{T}_L GS irresolute mapping
- (ii) $f^{-1}(B)$ is an IF \mathbb{T}_L GSOS in X for each IF \mathbb{T}_L GSOS B in Y
- (iii) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y.

Proof: (i) \Rightarrow (ii): Obviously true.

(ii) \Rightarrow (iii): Let B be any IFS in Y. Clearly $B \subseteq cl(B)$. Then $f^{-1}(B) \subseteq f^{-1}(cl(B))$. Since $cl(B)$ is an IFCS in Y, $cl(B)$ is an IF \mathbb{T}_L GSCS in Y. Therefore $f^{-1}(cl(B))$ is an IF \mathbb{T}_L GSCS in X, by hypothesis. Since X is an IF \mathbb{T}_L T_{1/2} space, $f^{-1}(cl(B))$ is an IFCS in X. Hence $cl(f^{-1}(B)) \subseteq cl(f^{-1}(cl(B))) = f^{-1}(cl(B))$. That is $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

(iii) \Rightarrow (i): Let B be an IF \mathbb{T}_L GSCS in Y. Since Y is an IF \mathbb{T}_L T_{1/2} space, B is an IFCS in Y and $cl(B) = B$.

Hence $f^{-1}(B) = f^{-1}(\text{cl}(B)) \supseteq \text{cl}(f^{-1}(B))$, by hypothesis. But clearly $f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$. Therefore, $\text{cl}(f^{-1}(B)) = f^{-1}(B)$.

This implies $f^{-1}(B)$ is an IFCS in X and hence it is an IF \mathbb{T} GSCS in X . Thus f is an IF \mathbb{T} GS irresolute mapping.

Theorem 3.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping and (Y, σ) is an IF \mathbb{T} $T_{1/2}$ space. Then the following statements are equivalent:

- (i) f is an IF \mathbb{T} GS irresolute mapping
- (ii) f is an IF \mathbb{T} GS continuous mapping

Proof: (i) \Rightarrow (ii): Follows from the theorem 3.2.

(ii) \Rightarrow (i): Let f be an IF \mathbb{T} GS continuous mapping. Let A be an IF \mathbb{T} GSCS in (Y, σ) . Since (Y, σ) is an IF \mathbb{T} $T_{1/2}$ space, A is an IFCS in (Y, σ) and by hypothesis $f^{-1}(A)$ is an IF \mathbb{T} GSCS in (X, τ) . Therefore f is an IF \mathbb{T} GS irresolute mapping.

Theorem 3.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into IFTS Y . Then the following conditions are equivalent

- (i) f is an IF \mathbb{T} GS irresolute mapping
- (ii) $f^{-1}(B)$ is an IF \mathbb{T} GSOS in X for every IF \mathbb{T} GSOS B in Y .

Proof: (i) \Rightarrow (ii): Let B be an IF \mathbb{T} GSOS in Y , then B^c is an IF \mathbb{T} GSCS in Y . Since f is an IF \mathbb{T} GS irresolute mapping, $f^{-1}(B^c)$ is an IF \mathbb{T} GSCS in X . But $f^{-1}(B^c) = (f^{-1}(B))^c$, implies $f^{-1}(B)$ is an IF \mathbb{T} GSOS in X .

(ii) \Rightarrow (i): Let B be an IF \mathbb{T} GSCS in Y . By our assumption $f^{-1}(B^c)$ is an IF \mathbb{T} GSOS in X for every IF \mathbb{T} GSOS B^c in Y . But $f^{-1}(B^c) = (f^{-1}(B))^c$, which implies $f^{-1}(B)$ is an IF \mathbb{T} GSCS in X . Hence f is an IF \mathbb{T} GS irresolute mapping.

Theorem 3.13: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T} GS irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF α continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IF \mathbb{T} GS continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IF α CS in Y , Since g is IF α continuous. Since every IF α CS is an IF \mathbb{T} GSCS, $g^{-1}(A)$ is an IF \mathbb{T} GSCS in Y . But f is an IF \mathbb{T} GS irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T} GSCS in X . Hence $g \circ f$ is an IF \mathbb{T} GS continuous mapping.

Theorem 3.14: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T} GS irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF α G continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IF \mathbb{T} GS continuous mapping.

Proof: Let A be an IFCS in Z . By assumption, $g^{-1}(A)$ is an IF α GCS in Y . Since every IF α GCS is an IF \mathbb{T} GSCS, $g^{-1}(A)$ is an IF \mathbb{T} GSCS in Y . But f is an IF \mathbb{T} GS irresolute mapping, implies $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T} GSCS in X . Hence $g \circ f$ is an IF \mathbb{T} GS continuous mapping.

Theorem 3.15: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T} GS irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFG continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IF \mathbb{T} GS continuous mapping.

Proof: Let A be an IFCS in Z . By assumption, $g^{-1}(A)$ is an IFGCS in Y . Since every IFGCS is an IF \mathbb{T} GSCS, $g^{-1}(A)$ is an IF \mathbb{T} GSCS in Y . But f is an IF \mathbb{T} GS irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T} GSCS in X . Hence $g \circ f$ is an IF \mathbb{T} GS continuous mapping.

Theorem 3.16: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF \mathbb{T} GS irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFGS continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IF \mathbb{T} GS continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFGSCS in Y , Since g is an IFGS continuous. Since every IFGSCS is an IF \mathbb{T} GSCS, $g^{-1}(A)$ is an IF \mathbb{T} GSCS in Y . Since f is an IF \mathbb{T} GS irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IF \mathbb{T} GSCS in X . Hence $g \circ f$ is an IF \mathbb{T} GS continuous mapping.

Definition 3.17: Let A be an IFS in an IFTS (X, τ) . Then \mathbb{T} -generalized Semi closure of A (\mathbb{T} gscl(A) in short) and \mathbb{T} -generalized semi interior of A (\mathbb{T} gsint(A) in short) are defined by

$$\mathbb{I}\mathbb{E}\text{gsint}(A) = \cup \{ G / G \text{ is an IF}\mathbb{I}\mathbb{E}\text{GSOS in } X \text{ and } G \subseteq A \}$$

$$\mathbb{I}\mathbb{E}\text{gscl}(A) = \cap \{ K / K \text{ is an IF}\mathbb{I}\mathbb{E}\text{GSCS in } X \text{ and } A \subseteq K \}.$$

Proposition 3.18: If A is an IFS in X, then $A \subseteq \mathbb{I}\mathbb{E}\text{gscl}(A) \subseteq \text{cl}(A)$.

Proof: The result follows from the definition.

Theorem 3.19: If A is an IF $\mathbb{I}\mathbb{E}$ GSCS in X then $\mathbb{I}\mathbb{E}\text{gscl}(A) = A$.

Proof: Since A is an IF $\mathbb{I}\mathbb{E}$ GSCS, $\mathbb{I}\mathbb{E}\text{gscl}(A)$ is the smallest IF $\mathbb{I}\mathbb{E}$ GSCS which contains A, which is nothing but A. Hence $\mathbb{I}\mathbb{E}\text{gscl}(A) = A$.

Theorem 3.20: If A is an IF $\mathbb{I}\mathbb{E}$ GSOS in X then $\mathbb{I}\mathbb{E}\text{gsint}(A) = A$.

Proof: Similar to above theorem.

Proposition 3.21: Let (X, τ) be any IFTS. Let A and B be any two intuitionistic fuzzy sets in (X, τ) . Then the intuitionistic fuzzy $\mathbb{I}\mathbb{E}$ -generalized Semi closure operator satisfies the following properties.

- (i) $A \subseteq \mathbb{I}\mathbb{E}\text{gscl}(A)$
- (ii) $\mathbb{I}\mathbb{E}\text{gsint}(A) \subseteq A$
- (iii) $A \subseteq B \Rightarrow \mathbb{I}\mathbb{E}\text{gscl}(A) \subseteq \mathbb{I}\mathbb{E}\text{gscl}(B)$
- (iv) $A \subseteq B \Rightarrow \mathbb{I}\mathbb{E}\text{gsint}(A) \subseteq \mathbb{I}\mathbb{E}\text{gsint}(B)$

Theorem 3.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IF $\mathbb{I}\mathbb{E}$ GS irresolute mapping, then $f(\mathbb{I}\mathbb{E}\text{gscl}(A)) \subseteq \text{cl}(f(A))$ for every IFS A of X.

Proof: Let A be an IFCS of X. Then $\text{cl}(f(A))$ is an IFCS of Y. Since every IFCS is an IF $\mathbb{I}\mathbb{E}$ GSCS, $\text{cl}(f(A))$ is an IF $\mathbb{I}\mathbb{E}$ GSCS

in Y. Since f is IF $\mathbb{I}\mathbb{E}$ GS irresolute, $f^{-1}(\text{cl}(f(A)))$ is IF $\mathbb{I}\mathbb{E}$ GSCS in X. Clearly $A \subseteq f^{-1}(\text{cl}(f(A)))$.

Therefore $\mathbb{I}\mathbb{E}\text{gscl}(A) \subseteq \mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Hence $f(\mathbb{I}\mathbb{E}\text{gscl}(A)) \subseteq \text{cl}(f(A))$ for every IFS A of X.

Theorem 3.23: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is IF $\mathbb{I}\mathbb{E}$ GS irresolute, then $\mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y.

Proof: Let B be an IFS of Y. Then $\text{cl}(B)$ is an IFCS of Y. since every IFCS is an IF $\mathbb{I}\mathbb{E}$ GSCS, $\text{cl}(B)$ is an IF $\mathbb{I}\mathbb{E}$ GSCS in Y. By hypothesis, $f^{-1}(\text{cl}(B))$ is IF $\mathbb{I}\mathbb{E}$ GSCS in X. Clearly $B \subseteq \text{cl}(B)$ implies $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B))$. Therefore, $\mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B)) \subseteq \mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$. Hence $\mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y.

Theorem 3.24: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into IFTS Y. Then the following conditions are equivalent

- (i) f is an IF $\mathbb{I}\mathbb{E}$ GS irresolute mapping
- (ii) $f^{-1}(B)$ is an IF $\mathbb{I}\mathbb{E}$ GSOS in X, for each IF $\mathbb{I}\mathbb{E}$ GSOS in Y
- (iii) $f^{-1}(\mathbb{I}\mathbb{E}\text{gsint}(B)) \subseteq \mathbb{I}\mathbb{E}\text{gsint}(f^{-1}(B))$
- (iv) $\mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y.

Proof:

(i) \Rightarrow (ii): is obviously true.

(ii) \Rightarrow (iii): Let B be an IF $\mathbb{I}\mathbb{E}$ GSCS in Y and $\mathbb{I}\mathbb{E}\text{gsint}(B) \subseteq B$. Then $f^{-1}(\mathbb{I}\mathbb{E}\text{gsint}(B)) \subseteq f^{-1}(B)$. Since $\mathbb{I}\mathbb{E}\text{gsint}(B)$ is an IF $\mathbb{I}\mathbb{E}$ GSOS in Y, $f^{-1}(\mathbb{I}\mathbb{E}\text{gsint}(B))$ is an IF $\mathbb{I}\mathbb{E}$ GSOS in X, by hypothesis. Hence $f^{-1}(\mathbb{I}\mathbb{E}\text{gsint}(B)) \subseteq \mathbb{I}\mathbb{E}\text{gsint}(f^{-1}(B))$.

(iii) \Rightarrow (iv): is obvious by taking complement in (iii).

(iv) \Rightarrow (i): Let B be an IF $\mathbb{I}\mathbb{E}$ GSCS in Y and $\mathbb{I}\mathbb{E}\text{gscl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\mathbb{I}\mathbb{E}\text{gscl}(B)) \supseteq \mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B))$. Therefore, $\mathbb{I}\mathbb{E}\text{gscl}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IF $\mathbb{I}\mathbb{E}$ GSCS in X. Thus f is an IF $\mathbb{I}\mathbb{E}$ GS irresolute mapping.

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