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# (1, 2)*- $\ddot{g}$ -CLOSED SETS AND DECOMPOSITION OF (1, 2)*-CONTINUITY

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***ABSTRACT***

*There are various types of generalization of (1, 2)*-continuous maps in the development of bitopological spaces. Recently some decompositions of (1,2)*-continuity are obtained by various authors with the help of generalized (1,2)*-continuous maps in bitopological spaces. In this paper we obtain a decomposition of (1,2)*-continuity using a generalized (1,2)*-continuity called (1,2)*-* $\ddot{g}$ *-continuity in bitopological space. We also obtain characterizations of (1, 2)*-* $\ddot{g}$ *-continuity in bitopological spaces.*

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***Key words and Phrases:*** *(1,2)*-* $\ddot{g}$ *-closed set, (1,2)*-* $sglc^*$ *-set, (1,2)*-* $\ddot{g}$ *-continuous map, (1,2)*-* $sglc^{\#}$*-continuous map.*

## 1. INTRODUCTION

Different types of generalizations of continuous maps were introduced and studied by various authors in the recent development of topology. The decomposition of continuity is one of the many problems in general topology. Tong [13] introduced the notions of A-sets and A-continuity and established a decomposition of continuity. Also Tong [14] introduced the notions of B-sets and B-continuity and used them to obtain another decomposition of continuity and Ganster and Reilly [3] have improved Tong's decomposition result. Przemski [6] obtained some decompositions of continuity. Hatir et. al. [4] also obtained a decomposition of continuity. Dontchev and Przemski [2] obtained some decompositions of continuity. In this paper, we obtain a decomposition of continuity in bitopological spaces using (1,2)*- $\ddot{g}$ -continuity. We also obtain characterizations of (1,2)*- $\ddot{g}$ -continuous map.

## 2. PRELIMINARIES

Throughout this paper, $(X, \tau_1, \tau_2)$, $(Y, \sigma_1, \sigma_2)$ and $(Z, \eta_1, \eta_2)$ (briefly, X, Y and Z) will denote bitopological spaces.

**Definition 2.1:** Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [7] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

**Definition 2.2 [7]:** Let S be a subset of a bitopological space X. Then
(1) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap\{F : S \subseteq F$ and F is $\tau_{1,2}$-closed $\}$.
(2) the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup\{F : F \subseteq S$ and F is $\tau_{1,2}$-open $\}$.

**Definition 2.3:** A subset A of a bitopological space X is called
(1, 2)*-semi-open set [8] if $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$.

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The complement of (1,2)*-semi-open set is said to be (1,2)*-semi-closed.

The (1, 2)*-semi-closure of a subset A of X is, denoted by (1, 2)*-scl(A), defined to be the intersection of all (1,2)*-semi-closed sets of X containing A.

**Definition 2.4:** A subset A of a bitopological space X is called

(i) (1,2)*-semi-generalized closed (briefly (1, 2)*-sg-closed) set [9] if (1, 2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.

The complement of (1, 2)*-sg-closed set is called (1, 2)*-sg-open set;

(ii) (1,2)*- $\ddot{g}$ -closed set [5] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-sg-open in X.

The complement of (1, 2)*- $\ddot{g}$ -closed set is called (1, 2)*- $\ddot{g}$ -open set.

The collection of all (1, 2)*- $\ddot{g}$ -closed (resp. (1, 2)*- $\ddot{g}$ -open) sets of X is denoted by (1, 2)*- $\ddot{G}$ C(X) (resp. (1, 2)*- $\ddot{G}$ O(X)).

## 3. CHARACTERIZATIONS OF (1,2)*- $\ddot{g}$ -CONTINUOUS MAPS

We introduce the following definition.

**Definition 3.1:** For every set A $\subseteq$ X, we define the (1, 2)*- $\ddot{g}$ -closure of A to be the intersection of all (1, 2)*- $\ddot{g}$ -closed sets containing A.

In symbols, (1, 2)*- $\ddot{g}$ -cl(A) = $\cap$ {F : A $\subseteq$ F $\in$ (1,2)*- $\ddot{G}$ C(X)}.

**Definition 3.2:** A map f: (X, $\tau_1$, $\tau_2$) $\rightarrow$ (Y, $\sigma_1$, $\sigma_2$) is called
(i) (1,2)*- $\ddot{g}$ -continuous if for each $\sigma_{1,2}$-closed set V of Y, $f^{-1}$(V) is (1,2)*- $\ddot{g}$ -closed in X.
(ii) (1,2)*-continuous [ 10] if for each $\sigma_{1,2}$-closed set V of Y, $f^{-1}$(V) is $\tau_{1,2}$-closed in X.

**Definition 3.3:** Let (X, $\tau_1$, $\tau_2$ ) be a bitopological space. Let x be a point of X and G be a subset of X. Then G is called an (1,2)*- $\ddot{g}$ -neighborhood of x (briefly, (1,2)*- $\ddot{g}$ -nbhd of x) in X if there exists an (1,2)*- $\ddot{g}$ -open set U of X such that x $\in$ U $\subset$ G.

**Theorem 3.4:** Every $\tau_{1,2}$-closed set is (1,2)*- $\ddot{g}$ -closed but not conversely.

**Proof:** If A is any $\tau_{1,2}$-closed set in X and G is any (1,2)*-sg-open set containing A, then G $\supseteq$ A = $\tau_{1,2}$-cl(A). Hence A is (1, 2)*- $\ddot{g}$ -closed.

**Example 3.5:** Let X = {a, b, c}, $\tau_1$ = {$\phi$, X, {a, b}} and $\tau_2$ = {$\phi$, X}. Then the sets in {$\phi$, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {$\phi$, X, {c}} are called $\tau_{1,2}$-closed. Also the sets in {$\phi$, X, {c}, {a, c}, {b, c}} are called (1, 2)*- $\ddot{g}$ -closed. Notice that the set {a, c} is (1, 2)*- $\ddot{g}$ -closed set but not $\tau_{1,2}$-closed.

**Proposition 3.6:** Every (1, 2)*-continuous map is (1, 2)*- $\ddot{g}$ -continuous but not conversely.

**Proof:** The proof follows from Theorem 3.4.

**Example 3.7:** Let X = Y = {a, b, c}, $\tau_1$ = {$\phi$, X, {a, b}} and $\tau_2$ = {$\phi$, X}. Then the sets in {$\phi$, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {$\phi$, X, {c}} are called $\tau_{1,2}$-closed. Let $\sigma_1$ = {$\phi$, Y} and $\sigma_2$ = {$\phi$, Y, {a}}.

Then the sets in {$\phi$, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {$\phi$, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {$\phi$, X, {c}, {a, c}, {b, c}} are called (1, 2)*- $\ddot{g}$ -closed. Let f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$) be the identity map. Then f is (1, 2)*- $\ddot{g}$ -continuous map but not (1,2)*-continuous.

**Proposition 3.8:** A map f : $(X, \tau_1,\tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*- $\ddot{g}$ -continuous if and only if $f^{-1}(U)$ is (1,2)*- $\ddot{g}$ -open in X, for every $\sigma_{1,2}$-open set U in Y.

**Proposition 3.9:** For any $A \subseteq X$, the following hold.

(i) (1, 2)*- $\ddot{g}$ -cl(A) is the smallest (1,2)*- $\ddot{g}$ -closed set containing A.

(ii) A is (1, 2)*- $\ddot{g}$ -closed if and only if (1,2)*- $\ddot{g}$ -cl(A) = A.

**Proposition 3.10:** Let A be a subset of a bitopological space X. Then $x \in$ (1,2)*- $\ddot{g}$ -cl(A) if and only if for any (1,2)*- $\ddot{g}$ -nbhd $G_x$ of x in X, $A \cap G_x \neq \phi$.

**Proof:** Necessity. Assume $x \in$ (1, 2)*- $\ddot{g}$ -cl(A). Suppose that there is an (1, 2)*- $\ddot{g}$ -nbhd G of the point x in X such that $G \cap A = \phi$. Since G is (1, 2)*- $\ddot{g}$ -nbhd of x in X, by Definition 3.3, there exists an (1, 2)*- $\ddot{g}$ -open set $U_x$ such that $x \in U_x \subseteq G$. Therefore, we have $U_x \cap A = \phi$ and so $A \subseteq (U_x)^c$. Since $(U_x)^c$ is an (1,2)*- $\ddot{g}$ -closed set containing A, we have by Definition 3.1, (1,2)*- $\ddot{g}$ -cl(A) $\subseteq (U_x)^c$ and therefore $x \notin$ (1,2)*- $\ddot{g}$ -cl(A), which is a contradiction.

Sufficiency. Assume for each (1, 2)*- $\ddot{g}$ -nbhd Gx of x in X, $A \cap G_x \neq \phi$. Suppose $x \notin$ (1, 2)*- $\ddot{g}$ -cl(A). Then by Definition 3.1, there exists a (1, 2)*- $\ddot{g}$ -closed set F of X such that $A \subseteq F$ and $x \notin F$. Thus $x \in F^c$ and $F^c$ is (1, 2)*- $\ddot{g}$ -open in X and hence $F^c$ is a (1,2)*- $\ddot{g}$ -nbhd of x in X. But $A \cap F^c = \phi$, which is a contradiction.

In the next theorem we explore certain characterizations of (1, 2)*- $\ddot{g}$ -continuous functions.

**Theorem 3.11:** Suppose the collection of all (1, 2)*- $\ddot{g}$ -open sets of X is closed under arbitrary union.
Let f: $(X,\tau_1,\tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map from a bitopological space $(X, \tau_1, \tau_2)$ into a bitopological space $(Y, \sigma_1, \sigma_2)$. Then the following statements are equivalent.

(1) The function f is (1, 2)*- $\ddot{g}$ -continuous.

(2) The inverse of each $\sigma_{1,2}$-open set is (1,2)*- $\ddot{g}$ -open.

(3) For each point x in X and each $\sigma_{1,2}$-open set V in Y with $f(x) \in V$, there is an (1,2)*- $\ddot{g}$ -open set U in X such that $x \in U$, $f(U) \subseteq V$.

(4) The inverse of each $\sigma_{1,2}$-closed set is (1,2)*- $\ddot{g}$ -closed.

(5) For each x in X, the inverse of every neighborhood of f(x) is an (1,2)*- $\ddot{g}$ -nbhd of x.

(6) For each x in X and each neighborhood N of f(x), there is an (1, 2)*- $\ddot{g}$ -nbhd G of x such that $f(G) \subseteq N$.

(7) For each subset A of X, f ((1, 2)*- $\ddot{g}$ -cl(A)) $\subseteq \sigma_{1,2}$-cl(f(A)).

(8) For each subset B of Y, (1, 2)*- $\ddot{g}$ -cl($f^{-1}(B)$) $\subseteq f^{-1}(\sigma_{1,2}$-cl(B)).

**Proof:**
(1) $\Leftrightarrow$ (2). This follows from Proposition 3.8.

(1) $\Leftrightarrow$ (3). Suppose that (3) holds and let V be an $\sigma_{1,2}$-open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists an (1, 2)*- $\ddot{g}$ -open set $U_x$ such that $x \in U_x$ and $f(U_x) \subseteq V$. Now, $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Then $f^{-1}(V)$ is (1,2)*- $\ddot{g}$ -open in X and therefore f is (1,2)*- $\ddot{g}$ -continuous.

Conversely, suppose that (1) holds and let $f(x) \in V$ where V is $\sigma_{1,2}$-open in Y. Then $f^{-1}(V) \in$ (1, 2)*- $\ddot{G}$ O(X), since f is (1, 2)*- $\ddot{g}$ -continuous. Let $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$.

(2) $\Leftrightarrow$ (4). This result follows from the fact if A is a subset of Y, then $f^{-1}(A^c) = (f^{-1}(A))^c$.

(2) ⇒ (5). For x in X, let N be a neighborhood of f(x). Then there exists an $\sigma_{1,2}$-open set U in Y such that f(x) ∈ U ⊆ N. Consequently, $f^{-1}(U)$ is an (1,2)*- $\ddot{g}$ -open set in X and x ∈ $f^{-1}(U) \subseteq f^{-1}(N)$. Thus $f^{-1}(N)$ is an (1,2)*- $\ddot{g}$ -nbhd of x.

(5) ⇒ (6). Let x ∈ X and let N be a neighborhood of f(x). Then by assumption, G = $f^{-1}(N)$ is an (1,2)*- $\ddot{g}$ -nbhd of x and f(G) = $f(f^{-1}(N)) \subseteq N$.

(6) ⇒ (3). For x in X, let V be an $\sigma_{1,2}$-open set containing f(x). Then V is a neighborhood of f(x). So by assumption, there exists an (1,2)*- $\ddot{g}$ -nbhd G of x such that f(G) ⊆ V. Hence there exists an (1,2)*- $\ddot{g}$ -open set U in X such that x ∈ U ⊆ G and so f(U) ⊆ f(G) ⊆ V.

(7) ⇔ (4). Suppose that (4) holds and let A be a subset of X. Since A ⊆ $f^{-1}(f(A))$, we have A ⊆ $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Since $\sigma_{1,2}$-cl(f(A)) is a $\sigma_{1,2}$-closed set inY, by assumption $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ is an (1,2)*- $\ddot{g}$ -closed set containing A. Consequently, (1,2)*- $\ddot{g}$ -cl(A) ⊆ $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Thus f((1,2)*- $\ddot{g}$ -cl(A)) ⊆ $f(f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))) \subseteq \sigma_{1,2}\text{-cl}(f(A))$.

Conversely, suppose that (7) holds for any subset A of X. Let F be a $\sigma_{1,2}$-closed subset of Y. Then by assumption, f((1,2)*- $\ddot{g}$ -cl($f^{-1}(F)$)) ⊆ $\sigma_{1,2}\text{-cl}(f(f^{-1}(F))) \subseteq \sigma_{1,2}\text{-cl}(F)$ = F. That is (1, 2)*- $\ddot{g}$ -cl($f^{-1}(F)$) ⊆ $f^{-1}(F)$ and so $f^{-1}(F)$ is (1,2)*- $\ddot{g}$ -closed.

(7) ⇔ (8). Suppose that (7) holds and B be any subset of Y. Then replacing A by $f^{-1}(B)$ in (7), we obtain f((1,2)*- $\ddot{g}$ -cl($f^{-1}(B)$)) ⊆ $\sigma_{1,2}\text{-cl}(f(f^{-1}(B))) \subseteq \sigma_{1,2}\text{-cl}(B)$. That is (1,2)*- $\ddot{g}$ -cl($f^{-1}(B)$) ⊆ $f^{-1}(\sigma_{1,2}\text{-cl}(B))$.

Conversely, suppose that (8) holds. Let B = f(A) where A is a subset of X. Then we have, (1,2)*- $\ddot{g}$ -cl(A) ⊆ (1,2)*- $\ddot{g}$ -cl($f^{-1}(B)$) ⊆ $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ and so f((1,2)*- $\ddot{g}$ -cl(A)) ⊆ $\sigma_{1,2}$-cl(f(A)). This completes the proof of the theorem.

## 4. DECOMPOSITION OF (1,2)*-CONTINUITY

In this section by using (1, 2)*- $\ddot{g}$ -continuity we obtain a decomposition of (1, 2)*-continuity in bitopological spaces.

To obtain a decomposition of (1,2)*-continuity, we introduce the notion of (1,2)*- $sglc^{\#}$-continuous map in bitopological spaces and prove that a map is (1,2)*-continuous if and only if it is both (1,2)*- $\ddot{g}$ -continuous and (1,2)*- $sglc^{\#}$-continuous.

**Definition 4.1:** A subset A of a bitopological space X is called (1,2)*- $sglc^{*}$ -set if A = M ∩ N, where M is (1,2)*-sg-open and N is $\tau_{1,2}$-closed in X.

**Example 4.2:** Let X = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {c}}. Then the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-closed. Then {a} is (1,2)*- $sglc^{*}$ -set in X.

**Remark 4.3:** Every $\tau_{1,2}$-closed set is (1,2)*- $sglc^{*}$ -set but not conversely.

**Example 4.4:** Let X = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {a}}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Then {a, b} is (1, 2)*- $sglc^{*}$ -set but not $\tau_{1,2}$-closed in X.

**Remark 4.5:** (1, 2)*- $\ddot{g}$ -closed sets and (1, 2)*- $sglc^{*}$ -sets are independent of each other.

**Example 4.6:** Let X = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-closed. Then {b, c} is a (1,2)*- $\ddot{g}$ -closed set but not (1,2)*- $sglc^{*}$ -set in X.

**Example 4.7:** Let X = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {b}}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed. Then {a, b} is an (1, 2)*- $sglc^{*}$ -set but not (1, 2)*- $\ddot{g}$ -closed set in X.

 

**Proposition 4.8:** Let X be a bitopological space. Then a subset A of X is $\tau_{1,2}$-closed if and only if it is both (1,2)*- $\ddot{g}$ -closed and (1,2)*- $sglc^*$ -set.

**Proof:** Necessity is trivial. To prove the sufficiency, assume that A is both (1, 2)*- $\ddot{g}$ -closed and (1, 2)*- $sglc^*$ -set. Then A = M $\cap$ N, where M is (1, 2)*-sg-open and N is $\tau_{1,2}$-closed in X. Therefore, A $\subseteq$ M and A $\subseteq$ N and so by hypothesis, $\tau_{1,2}$- cl(A) $\subseteq$ M and $\tau_{1,2}$-cl(A) $\subseteq$ N. Thus $\tau_{1,2}$-cl(A) $\subseteq$ M $\cap$ N = A and hence $\tau_{1,2}$-cl(A) = A. That is A is $\tau_{1,2}$-closed in X.

We introduce the following definition.

**Definition 4.9:** A map f : (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) is said to be (1,2)*- $sglc^{\#}$-continuous if for each $\sigma_{1,2}$-closed set V of Y, $f^{-1}(V)$ is an (1,2)*- $sglc^*$ -set in X.

**Example 4.10:** Let X = Y = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {a}}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed in X. Let $\sigma_1$ = {ϕ, Y, {a}} and $\sigma_2$ = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-closed. Let f : (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) be the identity map. Then f is (1,2)*- $sglc^{\#}$-continuous map.

**Remark 4.11:** From the Remark 4.3, it is clear that every (1,2)*-continuous map is (1,2)*- $sglc^{\#}$-continuous but not conversely.

**Example 4.12:** Let X = Y = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {b}}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed. Let $\sigma_1$ = {ϕ, Y, {b}} and $\sigma_2$ = {ϕ, Y, {a, c}}. Then the sets in {ϕ, Y, {b}, {a, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}, {a, c}} are called $\sigma_{1,2}$-closed. Let f : (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) be the identity map. Then f is (1,2)*- $sglc^{\#}$-continuous map but not (1,2)*-continuous. Since for the $\sigma_{1,2}$-closed set {b} in Y, $f^{-1}(\{b\})$ = {b}, which is not $\tau_{1,2}$-closed in X.

**Remark 4.13:** (1,2)*- $\ddot{g}$ -continuity and (1,2)*- $sglc^{\#}$-continuity are independent of each other.

**Example 4.14:** Let X = Y = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed. Let $\sigma_1$ = {ϕ, Y} and $\sigma_2$ = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Let f : (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) be the identity map. Then f is (1,2)*- $\ddot{g}$ -continuous map but not (1,2)*- $sglc^{\#}$-continuous.

**Example 4.15:** Let X = Y = {a, b, c}, $\tau_1$ = {ϕ, X} and $\tau_2$ = {ϕ, X, {a}}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let $\sigma_1$ = {ϕ, Y} and $\sigma_2$ = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-closed. Let f : (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) be the identity map. Then f is (1, 2)*- $sglc^{\#}$-continuous map but not (1,2)*- $\ddot{g}$ -continuous.

We have the following decomposition for (1,2)*-continuity.

**Theorem 4.16:** A map f: (X, $\tau_1,\tau_2$) $\to$ (Y, $\sigma_1$, $\sigma_2$) is (1,2)*-continuous if and only if it is both (1,2)*- $\ddot{g}$ -continuous and (1,2)*- $sglc^{\#}$-continuous.

**Proof:** Assume that f is (1, 2)*-continuous. Then by Proposition 3.6 and Remark 4.11, f is both (1, 2)*- $\ddot{g}$ -continuous and (1, 2)*- $sglc^{\#}$-continuous.

Conversely, assume that f is both (1,2)*- $\ddot{g}$ -continuous and (1,2)*- $sglc^{\#}$-continuous. Let V be a $\sigma_{1,2}$-closed subset of Y. Then $f^{-1}(V)$ is both (1, 2)*- $\ddot{g}$ -closed set and (1, 2)*- $sglc^*$ -set. By Proposition 4.8, $f^{-1}(V)$ is a $\tau_{1,2}$-closed set in X and so f is (1,2)*-continuous.

 

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