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OPTIMAL STRATEGY ANALYSIS OF AN N-POLICY TWO-PHASE M/E$_K$/1 QUEUEING SYSTEM WITH SERVER STARTUP AND BREAKDOWNS

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ABSTRACT

This paper studies a two-phase M/E$_k$/1 queueing system with N-policy for exhaustive batch service, server startup and breakdowns. The customers arrive individually according to a Poisson process and waiting customers receive batch service all at a time in the first phase and are served individually in the second phase. The server is turned off each time the system empties. When the queue length reaches or exceeds N (threshold), the batch service starts. Before the batch service, the system requires a random startup time for pre-service. As soon as the startup period is over, the server starts the batch service which is followed by individual service to all customers in the batch. It is assumed that the server may breakdown during individual service following Poisson process and the repair time follows an exponential distribution. Explicit expressions for the steady state distribution of the number of customers in the system and expressions for the expected system length are derived. The total expected cost function is developed to determine the optimal threshold of N at a minimum cost. Sensitivity analysis is carried out through numerical illustrations.

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Key words: Vacation, N-policy, Queueing System, Two-phase, Startup, Breakdowns.

1. INTRODUCTION:

Queueing models with two phases of service and server vacations have several applications in many areas such as computer network administration and telecommunication systems, where the messages are processed in two phases by a single server. In inventory control processes, due date, quantity and quality are analyzed initially in batch mode followed by individual service of the batch.

Out of the situations discussed above and in the situation discussed in this paper, computer controlled manufacturing and pharmaceutical industry are classic examples as explained below:

1. Consider a process where pipes are molded to a given specification and are subsequently sent to another machine for authentication and validity of the specification and then released to the market for sale. Initially all pipes are molded as per specification on a machine and then, they are subsequently transferred to another machine for testing of quality and validity of the prescribed specifications. Any item, which is not in conformity with the quality specification, would be sent to recycling. The process continues so till the ordered quantity is exhausted. The process halts when there are no orders and restarts again when the orders reach a specific level.

2. The case of manufacturing bulk drugs in pharmaceutical industry is another classic example where the production cannot commence until the ordered quantity reaches specific requirement. Once the specific requirement condition is met, the drug is manufactured in the first phase and then it is tested to the specifications in the second phase well before it is released to the customer or else for onward processing. After the second phase, the production process commences even if there is a single order, failing which the system remains in idle or no production state. The production cannot commence until the ordered quantity reaches specific limit. The production process needs startup period after an idle state.

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The examples are many. However, the above said examples are only a few out of many that are encountered in several industrial and real life situations. All such examples provide an insight into the optimal control policy at every stage of processing. Several attempts have been made by many investigators to provide a valid and a meaningful solution so as to estimate the optimal control policy. Out of those who have made an attempt to study the system, Krishna and Lee [8], Doshi [3] studied distributed systems where all customers receive batch service in first phase followed by individual service in second phase. Selvam and Sivasankaran [10] introduced two-phase queuing system with server vacation.

For the control policy of vacation queues it is usually assumed that the server availability depends completely on the number of customers in the system. Every time the system is empty, the server goes on a vacation. The time the server finds at least N customers in the system, it comes back from vacation and begins serving immediately and exhaustively. This type of control policy is also called N-policy queueing system with vacations. Kim and Chae [7] analyzed the Two-Phase queueing system with N-policy. Vasanta Kumar and Chandan [11], [12] presented the optimal control policy of two-phase M/M/1 and $M^X/E_k/1$ queueing systems with N-policy.

The server-startup corresponds to the preparatory work of the server before starting the service. In real time situations, the server often requires a preparatory time before starting each service period. Concerning queueing systems with startup time, Baker [2] first proposed the N-policy M/M/1 queueing system. The N-policy M/G/1 queueing system with startup time was first studied by Minh [9] and was investigated by several other researchers. Hur and Paik [4] examined the operating characteristics of M/G/1 queueing systems under N-policy with server startup and explained how the system's optimal policy and cost parameters behave for various arrival rates.

Queueing systems with server breakdowns and vacation have also been investigated in different frame works in recent past. The unreliable server is commonly found in computer systems, communication systems and in manufacturing systems and other day to day realistic queueing problems. Wang [13] first proposed a Markovian queueing system under N-policy with server breakdowns. Wang [14], Wang et al. [15] and Wang et al. [16] extended the model proposed by Wang [13] to $M/E_k/1$, $M/H_2/1$ and $M/H_k/1$ queueing systems respectively. They developed closed form solutions and provided a sensitivity analysis. Ke [5] studied the control policy of a removable and unreliable server for an $M^X/M/1$ queueing system, where the removable server operates an N-policy. Ke [6] presented the control policies of an M/G/1 queueing system with a startup time and unreliable server, in which the length of the vacation period is controlled either by the number of arrivals during the idle period or by a timer. Anantha Lakshmi *et al*. [1] presented the optimal control strategy of an N-policy bulk arrival queueing system with server startup and breakdowns.

This paper considers the optimal control policy for two-phase $M/E_k/1$ queueing system with N-policy, server startups and breakdowns.

2. THE MATHEMATICAL MODEL:

We will now consider an $M/E_k/1$ queueing system with server startup, two-phases of service and an unreliable server under N-policy.

(i) The arrival process is a Poisson process with arrival rate λ. Customers have access to the service according to the first come first serve order.
(ii) The service is in two-phases, the first phase of service is batch service to all customers waiting in the queue. On completion of batch service, the server proceeds to the second phase to serve all customers individually. Batch service time is assumed to be exponentially distributed with mean $1/\beta$ and is independent of batch size. Individual service is in *k* exponential phases each with mean $1/k\mu$. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If customers are waiting, the server starts the batch service followed by individual service to each customer in the batch. If no customer is waiting, the server goes on vacation.
(iii) Whenever the system becomes empty, the server is turned off. As soon as the total number of arrivals in the queue reaches to predetermined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. Before the batch service, the server needs a startup time which follows an exponential distribution with mean $1/\theta$. After the server finishes startup, it starts serving the first phase of waiting customers.
(iv) The customers who arrive during the pre-service and batch service are also allowed to enter the same batch which is in service. This criterion is called exhaustive service without gating.
(v) The server is subject to breakdown at any time during individual service with Poisson breakdown rate α. Whenever the server fails, it is sent for repair immediately and during that period the server stops providing service. The repair time is assumed to be exponentially distributed with mean $1/\gamma$.
(vi) In case the server breaks down while serving customers, it is sent for repair and that particular customer who was just being served should wait for the server to come back to complete the remaining service. Immediately after the

server is repaired, it starts to serve and the service time is sequential. A customer who arrives and finds the server busy or broken down must wait in the queue until the server is available. Customers continue to arrive during the repair period of the broken server.

The four main objectives for which the analysis has been carried out in this paper are:

i. To determine the steady state probability distribution of the number of units in the system.
ii. To derive expected number of units in the system.
iii. To formulate the total expected cost function for the system, and determine the optimal value of the control parameter N.
iv. To carryout sensitivity analysis on the optimal value of N and the minimum expected cost through numerical illustrations.

3. STEADY STATE RESULTS:

In steady state the following notations are used.

$P_{0,i,0}$ ≡ The probability that there are i service phases in the batch queue when the server is on vacation, where i = 0, k, 2k, 3k, . . . , (N-1)k **.**

$P_{1,i,0}$ ≡ The probability that there are i service phases in the batch queue when the server is doing pre-service (startup work), where i = Nk, (N+1)k, (N+2)k,….

$P_{2,i,0}$ ≡ The probability that there are i service phases in the batch queue when the server is in batch service, where i = k, 2k, 3k . . .

$P_{3,i,j}$ ≡ The probability that there are i service phases in the batch queue and j service phases in individual queue when the server is in individual service, where i = 0, k, 2k, 3k . . . and j = 1, 2, 3 . . .

$P_{4,i,j}$ ≡ The probability that there are i service phases in the batch queue and j service phases in individual queue when the server is in individual queue but found to be broken down, where i = 0, k, 2k, . . . and j = 1, 2, 3, . . . **.**

The steady-state equations satisfied by the system size probabilities are as follows:

$$\lambda P_{0,0,0} = k\mu P_{3,0,1}. \tag{1}$$

$$\lambda P_{0,i,0} = \lambda P_{0,i-k,0}, \quad i = k, 2k, 3k, \dots (N-1)k. \tag{2}$$

$$(\lambda+\theta) P_{1,Nk,0} = \lambda P_{0,(N-1)k,0}. \tag{3}$$

$$(\lambda+\theta) P_{1,i,0} = \lambda P_{1,i-k,0} + k\mu P_{3,i,1}, \quad i = (N+1)k, (N+2)k, (N+3)k, \dots . \tag{4}$$

$$(\lambda+\beta) P_{2,i,0} = \lambda P_{2,i-k,0} + k\mu P_{3,i,1}, \quad i = k, 2k, 3k, \dots, (N-1)k. \tag{5}$$

$$(\lambda+\beta) P_{2,i,0} = \lambda P_{2,i-k,0} + k\mu P_{3,i,1} + \theta P_{1,i,0}, \quad i = Nk, (N+1)k, (N+2)k, \dots \tag{6}$$

$$(\lambda+\alpha+k\mu) P_{3,0,j} = k\mu P_{3,0,j+1} + \beta P_{2,j,0} + \gamma P_{4,0,j}, \quad j \ge 1. \tag{7}$$

$$(\lambda+\alpha+k\mu) P_{3,i,j} = k\mu P_{3,i,j+1} + \lambda P_{3,i-k,j} + \gamma P_{4,i,j}, \quad i \ge k, \quad j \ge 1. \tag{8}$$

$$(\lambda+\gamma) P_{4,0,j} = \alpha P_{3,0,j}, \quad j \ge 1. \tag{9}$$

$$(\lambda+\gamma) P_{4,i,j} = \alpha P_{3,i,j} + \lambda P_{4,i-k,j}, \quad i \ge k, \quad j \ge 1. \tag{10}$$

The following probability generating functions are defined

$$G_0(z) = \sum_{i=0}^{(N-1)k} P_{0,i,0} z^i, \quad G_1(z) = \sum_{i=Nk}^{\infty} P_{1,i,0} z^i, \quad G_2(z) = \sum_{i=k}^{\infty} P_{2,i,0} z^i,$$

$$G_3(z,y) = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} P_{3,i,j} z^i y^j, \quad G_4(z,y) = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} P_{4,i,j} z^i y^j, \quad R_j(z) = \sum_{i=0}^{\infty} P_{3,i,j} z^i,$$

and $S_j(z) = \sum_{i=0}^{\infty} P_{4,i,j} z^i$, where $|z| \le 1$ and $|y| \le 1$.

Multiplication of equation (2) by z^i and adding over i = k, 2k, 3k, . . . ,(N-1)k yields

$$G_o(z) = \frac{(1-z^{Nk})}{(1-z^k)} P_{0,0,0}. \tag{11}$$

Multiplication of equations (3) and (4) by z^i and adding over i = Nk, (N+1)k, (N+2)k,…

yields

$$G_1(z) = \frac{\lambda z^{Nk}}{(\lambda(1-z^k)+\theta)} P_{0,0,0}. \tag{12}$$

Multiplication of equations (5) and (6) by z^i and adding over i = k, 2k, 3k, . . . yields

$$G_2(z) = \frac{[k\mu R_1(z) + \theta G_1(z) - \lambda P_{0,0,0}]}{\left(\lambda(1-Z^k)+\beta\right)}. \tag{13}$$

Multiplication of equation (8) by z^i and adding over i = k, 2k, 3k, . . . and using (7)

$$(\lambda(1-z^k) + \alpha + k\mu) R_j(z) = k\mu R_{j+1}(z) + \gamma S_j(z) + \beta P_{2,j,0}.$$

Multiplication of this equation by y^j and adding over j = 1, 2, 3 . . . yields

$$[\lambda y(1-z^k) + \alpha y + k\mu(y-1)] G_3(z,y) = \gamma y G_4(z,y) + \beta y G_2(y) - k\mu y R_1(z) \tag{14}$$

Multiplication of equation (10) by z^i and adding over i = k, 2k, 3k . . . and using (9)

$$(\lambda(1-z^k) + \gamma) S_j(z) = \alpha R_j(z).$$

Multiplication of this equation by y^j and adding over j = 1, 2, 3 . . . yields

$$G_4(z,y) = \frac{\alpha}{\left[\lambda(1-z^k)+\gamma\right]} G_3(z,y). \tag{15}$$

The total probability generating function G (z, y) is given by

$$G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z,y) + G_4(z,y).$$

The normalizing condition is

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1,1) + G_4(1,1) = 1. \tag{16}$$

From equations (11) to (15)

$$G_0(1) = N P_{0,0,0}, \tag{17}$$

$$G_1(1) = (\lambda/\theta) P_{0,0,0}, \tag{18}$$

$$G_2(1) = k\mu R_1(1)/\beta, \tag{19}$$

$$G_3(1,1) = \frac{[\theta\beta G_1'(1) + \lambda\mu k^2 R_1(1)]\gamma}{k\left[\mu\gamma - \lambda(\alpha+\gamma)\beta\right]}, \tag{20}$$

and $G_4(1,1) = (\alpha/\gamma) G_3(1,1)$, (21)

where $P_{0,0,0} = \left[1 - \frac{\lambda}{\mu}\left(1+\frac{\alpha}{\gamma}\right) - \frac{\lambda}{\beta}\right]\frac{\theta}{(\lambda+N\theta)}$. (22)

Normalizing condition (16) yields

$$R_1(1) = \lambda / k\mu .$$

Substituting the value of $R_1(1)$ in (19), (20) and (21) yields

$$G_2(1) = \lambda / \beta, \quad G_3(1,1) = \lambda / \mu \quad \text{and} \quad G_4(1,1) = (\alpha/\gamma)(\lambda/\mu).$$

Under steady state conditions, let P_0, P_1, P_2, P_3 and P_4 be the probabilities that the server is in vacation, in startup, in batch service, in individual service and breakdown states respectively. Then,

$$P_0 = G_0(1) = N P_{0,0,0}, \quad P_1 = G_1(1) = (\lambda/\theta) P_{0,0,0}, \quad P_2 = G_2(1) = \lambda/\beta,$$

$$P_3 = G_3(1,1) = \lambda/\mu \quad \text{and} \quad P_4 = G_4(1,1) = (\alpha/\gamma)(\lambda/\mu).$$

3.1 Expected number of customers in the system:

Let L_0, L_1, L_2, L_3 and L_4 be the expected number of service phases in the system when the server is in vacation, in startup, in batch service, in individual service and breakdown states respectively.

$$\text{Then } L_0 = \sum_{i=0}^{(N-1)k} i P_{0,i,0} = G_0^1(1) = \frac{N(N-1)k}{2} P_{0,0,0}. \tag{23}$$

$$L_1 = \sum_{i=Nk}^{\infty} i P_{1,i,0} = G_1^1(1) = \frac{\lambda(\lambda+N\theta)k}{\theta^2} P_{0,0,0}. \tag{24}$$

$$L_2 = \sum_{i=k}^{\infty} i P_{2,i,0} = G_2^1(1) = \lambda k/\beta. \tag{25}$$

$$L_3 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} (i+j) P_{3,i,j} = G_3^1(1,1). \tag{26}$$

$$= \rho \left[1 + \frac{\rho_1}{1-\rho_1}\left(\frac{k+1}{2}\right) + \frac{\lambda\alpha k\rho}{\gamma^2(1-\rho_1)} + \frac{\lambda k}{\beta(1-\rho_1)} + \frac{\lambda(k-1)}{2\beta(1-\rho_1)} + \frac{1}{1-\rho_1}\left\{ \frac{\lambda(\lambda+N\theta)k}{\theta^2} + \frac{N(Nk-1)}{2} + \frac{\lambda}{\theta}\left(\frac{k-1}{2}\right) \right\} P_{0,0,0} \right]. \tag{27}$$

$$L_4 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} (i+j) P_{4,i,j} = G_4^1(1,1) = \frac{\alpha}{\gamma}\left[G_3^1(1,1) + \frac{\lambda k\rho}{\gamma} \right]. \tag{28}$$

The expected number of service phases in the system

$$L(P) = L_0 + L_1 + L_2 + L_3 + L_4$$

$$= \frac{N(N-1)k}{2} P_{0,0,0} + \frac{N(Nk-1)\rho_1}{2(1-\rho_1)} P_{0,0,0} + \frac{\lambda(\lambda+N\theta)k}{\theta^2(1-\rho_1)} P_{0,0,0} + \rho_1\left[1 + \frac{\rho_1}{1-\rho_1}\left(\frac{k+1}{2}\right)\right] + \frac{\lambda k}{\beta(1-\rho_1)} + \frac{\lambda\rho\alpha k}{\gamma^2(1-\rho_1)} + \frac{\lambda k(k-1)\rho_1}{2\beta(1-\rho_1)} + \frac{\lambda(k-1)}{2\theta(1-\rho_1)} P_{0,0,0} \tag{29}$$

where $\rho = \lambda/\mu$ and $\rho_1 = \lambda/\mu\,(1+\alpha/\gamma)$.

Expected number of customers in the system is given by

$$L(N) = \frac{1}{k}\left[L(P) - \left(\frac{k+1}{2}\right)\left(\frac{\lambda}{\mu}\left(1+\frac{\alpha}{\gamma}\right)+\frac{\lambda}{\beta}\right)\right] + \left(\frac{\lambda}{\mu}\left(1+\frac{\alpha}{\gamma}\right)+\frac{\lambda}{\beta}\right). \tag{30}$$

3.2. Characteristic features of the system:

Let E_0, E_1, E_2, E_3 and E_4 denote the expected length of vacation period, startup period, batch service period, individual service period and breakdown period respectively. Then the expected length of a cycle is given by

$$E_C = E_0 + E_1 + E_2 + E_3 + E_4 \tag{31}$$

The long run fractions of time the server is in vacation, in startup, in batch service, in individual service and breakdown states are respectively given by

$$E_0 / E_C = P_0 = N\, P_{0,0,0}, \tag{32}$$

$$E_1 / E_C = P_1 = (\lambda/\theta)\, P_{0,0,0}, \tag{33}$$

$$E_2 / E_C = P_2 = \lambda/\beta, \tag{34}$$

$$E_3 / E_C = P_3 = \rho, \tag{35}$$

and

$$E_4 / E_C = P_4 = \rho(\alpha/\gamma). \tag{36}$$

Expected length of vacation period $E_0 = N / \lambda$.

Substituting this in equation (31)

$$E_C = \frac{(N+\lambda/\theta)}{\lambda(1-\rho_1-\lambda/\beta)}. \tag{37}$$

4. OPTIMAL CONTROL POLICY:

We develop a steady state total expected cost function per unit time for the N-policy two-phase M/E$_k$/1 queueing system with server startup and breakdowns, in which N is a decision variable. With the cost structure being constructed, the objective is to determine the optimal operating N-policy so as to minimize this function.

Let
$C_h \equiv$ holding cost per unit time for each customer present in the system,
$C_o \equiv$ cost per unit time for keeping the server on and in operation,
$C_m \equiv$ startup cost per unit time per cycle,
$C_s \equiv$ setup cost per cycle,
$C_b \equiv$ breakdown cost per unit time,
$C_r \equiv$ reward per unit time for the server being on vacation.

The total expected cost function per unit time is given by

$$T(N) = C_h L(N) + C_0\left(\frac{E_2+E_3}{E_C}\right) + C_m\left(\frac{E_1}{E_C}\right) + C_b\left(\frac{E_4}{E_C}\right) + C_s\left(\frac{1}{E_C}\right) - C_r\left(\frac{E_0}{E_C}\right) \tag{38}$$

From (34) to (36), it is observed that E_2 / E_C, E_3 / E_C and E_4 / E_C are not a function of decision variable N. Hence for the determination of the optimal operating N-policy, minimizing T(N) in (38) is equivalent to minimizing

$$T_1(N) = C_h L(N) + C_m\left(\frac{E_1}{E_C}\right) + C_s\left(\frac{1}{E_C}\right) - C_r\left(\frac{E_0}{E_C}\right) \tag{39}$$

Differentiating T_1 (N) with respect to N and setting the result to zero, we obtain the optimal value N^* of N.

Hence, $$N^* = \left[\sigma^2 + \frac{\sigma}{k}\big((2k-1)-(k-1)\rho_1\big) + \frac{2\sigma(1-\rho_1)}{C_h}\,(C_m + \theta C_s + C_r)\right]^{1/2} - \sigma\ , \qquad (40)$$

where $\sigma = \lambda/\theta$ (mean number of arrivals during startup times).

5. SENSITIVITY ANALYSIS:

In the course of analysis, sensitivity analysis has been carried out to find the optimum value of N (i.e., N*), expected system length and minimum cost based on changes in the system parameters. In order to achieve this, the system parameters are classified into two categories. The non-monetary parameters ($\lambda, \mu, \beta, \theta, k, \alpha$ and γ) and the monetary parameters (C_h, C_o, C_b, C_m, C_r and C_s).

In order to arrive at the conclusions, the following arbitrary values of the system parameters are considered.

λ =1.0, μ = 10, β = 5, θ =3, $\gamma = 3$, $k = 2$, C_h =10, C_o =200, C_m = 200, C_b = 100, C_s = 1000, C_r = 50.

Using equations (30), (38) and (40), the values of N^*, L (N^*) and T (N^*) have been tabulated.

Case i: Effect of the non-monetary parameters

It is noticed from the results in Table 1 that as λ increases, the value of N^* and expected cost increases. Further, it can be seen that length of the queue is clearly convex and it is a function of λ.

Table 1: Effect of λ on N^*, expected system length and expected cost.

λ	0.5	1.0	1.5	2.0	2.5
N^*	9	13	16	17	19
L(N^*)	3.8483	5.1916	5.6177	4.9880	4.1736
T(N^*)	76.51	134.25	173.36	199.30	213.60

Computed values in Table 2 show that N* increases for smaller values of μ and does not change for larger values of μ, however the expected cost decreases and queue length increases.

Table 2: Effect of μ on N*, expected system length and expected cost.

μ	2	4	6	8	10	12
N*	9	12	13	13	13	13
L(N*)	3.8874	4.8040	5.1836	5.1891	5.1916	5.1930
T(N*)	190.83	159.89	145.73	138.66	136.25	131.37

It can be observed from table 3 that N* is unaltered and the expected cost decreases and queue length increases with increase in β.

Table 3: Effect of β on N*, expected system length and expected cost.

β	2	4	6	8	10	12
N*	13	13	13	13	13	13
L(N*)	3.5259	4.9148	5.3767	5.6080	5.7468	5.8394
T(N*)	168.22	139.91	130.40	125.76	122.93	121.04

Computed values in Tables 4 and 5 indicate that with increase in the values of θ and γ, not much of significant variation is noticed in the value of N^*. However, the expected cost and expected system length decreases.

Table 4. Effect of θ on N*, expected system length and expected cost.

θ	1	3	5	7	9
N*	13	13	13	13	13
L(N*)	5.8772	5.1916	5.1340	5.1096	5.0961
T(N*)	142.55	134.25	132.49	131.73	131.38

Table 5: Effect of γ on N*, expected system length and expected cost.

γ	1	3	5	7	9
N*	13	13	13	13	13
L(N*)	5.2103	5.1916	5.1903	5.1900	5.1899
T(N*)	135.35	134.25	134.05	133.97	133.93

Computed values in Tables 6 and 7 show that with increase in the values of α and k, no variation is noticed in the value of N^*. However, the expected cost and expected system length increase slightly.

Table 6: Effect of α on N*, expected system length and expected cost.

α	0.1	0.3	0.5	0.7	0.9
N*	13	13	13	13	13
L(N*)	5.1907	5.1925	5.1943	5.1968	5.1978
T(N*)	134.01	134.49	134.96	135.44	135.92

Table 7: Effect of *k* on N*, expected system length and expected cost.

k	1	2	3	4	5
N*	13	13	13	13	13
L(N*)	5.1440	5.1916	5.2072	5.2150	5.2197
T(N*)	133.78	134.25	134.41	134.48	134.53

Case ii: Effect of the monetary parameters

Computed values in Table 8 show that with increase in the values of C_h, the value of N^* and expected system length decreases. Further, the expected cost increases.

Table 8: Effect of C_h on N*, expected system length and expected cost.

C_h	5	10	15	20	25
N*	19	13	11	9	8
L(N*)	7.5759	5.1916	4.4178	3.6449	3.2591
T(N*)	102.43	134.25	158.54	179.37	197.51

Data in Tables 9 and 10 suggest that with increase in the values of C_o and C_b, there is no variation in the value of N^* and expected system length. Further, the expected cost increases.

Table 9: Effect of C_0 on N*, expected system length and expected cost.

C_0	100	200	300	500	1000
N*	13	13	13	13	13
L(N*)	5.1916	5.1916	5.1916	5.1916	5.1916
T(N*)	104.25	134.25	164.25	224.25	374.25

Table 10: Effect of C_b on N*, expected system length and expected cost.

C_b	100	200	300	500	1000
N*	13	13	13	13	13
L(N*)	5.1916	5.1916	5.1916	5.1916	5.1916
T(N*)	134.25	134.92	135.50	136.92	140.25

Computed values in Table 11 indicate that with increase in the values of C_m, not much of significant variation is noticed in the value of N^* and expected system length, where as the expected cost increases.

Table 11: Effect of C_m on N*, expected system length and expected cost.

C_m	100	200	300	500	1000
N*	13	13	13	14	15
L(N*)	5.1916	5.1916	5.1916	5.5783	5.9660
T(N*)	132.52	134.25	135.98	139.03	146.70

It is noticed from the results in Table 12 that as C_r increases, there is no change in the value of N^* and expected system length. However, the expected cost decreases.

Table 12: Effect of C_r on N*, expected system length and expected cost.

C_r	40	50	70	90	100
N*	13	13	13	13	13
T(N*)	5.1916	5.1916	5.1916	5.1916	5.1916
L(N*)	141.01	134.25	120.73	107.21	100.45

It is noticed from the results in Table 13 that as C_s increases, the value of N^*, the expected system length and expected cost increase.

Table 13: Effect of C_s on N*, expected system length and expected cost.

C_s	500	1000	1500	2000	2500
N*	9	13	16	18	21
L(N*)	3.6321	5.1916	6.3421	7.1225	8.1862
T(N*)	105.78	134.25	156.74	176.07	192.07

7. CONCLUSIONS:

Optimal strategy analysis of N-policy two-phase $M/E_k/1$ queuing system with startup time and server breakdowns is studied. System performance measures are derived. A cost function is formulated to determine the optimal value of N. Sensitivity analysis is carried out through numerical illustrations. These numerical values will be useful in analyzing practical queueing systems and make decisions.

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