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On α - Generalized Star Regular Closed Sets in Bitopological Spaces

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ABSTRACT

*The aim of this paper is to introduce the concept of αg*r-closed sets in bitopological spaces and the newly related concept of pairwise αg*r-continuous mappings. Also αG*RO-connectedness and αG*RO-compactness are introduced in bitopological spaces and some of their properties are established.*

Keywords: *αg*r-closed sets, pairwise αg*r-continuous, αG*RO-connectedness, αG*RO-compactness.*

Mathematics Subject Classification: *54C10, 54C08, 54C05, 54E55.*

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1 INTRODUCTION

The concept of bitopological spaces was introduced by Kelly [4] in 1963. Separation axioms in bitopological spaces were first studied by him. Fukutake [3] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces in 1986. On the other hand Chandrasekhara Rao and Kannan [1] introduced the concept of generalized star regular closed sets in bitopological spaces. Vadivel, Vijayalakshmi and Krishnaswamy [7] introduced the concepts of α-generalized star closed sets in bitopological spaces. The connectedness and components were introduced by Pervin [5] in bitopological spaces. A detailed study of connectedness in bitopological spaces was carried out by Reilly [6]. Fletcher, Hoyle III, and Patty introduced the notion of a pairwise compact bitopological spaces and proved that every pairwise Hausdorff pairwise compact space is pairwise regular.

In this paper, we introduce the concept of αg*r-closed sets in bitopological spaces and the newly related concept of pairwise αg*r-continuous mappings. Also αG*RO-connectedness and αG*RO-compactness are introduced in bitopological spaces and some of their properties are established.

2 PRELIMINARIES

Throughout this paper we shall denote by (X, τ_1, τ_2) a bitopological space. For any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior of A and the closure of A with respect to τ_i for i = 1, 2.

We shall require the following known definitions:

Definition 2.1: Let (X, τ) be a topological spaces. A subset A of X is called

- a *regular closed* set if A = cl(int(A)).
- an *α-open* set if $A \subseteq$ int(cl(int(A))).

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, the intersection (resp. union) of all τ_i-regular closed sets containing A (resp. τ_i-regular open sets contained in A) is called the *τ_i-regular closure* (resp. *τ_i-regular interior*) of A, denoted by τ_i-rcl(A) (resp. τ_i-rint(A)), i = 1, 2. The regular closure and regular interior of *B* relative to *A* with respect to the topology τ_i are written as τ_i-rcl$_A$(B) and τ_i-rint$_A$(B) for i = 1, 2, respectively. The set of all τ_i-regular closed (resp. τ_i-regular open) sets in X is denoted by τ_i-RC(X, τ_1, τ_2), (resp. τ_i-RO(X, τ_1, τ_2)), i = 1, 2. The set of all τ_i-α-open sets in X is denoted by τ_i-αO(X, τ_1, τ_2).

Definition 2.2: Let (X, τ_1, τ_2) be a bitopological space. A subset A of X is called

- *$\tau_1\tau_2$-α generalized star closed* (briefly, $\tau_1\tau_2$-αg* closed) in X if τ_2-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is τ_1-α-open in X.

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- *$\tau_1\tau_2$-α generalized star open* (briefly, $\tau_1\tau_2$-αg* open) in X if U ⊆ τ_2-int(A) whenever U ⊆ A and U is τ_1-α-closed in X.
- *$\tau_1\tau_2$-generalized star regular closed* (briefly, $\tau_1\tau_2$-g*r closed) in X if τ_2-rcl(A) ⊆ U whenever A ⊆ U and U is τ_1-open in X.
- *$\tau_1\tau_2$-generalized star regular open* (briefly, $\tau_1\tau_2$-g*r open) in X if U ⊆ τ_2-rint (A) whenever U ⊆ A and U is τ_1-closed in X.

Definition 2.3: Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces. A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is *pairwise continuous* if f : (X, τ_1) → (Y, σ_1) and f : (X, τ_2) → (Y, σ_2) are continuous.

3 $\tau_1\tau_2$-α generalized star regular closed sets

Definition 3.1: A set A of bitopological space (X, τ_1, τ_2) is called *τ_i-α generalized star regular closed* (briefly, τ_i-αg*r closed) if τ_i-rcl(A) ⊆ U whenever A ⊆ U and U is τ_i-α-open in X, i = 1, 2. The complement of a τ_i-αg*r closed set is said to be *τ_i-αg*r open*.

For any subset A ⊆ X, the intersection (resp. union) of all τ_i-αg*r closed sets containing A (resp. τ_i-αg*r open sets contained in A) is called the *τ_i-αg*r closure* (resp. *τ_i-αg*r interior*) of A, denoted by τ_i-αg*rcl(A) (resp. τ_i-αg*rint(A)), i = 1, 2. The set of all τ_i-αg*r closed (resp. τ_i-αg*r open) sets in X is denoted by τ_i-αG*RC(X, τ_1, τ_2) (resp. τ_i- αG*RO(X, τ_1, τ_2)), i = 1, 2.

Theorem 3.2: Every τ_i-regular closed set is τ_i-αg*r closed.

Definition 3.3: A set A of a bitopological space (X, τ_1, τ_2) is called *$\tau_1\tau_2$-α generalized star regular closed* (briefly, $\tau_1\tau_2$-αg*r closed) if τ_2-rcl(A) ⊆ U whenever A ⊆ U and U is τ_1-α-open in X. The complement of a $\tau_1\tau_2$-αg*r closed set is said to be *$\tau_1\tau_2$-αg*r open*. The set of all $\tau_1\tau_2$-αg*r closed sets in X is denoted by $\tau_1\tau_2$-αG*RC(X) and the set of all $\tau_1\tau_2$-αg*r open sets in X is denoted by $\tau_1\tau_2$-αG*RO(X).

Example 3.4: Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}. Then φ, X, {b, c} are $\tau_1\tau_2$-αg*r closed and {a}, {b}, {c}, {a, b}, {a, c} are not $\tau_1\tau_2$-αg*r closed.

Theorem 3.5: Every τ_2-regular closed set is $\tau_1\tau_2$-αg*r closed.

Proof: Let A be a τ_2-regular closed set. Let A ⊆ U and U be τ_1-α-open in X. Since A is τ_2-regular closed in X, we have τ_2-rcl(A) = A ⊆ U. Therefore, A is $\tau_1\tau_2$-αg*r closed set.

Theorem 3.6: If A and B are $\tau_1\tau_2$-αg*r closed sets then A ∪ B is $\tau_1\tau_2$-αg*r closed.

Proof: Suppose that A and B are $\tau_1\tau_2$-αg*r closed. Let U be τ_1-α-open and A ∪ B ⊆ U. Since A ∪ B ⊆ U, we have A ⊆ U and B ⊆ U. Since A and B are $\tau_1\tau_2$-αg*r closed sets, we have τ_2-rcl(A) ⊆ U and τ_2-rcl(B) ⊆ U. Therefore, [τ_2-rcl(A)] ∪ [τ_2-rcl(B)] ⊆ U. Since [τ_2-rcl(A)] ∪ [τ_2-rcl(B)] = τ_2-rcl(A ∪ B), we have τ_2-rcl(A ∪ B) ⊆ U. Hence A ∪ B is $\tau_1\tau_2$-αg*r closed.

Remark 3.7: The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.8: Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b, c}} and τ_2 = {φ, X, {a}, {b}, {a, b}}. Let A = {a} and B = {b}. Then A ∪ B = {a, b} is $\tau_1\tau_2$-αg*r closed. But A = {a} is not $\tau_1\tau_2$-αg*r closed.

Remark 3.9: Intersection of two $\tau_1\tau_2$-αg*r closed sets need not be $\tau_1\tau_2$-αg*r closed as can be seen from the following example.

Example 3.10: Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b, c}} and τ_2 = {φ, X, {a}, {b}, {a, b}}. Let A = {a, b} and B = {a, c}. Then A ∩ B = {a} is not $\tau_1\tau_2$-αg*r closed. But A = {a, b} and B = {a, c} are $\tau_1\tau_2$-αg*r closed.

Definition 3.11: Let (X, τ_1, τ_2) be a bitopological space. A collection of subsets of X is said to be *τ_i-regular locally finite*, if for each point x in X there is a τ_i-regular open set U containing X such that U intersects only finitely many of the sets in the collection.

Theorem 3.12: If {A_i, i ∈ I} is a τ_j-regular locally finite family, then τ_j-rcl[∪(A_i)] = ∪τ_j-rcl(A_i).

Theorem 3.13: The arbitrary union of $\tau_1\tau_2$-αg*r closed sets A_i, $i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-αg*r closed if the family $\{A_i, i \in I\}$ is τ_2-regular locally finite.

Proof: Let $\{A_i, i \in I\}$ be τ_2-regular locally finite and A_i be $\tau_1\tau_2$-αg*r closed in X for each $i \in I$. Let $\cup A_i \subseteq U$ and U be τ_1-α-open in X. Then $A_i \subseteq U$ and U is τ_1-α-open in X for each i. Since A_i is $\tau_1\tau_2$-αg*r closed in X for each $i \in I$, we have τ_2-rcl$(A_i) \subseteq U$. Consequently, $\cup[\tau_2\text{-rcl}(A_i)] \subseteq U$.

Since the family $\{A_i, i \in I\}$ is τ_2-regular locally finite, by Theorem 3.12, $\tau_2\text{-rcl}[\cup(A_i)] = \cup[\tau_2\text{-rcl}(A_i)] \subseteq U$. Therefore, $\cup A_i$ is $\tau_1\tau_2$-αg*r closed in X.

Theorem 3.14: The arbitrary intersection of $\tau_1\tau_2$-αg*r open sets A_i, $i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-αg*r open if the family $\{A_i^c, i \in I\}$ is τ_2-regular locally finite.

Proof: Let $\{A_i^c, i \in I\}$ be τ_2-regular locally finite and A_i be $\tau_1\tau_2$-αg*r open in X, for each $i \in I$. Then A_i^c is $\tau_1\tau_2$-αg*r closed in X, for each $i \in I$. Hence by Theorem 3.13, we have $\cup(A_i^c)$ is $\tau_1\tau_2$-αg*r closed in X. Consequently, $(\cap(A_i))^c$ is $\tau_1\tau_2$-αg*r closed in X. Therefore, $\cap A_i$ is $\tau_1\tau_2$-αg*r open in X.

Theorem 3.15: Let A be a subset of a bitopological space (X, τ_1, τ_2). If A is $\tau_1\tau_2$-αg*r closed then τ_2-rcl(A) – A contains no nonempty τ_1-α-closed set.

Proof: Suppose that A is $\tau_1\tau_2$-αg*r closed. Let F be a τ_1-α-closed set such that $F \subseteq \tau_2$-rcl(A) – A. We shall show that $F = \varphi$. Since $F \subseteq \tau_2$-rcl(A) – A, we have $A \subseteq F^c$ and $F \subseteq \tau_2$-rcl(A). Since F is a τ_1-α-closed set, we have F^c is a τ_1-α-open. Since A is $\tau_1\tau_2$-αg*r closed, we have τ_2-rcl(A) $\subseteq F^c$.

Thus $F \subseteq [\tau_2\text{-rcl}(A)]^c = X - [\tau_2\text{-rcl}(A)]$. Hence $F \subseteq [\tau_2\text{-rcl}(A)] \cap [X - [\tau_2\text{-rcl}(A)]] = \varphi$. Therefore, $F = \varphi$.
Hence τ_2-rcl(A) – A contains no nonempty τ_1-α-closed sets.

Theorem 3.16: Let A be a $\tau_1\tau_2$-αg*r closed set. Then A is τ_2-closed in X if and only if τ_2-cl(A) – A is τ_1-α-closed in X.

Proof: Suppose that A is $\tau_1\tau_2$-αg*r closed. Let A be τ_2-closed. Then τ_2-cl(A) = A. Therefore, τ_2-cl(A) – A = φ is τ_1-α-closed.

Conversely, suppose that A is $\tau_1\tau_2$-αg*r closed and τ_2-cl(A) – A is τ_1-α-closed. Since $\tau_2\text{-cl}(A) \subseteq \tau_2\text{-rcl}(A)$, we have $\tau_2\text{-cl}(A) - A \subseteq \tau_2\text{-rcl}(A) - A$, for any subset A of X. Since A is $\tau_1\tau_2$-αg*r closed, we have τ_2-cl(A) – A = φ, by Theorem 3.15. Hence A is τ_2-closed.

Theorem 3.17: Let A and B be subsets such that $A \subseteq B \subseteq \tau_2$-rcl(A). If A is $\tau_1\tau_2$-αg*r closed then B is $\tau_1\tau_2$-αg*r closed.

Proof: Let A and B be subsets such that $A \subseteq B \subseteq \tau_2$-rcl(A). Suppose that A is $\tau_1\tau_2$-αg*r closed. Let $B \subseteq U$ and U be τ_1-α-open in X. Since $A \subseteq B$ and $B \subseteq U$, we have $A \subseteq U$. Since A is $\tau_1\tau_2$-αg*r closed, we have τ_2-rcl(A) $\subseteq U$. Since $B \subseteq \tau_2$-rcl(A), we have $\tau_2\text{-rcl}(B) \subseteq \tau_2\text{-rcl}[\tau_2\text{-rcl}(A)] = \tau_2\text{-rcl}(A) \subseteq U$. Therefore B is $\tau_1\tau_2$-αg*r closed.

Theorem 3.18: If A is $\tau_1\tau_2$-αg*r closed and $A \subseteq B \subseteq \tau_2$-rcl(A) then τ_2-rcl(B) – B contains no nonempty τ_1-α-closed set.

Proof: Follows from Theorem 3.17 and 3.15.

Theorem 3.19: Suppose that $\tau_1\text{-}\alpha O(X, \tau_1, \tau_2) \subseteq \tau_2\text{-}RC(X, \tau_1, \tau_2)$. Then every subset of X is $\tau_1\tau_2$-αg*r closed.

Proof. Let A be a subset of X. Let $A \subseteq U$ and U be τ_1-α-open in X. Since $\tau_1\text{-}\alpha O(X, \tau_1, \tau_2) \subseteq \tau_2\text{-}RC(X, \tau_1, \tau_2)$, we have U is τ_2-regular closed in X. Therefore, τ_2-rcl(U) = U. Since $A \subseteq U$, we have $\tau_2\text{-rcl}(A) \subseteq \tau_2\text{-rcl}(U) = U$. Therefore, A is $\tau_1\tau_2$-αg*r closed.

Theorem 3.20: Let $B \subseteq A$ where A is τ_1-α-open and $\tau_1\tau_2$-αg*r closed. Then B is $\tau_1\tau_2$-αg*r closed relative to A if and only if B is $\tau_1\tau_2$-αg*r closed in X.

Proof: Let $B \subseteq A$ where A is τ_1-α-open and $\tau_1\tau_2$-αg*r closed. Suppose that B is $\tau_1\tau_2$-αg*r closed relative to A. We shall show that B is $\tau_1\tau_2$-αg*r closed in X. Let $B \subseteq U$ and U is τ_1-α-open in X. Since A and U are τ_1-α-open sets in X, we have $A \cap U$ is τ_1-α-open in A. Since $B \subseteq U$ and $B \subseteq A$, we have $B \subseteq U \cap A$. Since B is $\tau_1\tau_2$-αg*r closed relative to A, we have $\tau_2\text{-rcl}_A(B) \subseteq A \cap U$. Since $A \subseteq A$ and A is τ_1-α-open in X, we have $\tau_2\text{-rcl}(A) \subseteq A$ (since A is $\tau_1\tau_2$-αg*r closed in X). Since $B \subseteq A$, τ_2-rcl(B) $\subseteq$ τ_2-rcl(A). Hence τ_2-rcl(B) $\subseteq$ A. Therefore, τ_2-rcl(B) $\cap$ A = τ_2-rcl(B) $\Rightarrow$ τ_2-rcl$_A$(B) = τ_2-rcl(B). Hence τ_2-rcl(B) $\subseteq A \cap U$. Thus B is $\tau_1\tau_2$-αg*r closed.

Conversely, suppose that B is $\tau_1\tau_2$-αg*r closed in X. We shall show that B is $\tau_1\tau_2$-αg*r closed relative to A. Let $B \subseteq U$ and U be τ_1-α-open in A. Since U is τ_1-α-open in A, we have $U = V \cap A$, where V is τ_1-α-open in X.

Hence $B \subseteq U \subseteq V$. Since B is $\tau_1\tau_2$-αg*r closed in X, τ_2-rcl(B) $\subseteq$ V. Hence τ_2-rcl(B) $\cap$ A $\subseteq V \cap A$, which in turn implies that τ_2-rcl$_A$(B) $\subseteq V \cap A = U$. Therefore B is $\tau_1\tau_2$-αg*r closed relative to A.

Theorem 3.21: For each $x \in X$, the singleton {x} is either τ_1-α-closed or $\tau_1\tau_2$-αg*r open.

Proof: Let $x \in X$ and suppose that {x} is not τ_1-α-closed. Then X – {x} is not τ_1-α-open. Consequently, X is the only τ_1-α-open set containing the set X – {x}. Therefore X – {x} is $\tau_1\tau_2$-αg*r closed. Hence {x} is $\tau_1\tau_2$-αg*r open.

4 Pairwise αg*r-continuous Functions

Definition 4.1: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise αg*r-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-αg*r closed in X for each σ_j-closed set U in Y, $i \neq j$ and i, j = 1, 2.

Example 4.2: Let X = {a, b, c} and Y = {p, q, r}. Consider the topologies τ_1 = {φ, X, {a}, {b, c}}, τ_2 = {φ, X, {a}, {b}, {a, b}}, σ_1 = {φ, Y, {p}} and σ_2 = {φ, Y, {q}, {p}, {p, q}}. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by f(a) = q, f(b) = r, f(c) = p. Then f is pairwise αg*r-continuous.

Theorem 4.3: The following are equivalent for a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$

(a) f is pairwise αg*r-continuous
(b) $f^{-1}(A)$ is $\tau_i\tau_j$-αg*r open for each σ_j-open set A in Y, $i \neq j$ and i, j = 1, 2.

Proof: (a) ⇒ (b)
Suppose that f is pairwise αg*r-continuous. Let A be σ_j-open in Y. Then A^c is σ_j-closed in Y. Since f is pairwise αg*r-continuous, we have $f^{-1}(A^c)$ is $\tau_i\tau_j$-αg*r closed in X, $i \neq j$ and i, j = 1, 2. Consequently, $f^{-1}(A)$ is $\tau_i\tau_j$-αg*r open in X, $i \neq j$ and i, j = 1, 2.

(b) ⇒ (a)
Suppose that $f^{-1}(A)$ is $\tau_i\tau_j$-αg*r open for each σ_j-open set A in Y, $i \neq j$ and i, j = 1, 2. Let V be σ_j-closed in Y. Then V^c is σ_j-open in Y. Therefore, by our assumption, $f^{-1}(V^c)$ is $\tau_i\tau_j$-αg*r open in X, $i \neq j$ and i, j = 1, 2. Hence $f^{-1}(V)$ is $\tau_i\tau_j$-αg*r closed in X, $i \neq j$ and i, j = 1, 2. Therefore f is pairwise αg*r-continuous.

Definition 4.4: Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise regular continuous if $f^{-1}(U)$ is τ_i-regular closed in X for each σ_i-closed set U in Y, i = 1, 2.

Theorem 4.5: Every pairwise regular continuous function is pairwise αg*r-continuous.

Proof: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise regular continuous. Let U be a σ_j-closed set in Y. Then $f^{-1}(U)$ is τ_j-regular closed in X. Since every τ_j-regular closed set is $\tau_i\tau_j$-αg*r closed, $i \neq j$ and i, j = 1, 2, we have f is pairwise αg*r-continuous.

Definition 4.6: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise αg*r-irresolute if $f^{-1}(U)$ is $\tau_i\tau_j$-αg*r closed for each $\sigma_i\sigma_j$-αg*r closed set in Y, $i \neq j$ and i, j = 1, 2.

Example 4.7: Let X = {a, b, c} and Y = {p, q, r}. Consider the topologies τ_1 = {φ, X, {a}, {b, c}}, τ_2 = {φ, X, {a}, {b}, {a, b}}, σ_1 = {φ, Y, {p}}, σ_2 = {φ, Y, {p}, {q}, {p, q}}. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by f(a) = q, f(b) = r, f(c) = p. Then f is pairwise αg*r-irresolute.

Concerning the composition of functions, we have the following.

Theorem 4.8: Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be two functions.

(1) If f and g are pairwise αg*r-irresolute, then g∘f is also pairwise αg*r-irresolute.
(2) If f is pairwise αg*r-irresolute and g is pairwise αg*r-continuous then g∘f is pairwise αg*r-continuous.
(3) If f is pairwise αg*r-continuous and g is pairwise continuous then g∘f is pairwise αg*r-continuous.

Proof: (1) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be pairwise αg*r-irresolute. Let U be $\mu_i\mu_j$-αg*r closed in Z, $i \neq j$ and $i, j = 1, 2$. Since g is pairwise αg*r-irresolute, $g^{-1}(U)$ is $\sigma_i\sigma_j$-αg*r closed in Y, $i \neq j$ and $i, j = 1, 2$. Since f is pairwise αg*r-irresolute, $(g\circ f)^{-1} = f^{-1}(g^{-1}(U))$ is $\tau_i\tau_j$-αg*r closed in X, $i \neq j$ and $i, j = 1, 2$. Therefore, $g\circ f$ is pairwise αg*r-irresolute.

(2) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise αg*r-irresolute and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be pairwise αg*r-continuous. Let U be μ_j-closed in Z. Since g is pairwise αg*r-continuous, $g^{-1}(U)$ is $\sigma_i\sigma_j$-αg*r closed set in Y, $i \neq j$ and $i, j = 1, 2$. Since f is pairwise αg*r-irresolute, $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is $\tau_i\tau_j$-αg*r closed in X, $i \neq j$ and $i, j = 1, 2$. Therefore $g\circ f$ is pairwise αg*r-continuous.

(3) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise αg*r-continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be pairwise continuous. Let U be μ_j-closed in Z. Since g is pairwise continuous, $g^{-1}(U)$ is σ_j-closed in Y. Since f is pairwise αg*r-continuous, $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is $\tau_i\tau_j$-αg*r closed in X, $i \neq j$ and $i, j = 1, 2$. Therefore $g\circ f$ is pairwise αg*r-continuous.

Remark 4.9: The composition of two pairwise αg*r-continuous functions need not be a pairwise αg*r-continuous function as can be seen from the following example:

Example 4.10: Let X = {p, q, r}, Y = {a, b, c} and Z = {s, t, u}. Consider τ_1 = {φ, X, {p}}, τ_2 = {φ, X, {p}, {q}, {p, q}}, σ_1 = {φ, Y, {a}, {b, c}}, σ_2 = {φ, Y, {a}, {b}, {a, b}}, μ_1 = {φ, Z, {u}} and μ_2 = {φ, Z, {u}, {t}, {u, t}}. Then $\tau_1\tau_2$-αG*RC(X) = {φ, X, {r}, {p, r}, {q, r}} and $\sigma_1\sigma_2$-αG*RC(Y) = {φ, Y, {b}, {c}, {a, b}, {b, c}, {a, c}}. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by f(p) = h, f(q) = a, f(r) = c and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be a function defined by g(a) = t, g(b) = s, g(c) = u. Then f and g are αg*r-pairwise continuous. But $(g\circ f)^{-1}(\{s, t\}) = \{p, q\}$. Since {a, b} ∈ $\sigma_1\sigma_2$-αG*RC(Y) and {p, q} ∉ $\tau_1\tau_2$-αG*RC(X), we get $g\circ f$ is not αg*r-pairwise continuous.

5 Pairwise αG*RO-connected spaces

Definition 5.1: A bitopological space (X, τ_1, τ_2) is *pairwise αG*RO-connected* if X cannot be expressed as the union of two non empty disjoint sets A and B such that $[A \cap \tau_1\text{-}\alpha g^*rcl(B)] \cup [\tau_2\text{-}\alpha g^*rcl(A) \cap B] = \varphi$.

Suppose X can be so expressed then X is called *pairwise αG*RO-disconnected* and we write X = A|B and call this pairwise αG*RO-separation of X.

Example 5.2: Let X = {a, b, c, d}, τ_1 = {φ, X, {a}, {a, b}} and τ_2 = {φ, X, {a, b}, {a, b, c}, {a, b, d}. Then (X, τ_1, τ_2) is pairwise αG*RO-connected.

Example 5.3: Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b, c}} and τ_2 = {φ, X, {a}, {b}, {a, b}}. Then (X, τ_1, τ_2) is pairwise αG*RO-disconnected, since X = A|B gives a pairwise αG*RO-separation of X.

Theorem 5.4: The following conditions are equivalent for any bitopological space:

(a) X is pairwise αG*RO-connected.
(b) X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-αg*r open and B is τ_2-αg*r open
(c) X contains no nonempty proper subset which is both τ_1-αg*r open and τ_2-αg*r closed.

Proof: (a) ⇒ (b)
Assume that X is pairwise αG*RO-connected. Suppose that X can be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-αg*r open and B is τ_2-αg*r open. Then $A \cap B = \varphi$. Consequently $A \subseteq B^c$. Then τ_2-αg*rcl(A) ⊆ τ_2-αg*rcl $(B^c) = B^c$. Therefore, $\tau_2\text{-}\alpha g^*rcl(A) \cap B = \varphi$. Similarly we can prove $A \cap \tau_1\text{-}\alpha g^*rcl(B) = \varphi$. Hence $[A \cap \tau_1\text{-}\alpha g^*rcl(B)] \cup [\tau_2\text{-}\alpha g^*rcl(A) \cap B] = \varphi$. This is a contradiction. Hence X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-αg*r open and B is τ_2-αg*r open.

(b) ⇒ (c)
Assume that X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-αg*r open and B is τ_2-αg*r open. Suppose that X contains a nonempty proper subset A which is both τ_1-αg*r open and τ_2-αg*r closed. Then $X = A \cup A^c$, where A is τ_1-αg*r open, A^c is τ_2-αg*r open and A and A^c are disjoint. This is a contradiction to our assumption. Therefore, X contains no nonempty proper subset which is both τ_1-αg*r open and τ_2-αg*r closed.

(c) ⇒ (d)
Assume that X contains no nonempty proper subset which is both τ_1-αg*r open and τ_2-αg*r closed. Suppose that X is pairwise αG*RO-disconnected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1\text{-}\alpha g^*rcl(B)] \cup [\tau_2\text{-}\alpha g^*rcl(A) \cap B] = \varphi$. Since $A \cap B = \varphi$, we have $A = B^c$ and $B = A^c$.

Since τ_2-αg*rcl(A) ∩ B = φ, we have τ_2-αg*rcl(A) ⊆ B^c. Hence τ_2-αg*rcl(A) ⊆ A. Therefore, A is τ_2-αg*r closed. Similarly, B is τ_1-αg*r closed.

Since A = B^c, A is τ_1-αg*r open. Therefore, there exists a nonempty proper set A which is both τ_1-αg*r open and τ_2-αg*r closed. This is a contradiction to our assumption. Therefore X is pairwise αG*RO-connected.

Theorem 5.5: If A is a pairwise αG*RO-connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwise αG*RO-separation X = C|D, then A ⊆ C or A ⊆ D.

Proof: Suppose that (X, τ_1, τ_2) has the pairwise αG*RO-separation X = C|D. Then X = C ∪ D where C and D are two nonempty disjoint sets such that [C ∩ τ_1-αg*rcl(D)] ∪ [τ_2-αg*rcl(C) ∩ D] = φ. Since C ∩ D = φ, we have C = D^c and D = C^c. Now [(C ∩ A) ∩ τ_1-αg*rcl(D ∩ A)] ∪ [τ_2-αg*rcl(C ∩ A) ∩ (D ∩ A)] ⊆ [C ∩ τ_1-αg*rcl(D)] ∪ [τ_2-αg*rcl(C) ∩ D] = φ. Hence A = (C ∩ A)|(D ∩ A) is a pairwise αG*RO-separation of A. Since A is pairwise αG*RO-connected, we have either (C ∩ A) = φ or (D ∩ A) = φ. Consequently, A ⊆ C^c or A ⊆ D^c. Therefore, A ⊆ C or A ⊆ D.

Theorem 5.6: If A is pairwise αG*RO-connected and A ⊆ B ⊆ τ_1-αg*rcl(A) ∩ τ_2- αg*rcl(A) then B is pairwise αG*RO-connected.

Proof: Suppose that B is not pairwise αG*RO-connected. Then B = C ∪ D where C and D are two nonempty disjoint sets such that [C ∩ τ_1-αg*rcl(D)] ∪ [τ_2-αg*rcl(C) ∩ D] = φ. Since A is pairwise αG*RO-connected by Theorem 5.5, we have A ⊆ C or A ⊆ D. Suppose A ⊆ C. Then D = D ∩ B ⊆ D ∩ τ_2-αg*rcl(A) ⊆ D ∩ τ_2-αg*rcl(C) = φ. Consequently, D = φ. Similarly, we can prove C = φ if A ⊆ D. This is a contradiction to the fact that C and D are nonempty. Therefore, B is pairwise αG*RO-connected.

Theorem 5.7: The union of any family of pairwise αG*RO-connected sets having a nonempty intersection is pairwise αG*RO-connected.

Proof: Let I be an index set and i ∈ I. Let A = ∪A_i, where each A_i is pairwise αG*RO- connected with ∩A_i ≠ φ. Suppose that A is not pairwise αG*RO-connected. Then A = C ∪ D, where C and D are two nonempty disjoint sets such that [C ∩ τ_1-αg*rcl(D)] ∪ [τ_2-αg*rcl(C) ∩ D] = φ. Since A_i is pairwise αG*RO-connected and A_i ⊂ A, by Theorem 5.5, we have A_i ⊆ C or A_i ⊆ D. Since ∩A_i ≠ φ, there exists an element x ∈ ∩ A_i . Therefore, x ∈ A_i for all i. Suppose A_i ⊆ C for some i. Then x ∈ C since C ∩ D = φ , x ∉ D. Hence, we get A_j ⊆ C, for every j. Therefore, ∪A_i ⊆ C. Therefore, A ⊆ C. Hence D = φ. This is a contradiction. Therefore, A is pairwise αG*RO-connected.

6 Pairwise αG*RO-compact spaces

Definition 6.1: A nonempty collection $\mathcal{A}$ = {A_i, i ∈ I, an index set} is called a *pairwise regular open cover* of a bitopological space (X, τ_1, τ_2) if X = ∪A_i and $\mathcal{A}$ ⊆ τ_1-RO(X, τ_1, τ_2) ∪ τ_2-RO(X, τ_1, τ_2) and $\mathcal{A}$ contains at least one member of τ_1-RO(X, τ_1, τ_2) and one member of τ_2-RO(X, τ_1, τ_2).

Definition 6.2: A bitopological space (X, τ_1, τ_2) is *pairwise regular compact* if every pairwise regular open cover of X has a finite subcover.

Definition 6.3: A nonempty collection $\mathcal{A}$ = {A_i, i ∈ I, an index set} is called a τ_i-αg*r *open cover* of a bitopological space (X, τ_1, τ_2) if X = ∪A_i and $\mathcal{A}$ ⊆ τ_i-αG*RO(X, τ_1, τ_2), i = 1, 2.

Definition 6.4: A bitopological space (X, τ_1, τ_2) is *τ_i-αG*RO-compact* if every τ_i-αg*r open cover of X has a finite subcover.

Definition 6.5: A nonempty collection $\mathcal{A}$ = {A_i, i ∈ I, an index set} is called a *pairwise αg*r-open cover* of a bitopological space (X, τ_1, τ_2) if X = ∪A_i and $\mathcal{A}$ ⊆ τ_1-αG*RO(X, τ_1, τ_2) ∪ τ_2-αG*RO(X, τ_1, τ_2) and $\mathcal{A}$ contains at least one member of τ_1-αG*RO(X, τ_1, τ_2) and one member of τ_2-αG*RO(X, τ_1, τ_2).

Definition 6.6: A bitopological space (X, τ_1, τ_2) is *pairwise αG*RO-compact* if every pairwise αg*r-open cover of X has a finite subcover.

Theorem 6.7: Every pairwise αG*RO-compact space is pairwise regular compact.

Proof: Let (X, τ_1, τ_2) be pairwise αG*RO-compact. Let $\mathcal{A}$ = {A_i, i ∈ I} be a pairwise regular open cover of X. Then X = ∪A_i and $\mathcal{A}$ ⊆ τ_1-RO(X, τ_1, τ_2) ∪ τ_2-RO(X, τ_1, τ_2) and $\mathcal{A}$ contains at least one member of τ_1-RO(X, τ_1, τ_2) and one member of τ_2-RO(X, τ_1, τ_2). Since every τ_i-regular open set is τ_i-αg*r open, we have X = ∪A_i and $\mathcal{A}$ ⊆ τ_1-αG*RO(X, τ_1, τ_2) ∪ τ_2-αG*RO(X, τ_1, τ_2) and $\mathcal{A}$ contains at least one member of τ_1-αG*RO(X, τ_1, τ_2) and one member of τ_2-

$\alpha G^*RO(X, \tau_1, \tau_2)$. Therefore, $\mathcal{A}$ is a pairwise αg^*r-open cover of X. Since X is pairwise αG^*RO-compact, we have $\mathcal{A}$ has a finite subcover. Therefore, X is pairwise regular compact.

Theorem 6.8: If Y is a τ_1-αg^*r closed subset of a pairwise αG^*RO-compact space (X, τ_1, τ_2), then Y is τ_2-αG^*RO compact.

Proof: Let (X, τ_1, τ_2) be a pairwise αG^*RO-compact space. Let $\mathcal{A} = \{A_i, i \in I\}$ be a τ_2-αg^*r open cover of Y. Since Y is τ_1-αg^*r closed, Y^c is τ_1-αg^*r open. Then, $Y^c \cup \mathcal{A} = Y^c \cup \{A_i, i \in I\}$ is a pairwise αg^*r-open cover of X. Since X is pairwise αG^*RO-compact, $X = Y^c \cup A_1 \cup A_2 \cup \dots\dots \cup A_n$. Hence $Y = A_1 \cup A_2 \cup \dots\dots \cup A_n$. Therefore Y is τ_2-αG^*RO compact.

Theorem 6.9: If Y is a τ_1-regular closed subset of a pairwise αG^*RO-compact space (X, τ_1, τ_2), then Y is τ_2-αG^*RO compact.

Proof: Let Y be a τ_1-regular closed subset of (X, τ_1, τ_2). Since every τ_1-regular closed set is τ_1-αg^*r closed, we get Y is τ_1-αg^*r closed. Hence by Theorem 6.8, Y is τ_2-αG^*RO compact.

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