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DOMINATION NUMBERS OF GRID GRAPHS $P_{15} \times P_n$

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ABSTRACT

In this paper we deal with the domination numbers of the complete grid graphs $P_k \times P_n$ for $k = 15$ and $n \geq 1$. Establishing this numbers for various class of graphs is the content of many papers in literature. Some of the papers which deal with the dominating number of grid graphs are [3], [4], [5]. There is no general formula for the domination number of a graph. In this paper, we use the concept of transforming the domination from a vertex in a dominating set D of a graph G = (V, E) to a vertex in V – D, where G is a simple connected graph. We give an algorithm using this transformation to obtain a domination set of a graph G.

***Key words**: Dominating set, Domination number, Transformation of a dominating set.*

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1. INTRODUCTION

A graph *G = (V, E)* is a mathematical structure consisting of two sets, *V* (of vectors), and *E* (of edges) where *V* is a finite and nonempty and elements of *E* are unordered pairs *{u, v}* of distinct element of *V*. We simply write *uv* instead of *{u ,v}*. The *order* of the graph *G* is the cardinality of its vertex set V, the *size* of the graph *G* is the cardinality of its edge set *E* (see [1]). Two verities *u* and *v* of a graph G are said to be *adjacent* if $uv \in E$. For a vertex *v* of *G*, the neighborhood of *v*, *N (v)*, is defined as the set of all vertices of *G* which are adjacent to *v*. The closed neighborhood of *v* is $\overline{N}(v) = N(v) \cup \{v\}$. If *S* is a set of vertices of *G*, then the *neighborhood* of *S* is given by $N(S) = \bigcup_{v \in S} N(v)$ and $\overline{N}(S) = \cup \overline{N}(v)$. The *degree* of vertex v, is given by $d(v) = |N(v)|$, the number of vertices which are adjacent to v.

Let *G = (V, E)* be a graph. A set $D \subseteq V$ is called a *dominating set* of *G* if every vertex in *V-D* is adjacent to at least one vertex of D. A dominating set *D* of G is said to be a *minimal dominating set* of G if $|D| \leq |D_1|$ for any dominating set D_1 of *G*. In otherwords, dominating set is minimal if it contains no other dominating set as a proper subset. The cardinality of a minimal dominating set of *G* is known as the domination *number* of *G*, and is denoted by $\gamma(G)$.

Next we give some more definitions.

Let *D* be a dominating set of a graph $G = (V, E)$.

1. We define the *weight function* F_D on *V* as follows: $F_D : V \rightarrow \mathbb{N}$ given by $F_D(v)$ = the cardinality of the closed neighborhood of v, ie., $F_D(v) = |\overline{N}(v)|$.
   (Here $\mathbb{N}$ denotes the set of natural numbers).
2. We say that $v \in D$ has a *moving domination* if there exits a vertex $w \in N(v) - D$ such that $wu \in E$ for every vertex $u \in \{y \in N(v): F_D(y)\} = 1$.
3. We say that a vertex $v \in D$ is *redundant* of *D* if $F_D(w) \geq 2$ for every vertex $w \in \overline{N}(v)$.

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4. If $v \in D$ has moving domination, we say that *v* is *inefficient* if transforming the domination from *v* to any vertex in *N(v)* would not produce any redundant vertex.
5. For two vertices v_0 and v_n of a graph G, a v_0 - v_n *walk* is an alternating sequence of $n+1$ vertices and n edges v_0 , e_1 , v_1 , e_2 , v_2 , ….., e_n ,v_n such that consecutive vertices and edges are incident.
6. A *path* is defined to be a walk in which no vertices are repeated. A path with n vertices is denoted by P_n. It has $n-1$ edges and the *length* of P_n is $n-1$. The *grid graph* $P_k \times P_n$ is defined by $V(P_k \times P_n) = \{v_{i\,j} : 1 \le i \le k, 1 \le j \le n\}$ and two vertices $v_{i,j}$ and $v_{t.s}$ are adjacent either $i = t, j = s \pm 1 \; or \; i = t \pm 1, j = s$

Thus a grid graph can be treated as the Cartesian product of two paths P_k and P_n of path length $k-1$ and $n-1$.

Observe that if *D* is a dominating set of grid graph $P_k \times P_n$ which has no redundant vertex, then $v \in D$ has a moving domination if and only if one of the following two cases occurs:

Case 1: For every vertex $w \in N(v)$ we have $F_D(w) \ge 2$.

In this case the domination of *v* can be transformed to any vertex in $N(v) - D$.

Case 2: There exits exactly one vertex $u \in N(v)$ such that $F_D(u) = 1$.

In this case the domination of *v* can be transformed only to *u*.

2. AN ALGORITHM FOR FINDING A DOMINATING SET OF $P_k \times P_n$ USING TRANSFORMATION OF DOMINATION OF VERTICES.

In this section, we present an algorithm for finding dominating sets and consequently dominating numbers of graphs. In the next section we apply this to the grid graph $P_{15} \times P_n$.

Let G = *(V, E)* be a graph of order greater than 1, say, $|V| = m$. Let $D = V$ be a dominating set of G. Then for any vertex $v \in D$ we have $F_D(v) = d(v) + 1 \ge 2$. Pick a vertex v_1 of *D* and delete from *D* all vertices $w,\ w \in N(v_1)$. Then for $1 \le n \le \frac{m}{2}$, pick a vertex $v_n \in D - \bigcup_{i=1}^{n-1} \overline{N}(v_i)$ and delete from D all vertices $w, w \in N(v_n) - \bigcup_{i=1}^{n-1} \overline{N}(v_i)$. Next, if *D* contains a redundant vertex, then delete it. Repeat this process until *D* has no redundant vertices.

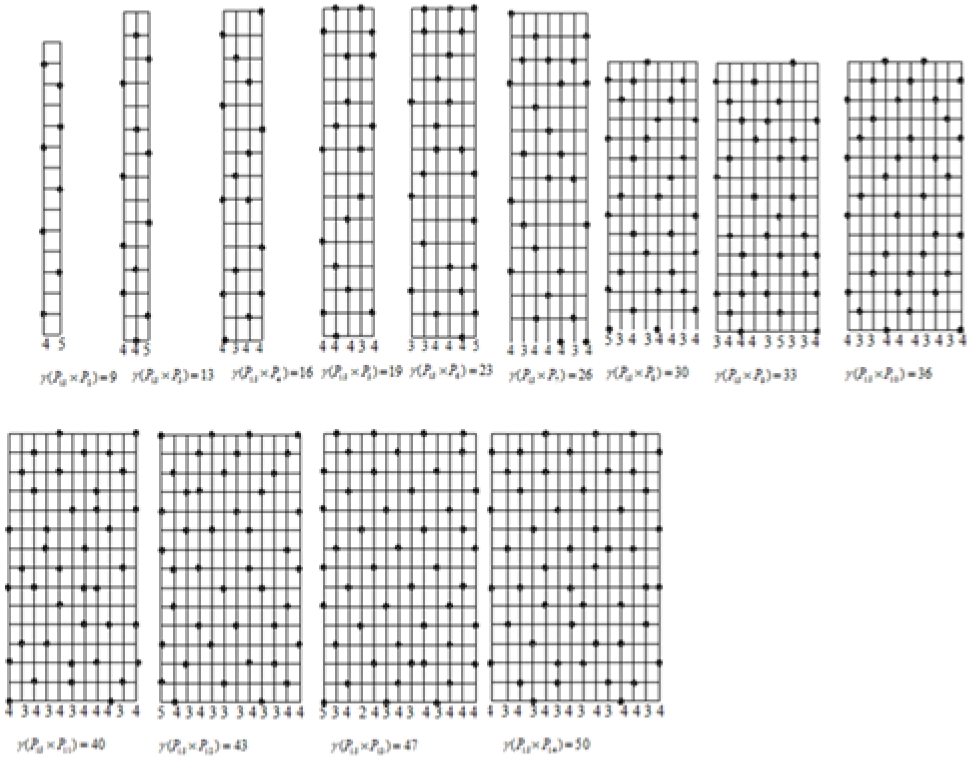
Next transform domination from vertices of D which have moving domination to vertices in $V - D$ to obtain redundant vertices and go to above step. If no redundant vertex can be obtained by a transformation of domination of vertices of *D*, then stop. This set is a dominating set *D* satisfying that for every $v \in D$ there is a $w \in N(v)$ such that $F_D(w) = 1$.

3. Example: In this section we give an example to illustrate the above algorithm, in case of the grid graph $G = P_{15} \times P_{14}$.

1. We have $|V| = 210$. Let *(k,n)* be the vertex in k^{th} row and in the n^{th} column of the graph *G.*
2. Let *D=V* be a dominating set of *G.*
3. Pick a vertex $v_1 = (1,3) \in D$ and delete from *D* all vertices $w,\ w \in N(v_1)$. Then for $1 \le n \le \frac{210}{2}$, pick a vertex $v_n,\ v_n \in D - \bigcup_{i=1}^{n-1} \overline{N}(v_i)$, and delete from *D* all vertices $w,\ w \in N(v_n) - \bigcup_{i=1}^{n-1} \overline{N}(v_i)$. We obtain a dominating set *D* (indicated by black circles in the figures below.)
4. Since for every vertex $v \in D, \exists w \in \overline{N}(v)$ such that $F_D(w)=1$, *D* has no redundant vertices.
5. Transform the domination from the vertex *(5,9)* to the vertex *(6,9)* and delete from *D*, and the resulting redundant vertex is *(7,9).* Therefore, the set *D* indicated in figure below (black circles) is a dominating set of $G=P_{15}\ X\ P_{14}$. Note that *D* is a minimal dominating set (see [3]).

(2). Domination numbers for $\gamma(P_{15} \times P_n)$ for $1 \le n \le 14$

Below we give some minimal dominating set of $P_{15} \times P_n$ up to $n = 14$ and consequently obtain the dominating numbers of these graphs



These results match with the known formula for dominating number of grid graphs $P_{15} \times P_n$,

$$\gamma(P_{15} \times P_n) = \begin{cases} \left\lfloor \dfrac{44n+27}{13} \right\rfloor & for\ n \equiv 5 \pmod{26} \\ \left\lfloor \dfrac{44n+40}{13} \right\rfloor & otherwise \end{cases}$$

See for eg [6].

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