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On αg-Separation Axioms

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Abstract

In this paper by using αg-open sets we define almost αg-normality and mild αg-normality also we continue the study of further properties of αg-normality. We show that these three axioms are regular open hereditary. We also define the class of almost αg-irresolute mappings and show that αg-normality is invariant under almost αg-irresolute M-αg-open continuous surjection.

***AMS Subject Classification**: 54D15, 54D10.*

***Key words and Phrases**: αg-open, semiopen, semipreopen, almost normal, midly normal, M-αg-closed, M-αg-open , rc-continuous.*

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1. Introduction

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P. Aruna Swathi Vyjayanthi studied *v*-Normal Almost- *v*-Normal, Mildly-*v*-Normal and *v*-US spaces. Inspired with these we introduce αg-Normal Almost- αg-Normal, Mildly- αg-Normal, αg-US, αg-S_1 and αg-S_2. Also we examine αg-convergence, sequentially αg-compact, sequentially αg-continuous maps, and sequentially sub αg-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. Preliminaries

Definition 2.1: $A\subset X$ is called
(i) g-closed if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(ii) αg-closed if $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is α-open in X.

Definition 2.2: A function f is said to be almost–pre-irresolute if for each x in X and each pre-neighborhood V of f(x), $pcl(f^{-1}(V))$ is a pre-neighborhood of x.

Definition 2.3: A space X is said to be
(i) T_1 (T_2) if for any $x \neq y$ in X, there exist (disjoint) open sets U; V in X such that $x\in U$ and $y\in V$.
(ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X.
(iii) normal[resp: mildly normal] if for any pair of disjoint [resp: regular-closed]closed sets F_1 and F_2, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.
(iv) almost normal if for each closed set A and each regular closed set B such that $A\cap B = \phi$, there exist disjoint open sets U and V such that $A\subset U$ and $B\subset V$.
(v) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \phi$, there exist disjoint open sets U and V such that $x \in U$ and $A\subset V$.

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(vi) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A.
(vii) R_0 if for any point x and a closed set F with $x \notin F$ in X, there exists a open set G containing F but not x.
(viii) R_1 iff for x, y $\in$ X with cl{x} $\neq$ cl{y}, there exist disjoint open sets U and V such that cl{x}$\subset$ U, cl{y}$\subset$V.
(ix) US-space if every convergent sequence has exactly one limit point to which it converges.
(x) pre-US space if every pre-convergent sequence has exactly one limit point to which it converges.
(xi) pre-S_1 if it is pre-US and every sequence $<x_n>$ pre-converges with subsequence of $<x_n>$ pre-side points.
(xii) pre-S_2 if it is pre-US and every sequence $<x_n>$ in X pre-converges which has no pre-side point.
(xiii) is weakly countable compact if every infinite subset of X has a limit point in X.
(xiv) Baire space if for any countable collection of closed sets with empty interior in X, their union also has empty interior in X.

Definition 2.4: Let $A\subset X$. Then a point x is said to be a
(i) limit point of *A* if each open set containing x contains some point y of *A* such that x $\neq$ y.
(ii) T_0–limit point of *A* if each open set containing x contains some point y of *A* such that cl{x} $\neq$ cl{y}, or equivalently, such that they are topologically distinct.
(iii) *pre*-T_0–limit point of *A* if each open set containing x contains some point y of *A* such that *p*cl{x} $\neq$ *p*cl{y}, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0–axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let X = {a, b, c, d} and τ = {{a}, {b, c}, {a, b, c}, X, ϕ}. Then b and c are the limit points but not the T_0–limit points of the set {b, c}. Further d is a T_0–limit point of {b, c}.

Example 2: Let X = (0, 1) and τ = {ϕ, X, and U_n = (0, 1–1⁄n), n = 2, 3, 4,. . . }. Then every point of X is a limit point of X. Every point of X~U_2 is a T_0–limit point of X, but no point of U_2 is a T_0–limit point of X.

Definition 2.5: A set *A* together with all its T_0–limit points will be denoted by T_0–cl*A*.

Note 2: i. Every T_0–limit point of a set *A* is a limit point of the set but the converse is not true in general.
ii. In T_0–space both are same.

Note 3: R_0–axiom is weaker than T_1–axiom. It is independent of the T_0–axiom. However $T_1 = R_0+T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1–space is weakly countable compact iff it is countable compact.

3. αg-T_0 LIMIT POINT:

Definition 3.01: In X, a point x is said to be a αg-T_0–limit point of *A* if each αg-open set containing x contains some point y of *A* such that *αg*cl{x} $\neq$ *αg*cl{y}, or equivalently; such that they are topologically distinct with respect to αg-open sets.

Example 3: *regular open set ⇒ open set ⇒ α-open set ⇒ αg-open set we have*
r-T_0–limit point ⇒ T_0–limit point ⇒ α-T_0–limit point ⇒ αg-T_0–limit point

Definition 3.02: A set A together with all its αg-T_0–limit points is denoted by T_0-*αg*cl(A)

Lemma 3.01: *If x is a αg-T_0–limit point of a set A then x is αg-limit point of A.*

Lemma 3.02: *If X is αg-T_0–space then every αg-T_0–limit point and every αg-limit point are equivalent.*

Corollary 3.03: *If X is r-T_0–space then every αg-T_0–limit point and every αg-limit point are equivalent.*

Theorem 3.04: *For $x \neq y \in X$,*
(i) *x is a αg-T_0–limit point of {y} iff $x\notin$αgcl{y} and $y\in$αgcl{x}.*
(ii) *x is not a αg-T_0–limit point of {y} iff either $x\in$αgcl{y}or αgcl{x} = αgcl{y}.*
(iii) *x is not a αg-T_0–limit point of {y} iff either $x\in$αgcl{y}or $y\in$αgcl{x}.*

Corollary 3.05:

(i) *If x is a αg-T_0–limit point of {y}, then y cannot be a αg-limit point of {x}.*

(ii) *If $\alpha gcl\{x\} = \alpha gcl\{y\}$, then neither x is a αg-T_0–limit point of {y} nor y is a αg-T_0–limit point of {x}.*

(iii) *If a singleton set A has no αg-T_0–limit point in X, then $\alpha gclA = \alpha gcl\{x\}$ for all $x \in \alpha gcl\{A\}$.*

Lemma 3.06: *In X, if x is a αg-limit point of a set A, then in each of the following cases x becomes αg-T_0–limit point of A ($\{x\} \neq A$).*

(i) $\alpha gcl\{x\} \neq \alpha gcl\{y\}$ *for* $y \in A$, $x \neq y$.

(ii) $\alpha gcl\{x\} = \{x\}$

(iii) *X is a αg-T_0–space.*

(iv) *A~{x} is αg-open*

Corollary 3.07: *In X, if x is a limit point of a set A, then in each of the following cases x becomes αg-T_0–limit point of A ($\{x\} \neq A$).*

(i) $\alpha gcl\{x\} \neq \alpha gcl\{y\}$ *for* $y \in A$, $x \neq y$.

(ii) $\alpha gcl\{x\} = \{x\}$

(iii) *X is a αg-T_0–space.*

(iv) *A~{x} is αg-open*

4. αg-T_0 AND αg-R_i AXIOMS, i = 0, 1:

In view of Lemma 3.6(iii), αg-T_0–axiom implies the equivalence of the concept of limit point of a set with that of αg-T_0–limit point of the set. But for the converse, if $x \in \alpha g$cl{y} then αgcl{x} $\neq$ αgcl{y} in general, but if x is a αg-T_0–limit point of {y}, then αgcl{x} = αgcl{y}

Lemma 4.01: *In a space X, a limit point x of {y} is a αg-T_0–limit point of {y} iff $\alpha gcl\{x\} \neq \alpha gcl\{y\}$.*

This lemma leads to characterize the equivalence of αg-T_0–limit point and αg-limit point of a set as the αg-T_0–axiom.

Theorem 4.02: *The following conditions are equivalent:*

(i) *X is a αg-T_0 space*

(ii) *Every αg-limit point of a set A is a αg-T_0–limit point of A*

(iii) *Every r-limit point of a singleton set {x} is a αg-T_0–limit point of {x}*

(iv) *For any x, y in X, $x \neq y$ if $x \in \alpha gcl\{y\}$, then x is a αg-T_0–limit point of {y}*

Note 5: In a αg-T_0–space X if every point of X is a r-limit point of X, then every point of X is αg-T_0–limit point of X. But a space X in which each point is a αg-T_0–limit point of X is not necessarily a αg-T_0–space

Theorem 4.03: *The following conditions are equivalent:*

(i) *X is a αg-R_0 space*

(ii) *For any x, y in X, if $x \in \alpha gcl\{y\}$, then x is not a αg-T_0–limit point of {y}*

(iii) *A point αg-closure set has no αg-T_0–limit point in X*

(iv) *A singleton set has no αg-T_0–limit point in X.*

Since every r-R_0–space is αg-R_0–space, we have the following corollary

Corollary 4.04: *The following conditions are equivalent:*

(i) *X is a r-R_0 space*

(ii) *For any x, y in X, if $x \in \alpha gcl\{y\}$, then x is not a αg-T_0–limit point of {y}*

(iii) *A point αg-closure set has no αg-T_0–limit point in X*

(iv) *A singleton set has no αg-T_0–limit point in X.*

Theorem 4.05: *In a αg-R_0 space X, a point x is αg-T_0–limit point of A iff every αg-open set containing x contains infinitely many points of A with each of which x is topologically distinct*

If αg-R_0 space is replaced by rR_0 space in the above theorem, we have the following corollaries:

Corollary 4.06: *In an rR_0–space X,*

(i) *If a point x is rT_0–limit point of a set then every αg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.*

(ii) *If a point x is αg-T_0–limit point of a set then every αg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.*

Theorem 4.07: *X is* αg-R_0 *space iff a set A of the form* $A = \cup\, \alpha gcl\{x_{i\ i=1\ to\ n}\}$ *a finite union of point closure sets has no* αg-T_0*–limit point.*

Corollary 4.08: *If X is* rR_0 *space and*

(i)If $A = \cup\, \alpha gcl\{x_i\}$ *i =1 to n, a finite union of point closure sets has no* αg-T_0*–limit point.*

(ii)If $X = \cup\, \alpha gcl\{x_i\}$ *i =1 to n ,then X has no* αg-T_0*–limit point.*

Theorem 4.09: *The following conditions are equivalent:*

(i) *X is* αg-R_0*–space*
(ii) *For any x and a set in X, x is a* αg-T_0*–limit point of A iff every* αg-*open set containing x contains infinitely many points of A with each of which x is topologically distinct.*

Various characteristic properties of αg-T_0–limit points studied so far is enlisted in the following theorem for a ready reference.

Theorem 4.10: *In a* αg-R_0*–space, we have the following:*

(i) *A singleton set has no* αg-T_0*–limit point in X.*
(ii) *A finite set has no* αg-T_0*–limit point in X.*
(iii) *A point* αg-*closure has no set* αg-T_0*–limit point in X*
(iv) *A finite union point* αg-*closure sets have no set* αg-T_0*–limit point in X.*
(v) *For* $x, y \in X$, $x \in T_0\text{–}\ \alpha gcl\{y\}$ *iff* $x = y$.
(vi) *For any* $x, y \in X$, $x \neq y$ *iff neither x is* αg-T_0*–limit point of {y}nor y is* αg-T_0*–limit point of {x}*
(vii) *For any* $x, y \in X$, $x \neq y$ *iff* $T_0\text{–}\ \alpha gcl\{x\} \cap T_0\text{–}\ \alpha gcl\{y\} = \phi$.
(viii)*Any point* $x \in X$ *is a* αg-T_0*–limit point of a set A in X iff every* αg-*open set containing x contains infinitely many points of A with each which x is topologically distinct.*

Theorem 4.11: *X is* αg-R_1 *iff for any* αg-*open set U in X and points x, y such that* $x \in X\sim U$, $y \in U$, *there exists a* αg-*open set V in X such that* $y \in V \subset U$, $x \notin V$.

Lemma 4.12: *In* αg-R_1 *space X, if x is a* αg-T_0*–limit point of X, then for any non empty* αg-*open set U, there exists a non empty* αg-*open set V such that* $V \subset U$, $x \notin \alpha gcl(V)$.

Lemma 4.13: *In a* αg- *regular space X, if x is a* αg-T_0*–limit point of X, then for any non empty* αg-*open set U, there exists a non empty* αg-*open set V such that* $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.

Corollary 4.14: *In a regular space X,*

(i) *If x is a* αg-T_0*–limit point of X, then for any non empty* αg-*open set U, there exists a non empty* αg-*open set V such that* $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.
(ii) *If x is a* T_0*–limit point of X, then for any non empty* αg-*open set U, there exists a non empty* αg-*open set V such that* $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.

Theorem 4.15: *If X is a* αg-*compact* αg-R_1*-space, then X is a Baire Space.*

Proof: Let $\{A_n\}$ be a countable collection of αg-closed sets of X, each A_n having empty interior in X. Take A_1, since A_1 has empty interior, A_1 does not contain any αg-open set say U_0. Therefore we can choose a point $y \in U_0$ such that $y \notin A_1$. For X is αg-regular, and $y \in (X\sim A_1) \cap U_0$, a αg-open set, we can find a αg-open set U_1 in X such that $y \in U_1$, $\alpha gcl(U_1) \subset (X\sim A_1) \cap U_0$. Hence U_1 is a non empty αg-open set in X such that $\alpha gcl(U_1) \subset U_0$ and $vcl(U_1) \cap A_1 = \phi$. Continuing this process, in general, for given non empty αg-open set U_{n-1}, we can choose a point of U_{n-1} which is not in the αg-closed set A_n and a αg-open set U_n containing this point such that $\alpha gcl(U_n) \subset U_{n-1}$ and $\alpha gcl(U_n) \cap A_n = \phi$. Thus we get a sequence of nested non empty αg-closed sets which satisfies the finite intersection property. Therefore $\cap\, \alpha gcl(U_n) \neq \phi$. Then some $x \in \cap\, \alpha gcl(U_n)$ which in turn implies that $x \in U_{n-1}$ as $\alpha gcl(U_n) \subset U_{n-1}$ and $x \notin A_n$ for each n.

Corollary 4.16: *If X is a compact* αg-R_1*-space, then X is a Baire Space.*

Corollary 4.17: *Let X be a* αg-*compact* αg-R_1*-space. If* $\{A_n\}$ *is a countable collection of* αg-*closed sets in X, each* A_n *having non-empty* αg-*interior in X, then there is a point of X which is not in any of the* A_n.

Corollary 4.18: *Let X be a* αg-*compact* R_1*-space. If* $\{A_n\}$ *is a countable collection of* αg-*closed sets in X, each* A_n *having non-empty* αg- *interior in X, then there is a point of X which is not in any of the* A_n.

Theorem 4.19: *Let X be a non empty compact* αg-R_1*-space. If every point of X is a* αg-T_0*–limit point of X then X is uncountable.*

Proof: Since X is non empty and every point is a αg-T_0-limit point of X, X must be infinite. If X is countable, we construct a sequence of αg- open sets $\{V_n\}$ in X as follows:

Let X = V_1, then for x_1 is a αg-T_0-limit point of X, we can choose a non empty αg-open set V_2 in X such that $V_2 \subset V_1$ and $x_1 \notin \alpha g\mathrm{cl}V_2$. Next for x_2 and non empty αg-open set V_2, we can choose a non empty αg-open set V_3 in X such that $V_3 \subset V_2$ and $x_2 \notin \alpha g\mathrm{cl}V_3$. Continuing this process for each x_n and a non empty αg-open set V_n , we can choose a non empty αg-open set V_{n+1} in X such that $V_{n+1} \subset V_n$ and $x_n \notin \alpha g\mathrm{cl}V_{n+1}$.

Now consider the nested sequence of αg-closed sets $\alpha g\mathrm{cl}V_1 \supset \alpha g\mathrm{cl}V_2 \supset \alpha g\mathrm{cl}V_3 \supset \ldots\ldots\ldots \supset \alpha g\mathrm{cl}V_n \supset \ldots$ Since X is αg-compact and $\{\alpha g\mathrm{cl}V_n\}$ the sequence of αg-closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x \in \alpha g\mathrm{cl}V_n$. Further $x \in X$ and $x \in V_1$, which is not equal to any of the points of X. Hence X is uncountable.

Corollary 4.20: *Let X be a non empty* αg-*compact* αg-R_1-*space. If every point of X is a* αg-T_0–*limit point of X then X is uncountable*

5. αg –T_0-IDENTIFICATION SPACES AND αg –SEPARATION AXIOMS

Definition 5.01: Let (X, τ) be a topological space and let $\Re$ be the equivalence relation on X defined by $x\Re y$ iff $\alpha gcl\{x\} = \alpha gcl\{y\}$

Problem 5.02: show that $x\Re y$ iff $\alpha gcl\{x\} = \alpha gcl\{y\}$ is an equivalence relation

Definition 5.03: The space $(X_0, Q(X_0))$ is called the αg-T_0–identification space of (X,τ), where X_0 is the set of equivalence classes of $\Re$ and $Q(X_0)$ is the decomposition topology on X_0.

Let P_X: $(X,\tau) \to (X_0, Q(X_0))$ denote the natural map

Lemma 5.04: *If $x \in X$ and $A \subset X$, then $x \in \alpha gclA$ iff every* αg-*open set containing x intersects A.*

Theorem 5.05: *The natural map $P_X:(X,\tau) \to (X_0, Q(X_0))$ is closed, open and $P_X^{-1}(P_X(O)) = O$ for all $O \in PO(X,\tau)$ and $(X_0, Q(X_0))$ is* αg-T_0

Proof: Let $O \in PO(X,\tau)$ and let $C \in P_X(O)$. Then there exists $x \in O$ such that $P_X(x) = C$. If $y \in C$, then $\alpha gcl\{y\} = \alpha gcl\{x\}$, which, by lemma, implies $y \in O$. Since $\tau \subset PO(X,\tau)$, then $P_X^{-1}(P_X(U)) = U$ for all $U \in \tau$, which implies P_X is closed and open.

Let G, $H \in X_0$ such that $G \neq H$; let $x \in G$ and $y \in H$. Then $\alpha gcl\{x\} \neq \alpha gcl\{y\}$, which implies $x \notin \alpha gcl\{y\}$ or $y \notin \alpha gcl\{x\}$, say $x \notin \alpha gcl\{y\}$. Since P_X is continuous and open, then $G \in A = P_X\{X \sim \alpha gcl\{y\}\} \notin PO(X_0, Q(X_0))$ and $H \notin A$

Theorem 5.06: *The following are equivalent:*
(i) X is αg R_0 (ii) $X_0 = \{\alpha gcl\{x\}: x \in X\}$ and (iii) $(X_0, Q(X_0))$ is αg T_1

Proof:
(i) ⇒ (ii) Let $C \in X_0$, and let $x \in C$. If $y \in C$, then $y \in \alpha gcl\{y\} = \alpha gcl\{x\}$, which implies $C \in \alpha gcl\{x\}$. If $y \in \alpha gcl\{x\}$, then $x \in \alpha gcl\{y\}$, since, otherwise, $x \in X \sim \alpha gcl\{y\} \in PO(X,\tau)$ which implies $\alpha gcl\{x\} \subset X \sim \alpha gcl\{y\}$, which is a contradiction. Thus, if $y \in \alpha gcl\{x\}$, then $x \in \alpha gcl\{y\}$, which implies $\alpha gcl\{y\} = \alpha gcl\{x\}$ and $y \in C$. Hence $X_0 = \{\alpha gcl\{x\}: x \in X\}$

(ii)⇒(iii) Let $A \neq B \in X_0$. Then there exists x, $y \in X$ such that $A = \alpha gcl\{x\}$; $B = \alpha gcl\{y\}$, and $\alpha gcl\{x\} \cap \alpha gcl\{y\} = \phi$. Then $A \in C = P_X(X \sim \alpha gcl\{y\}) \in PO(X_0, Q(X_0))$ and $B \notin C$. Thus $(X_0, Q(X_0))$ is αg-T_1

(iii) ⇒ (i) Let $x \in U \in \alpha GO(X)$. Let $y \notin U$ and C_x, $C_y \in X_0$ containing x and y respectively. Then $x \notin \alpha gcl\{y\}$, which implies $C_x \neq C_y$ and there exists αg-open set A such that $C_x \in A$ and $C_y \notin A$. Since P_X is continuous and open, then $y \in B = P_X^{-1}(A) \in x \in \alpha g\ O(X)$ and $x \notin B$, which implies $y \notin \alpha gcl\{x\}$. Thus $\alpha gcl\{x\} \subset U$. This is true for all $\alpha gcl\{x\}$ implies $\cap \alpha gcl\{x\} \subset U$. Hence X is αg-R_0

Theorem 5.07: *(X, τ) is* αg-R_1 *iff $(X_0, Q(X_0))$ is* αg-T_2

The proof is straight forward from using theorems 5.05 and 5.06 and is omitted

Theorem 5.08: *X is* αg-T_i; *i = 0, 1, 2. iff there exists a* αg-*continuous, almost–open, 1–1 function from (X, τ) into a* αg-T_i *space ; i = 0,1,2. respectively.*

Proof: If X is αg-T_i; i = 0,1,2., then the identity function on X satisfies the desired properties. The converse is (ii) part of Theorem 2.13.

The following example shows that if f: $(X, \tau) \rightarrow (Y, \sigma)$ is continuous, αg-open, bijective, $A \in PO(Y, \sigma)$, and (Y, σ) αg-T_i ; i = 0,1,2, then f^{-1} (A) need not be αg –open and (X, τ) need not be αg-T_i ; i = 0,1,2

Theorem 5.09: *If f: $(X, \tau) \rightarrow (Y, \sigma)$ is αg-continuous, αg-open, and $x, y \in X$ such that $\alpha gcl\{x\} = \alpha gcl\{y\}$, then $\alpha gcl\{f(x)\} = \alpha gcl\{f(y)\}$.*

Theorem 5.10: *The following are equivalent*

(i) (X, τ) is αg-T_0
(ii) Elements of X_0 are singleton sets and
(iii)There exists a αg-continuous, αg-open, 1–1 function f: $(X, \tau) \rightarrow (Y, \sigma)$, where (Y, σ) is αg-T_0

Proof: (i) is equivalent to (ii) and (i) $\Rightarrow$ (iii) are straight forward and is omitted.

(iii) $\Rightarrow$ (i) Let $x, y \in X$ such that $f(x) \neq f(y)$, which implies $\alpha gcl\{f(x)\} \neq \alpha gcl\{f(y)\}$. Then by theorem 5.09, $\alpha gcl\{x\} \neq \alpha gcl\{y\}$. Hence (X, τ) is αg-T_0

Corollary 5.11: *A space (X, τ) is αg-T_i ; i = 1,2 iff (X, τ) is αg-T_{i-1} ; i = 1,2, respectively, and there exists a αg-continuous , αg-open, 1–1 function f: (X, τ) into a αg-T_0 space.*

Definition 5.04: $f: X \rightarrow Y$ is point–αg-closure 1–1 iff for $x, y \in X$ such that αgcl$\{x\} \neq \alpha g$cl$\{y\}$, αgcl$\{f(x)\} \neq \alpha g$cl$\{f(y)\}$.

Theorem 5.12:
(i)If f: $(X, \tau) \rightarrow (Y, \sigma)$ is point– αg-closure 1–1 and (X, τ) is αg-T_0, then f is 1–1
(ii)If f: $(X, \tau) \rightarrow (Y, \sigma)$, where $(X, \tau)$and (Y, σ) are αg-T_0 then f is point– αg-closure 1–1 iff f is 1–1

Proof: omitted

The following result can be obtained by combining results for αg-T_0– identification spaces, αg-induced functions and αg-T_i spaces; i = 1,2.

Theorem 5.13: *X is αg-R_i ; i = 0,1 iff there exists a αg-continuous , almost–open point– αg-closure 1–1 function f: (X, τ) into a αg-R_i space; i = 0,1 respectively.*

6. αg-Normal; Almost αg-normal and Mildly αg-normal spaces

Definition 6.1: A space X is said to be αg-normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint αg-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 4: Let X = {a, b, c} and $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Then X is αg-normal.

Example 5: Let X = {a, b, c, d} and $\tau = \{\phi,\{b, d\},\{a, b, d\},\{b, c, d\}, X\}$. Then X is not αg-normal and is not normal.

We have the following characterization of αg-normality.

Theorem 6.1: For a space X the following are equivalent:

(i) X is αg-normal.
(ii) For every pair of open sets U and V whose union is X, there exist αg-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.
(iii) For every closed set F and every open set G containing F, there exists a αg-open set U such that $F \subset U \subset \alpha gcl(U) \subset G$.

Proof: (i)$\Rightarrow$(ii): Let U and V be a pair of open sets in a αg-normal space X such that $X = U \cup V$. Then $X–U$, $X–V$ are disjoint closed sets. Since X is αg-normal there exist disjoint αg-open sets U_1 and V_1 such that $X–U \subset U_1$ and $X-V \subset V_1$. Let $A = X–U_1$, $B = X–V_1$. Then A and B are αg-closed sets such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(b) $\Rightarrow$(c): Let F be a closed set and G be an open set containing F. Then $X–F$ and G are open sets whose union is X.

Then by (b), there exist αg-closed sets W_1 and W_2 such that $W_1 \subset X–F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X–W_1$, $X–G \subset X–W_2$ and $(X–W_1) \cap (X–W_2) = \phi$. Let $U = X–W_1$ and $V = X–W_2$. Then U and V are disjoint αg-open sets such that $F \subset U \subset X–V \subset G$. As $X–V$ is αg-closed set, we have $\alpha gcl(U) \subset X–V$ and $F \subset U \subset \alpha gcl(U) \subset G$.

(c) ⇒ (a): Let F_1 and F_2 be any two disjoint closed sets of X. Put $G = X–F_2$, then $F_1 \cap G = \phi$. $F_1 \subset G$ where G is an open set. Then by (c), there exists a αg-open set U of X such that $F_1 \subset U \subset \alpha gcl(U) \subset G$. It follows that $F_2 \subset X–\alpha gcl(U) = V$, say, then V is αg-open and $U \cap V = \phi$. Hence F_1 and F_2 are separated by αg-open sets U and V. Therefore X is αg-normal.

Theorem 6.2: A regular open subspace of a αg-normal space is αg-normal.

Proof: Let Y be a regular open subspace of a αg-normal space X. Let A and B be disjoint closed subsets of Y. As Y is regular open, A, B are closed sets of X. By αg-normality of X, there exist disjoint αg-open sets U and V in X such that A ⊂U and B⊂V, U∩Y and V∩Y are αg-open in Y such that A⊂U∩Y and B⊂V∩Y. Hence Y is αg-normal.

Example 6: Let X = {a, b, c} with τ = {ϕ, {a}, {b}, {a, b}, X} is αg-normal and αg-regular.

However we observe that every αg-normal αg-R_0 space is αg-regular.
Now, we define the following.

Definition 6.2: A function f: X → Y is said to be almost–αg-irresolute if for each x in X and each αg-neighborhood V of f(x), αgcl(f^{-1}(V)) is a αg-neighborhood of x.

Clearly every αg-irresolute map is almost αg-irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 6.1: f is almost αg-irresolute iff $f^{-1}(V) \subset$ αg-int($\alpha gcl(f^{-1}(V))$) for every V∈αgO(Y).

Now we prove the following.

Lemma 6.2: f is almost αg-irresolute iff $f(\alpha gcl(U)) \subset \alpha gcl(f(U))$ for every U∈$\alpha g\, O$(X).

Proof: Let U∈$\alpha g\, O$(X).Suppose y∉ $\alpha gcl(f(U))$. Then there exists V∈ αgO(y) such that V∩f(U) = ϕ. Hence f^{-1}(V)∩U= ϕ. Since U∈αgO(X), we have αg-int($\alpha gcl(f^{-1}(V))$) ∩ αgcl(U) = ϕ. Then by lemma 6.1, f^{-1}(V)∩ αgcl(U) = ϕ and hence V∩$f(\alpha gcl(U))$ = ϕ. This implies that y∉$f(\alpha g$cl(U)).

Conversely, if V∈$\alpha g\, O$(Y), then W = X- $\alpha gcl(f^{-1}(V))$∈ $\alpha g\, O$(X). By hypothesis, $f(\alpha gcl(W)) \subset \alpha gcl(f(W))$ and hence X- αg-int($\alpha gcl(f^{-1}(V))$) = $\alpha gcl(W) \subset f^{-1}(\alpha gcl(f(W))) \subset f(\alpha gcl[f(X-f^{-1}(V))]) \subset f^{-1}[\alpha gcl(Y-V)] = f^{-1}(Y-V) = X-f^{-1}(V)$.
Therefore, $f^{-1}(V) \subset$ αg-int($\alpha gcl(f^{-1}(V))$). By lemma 6.1, f is almost αg-irresolute.

Now we prove the following result on the invariance of αg-normality.

Theorem 6.3: If f is an M-αg-open continuous almost αg-irresolute function from a αg-normal space X onto a space Y, then Y is αg-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f, f^{-1}(A) is closed and f^{-1}(B) is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is αg-normal, there exists a αg-open set U in X such that $f^{-1}(A) \subset U \subset \alpha gcl(U) \subset f^{-1}(B)$. Then $f(f^{-1}(A)) \subset f(U) \subset f(\alpha gcl(U)) \subset f(f^{-1}(B))$. Since f is M-αg-open almost αg-irresolute surjection, we obtain A⊂ $f(U) \subset \alpha gcl(f(U)) \subset$ B. Then again by Theorem 6.1 the space Y is αg-normal.

Lemma 6.3: A mapping f is M-αg-closed if and only if for each subset B in Y and for each αg-open set U in X containing f^{-1}(B), there exists a αg-open set V containing B such that $f^{-1}(V) \subset U$.

Now we prove the following:

Theorem 6.4: If f is an M-αg-closed continuous function from a αg-normal space onto a space Y, then Y is αg-normal.

Proof of the theorem is routine and hence omitted.

Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

Theorem 6.5: If f is an M-αg-closed map from a weakly Hausdorff αg-normal space X onto a space Y such that $f^{-1}(y)$ is S-closed relative to X for each $y\in Y$, then Y is αg-T_2.

Proof: Let y_1 and y_2 be any two distinct points of Y. Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is αg-normal, there exist disjoint αg-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for i = 1, 2. Since f is M-αg-closed, there exist αg-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$
for i = 1, 2. Then it follows that $U_1\cap U_2 = \phi$. Hence Y is αg-T_2.

Theorem 6.6: For a space *X* we have the following:
(a) If *X* is normal then for any disjoint closed sets A and B, there exist disjoint αg-open sets U, V such that $A \subset U$ and $B \subset V$;
(b) If *X* is normal then for any closed set A and any open set V containing A, there exists an αg-open set U of X such that $A\subset U\subset \alpha g\mathrm{cl}(U) \subset V$.

Definition 6.2: X is said to be almost αg-normal if for each closed set A and each regular closed set B such that $A\cap B = \phi$, there exist disjoint αg-open sets U and V such that $A\subset U$ and $B\subset V$.

Clearly, every αg-normal space is almost αg-normal, but not conversely in general.

Example 7: Let X = {a, b, c} and τ = {ϕ,{a}, {a, b}, {a, c}, X}.Then X is almost αg-normal and not αg-normal.

Now, we have characterization of almost αg-normality in the following.

Theorem 6.7: For a space X the following statements are equivalent:
(i) X is almost αg-normal
(ii) For every pair of sets U and V , one of which is open and the other is regular open whose union is X, there exist αg-closed sets G and H such that $G\subset U$,$H\subset V$ and $G\cup H = X$.
(iii) For every closed set A and every regular open set B containing A, there is a αg-open set V such that $A \subset V\subset \alpha gcl(V) \subset B$.

Proof:
(a)⇒(b) Let U be an open set and V be a regular open set in an almost αg-normal space X such that $U\cup V = X$.
Then (X-U) is closed set and (X-V) is regular closed set with $(X-U)\cap(X-V) = \phi$. By almost αg-normality of X, there exist disjoint αg-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$. Let $G = X- U_1$ and $H = X-V_1$. Then G and H are αg-closed sets such that $G\subset U$, $H\subset V$ and $G\cup H = X$.

(b) ⇒ (c) and **(c) ⇒ (a)** are obvious.

One can prove that almost αg-normality is also regular open hereditary.

Almost αg-normality does not imply almost αg-regularity in general. However, we observe that every almost αg-normal αg-R_0 space is almost αg-regular.

Next, we prove the following.

Theorem 6.8: Every almost regular, *v*-compact space X is almost αg-normal.

Recall that a function f: $X\to Y$ is called rc-continuous if inverse image of regular closed set is regular closed.

Now, we state the invariance of almost αg-normality in the following.

Theorem 6.9: If f is continuous M-αg-open rc-continuous and almost αg-irresolute surjection from an almost αg-normal space X onto a space Y, then Y is almost αg-normal.

Definition 6.3: A space X is said to be mildly αg-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint αg-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 8: Let X = {a, b, c} and τ = {ϕ,{b},{a, b},{b, c}, X}. Then X is mildly αg-normal.

We have the following characterization of mild αg-normality.

Theorem 6.10: For a space X the following are equivalent.

(i) X is mildly αg-normal.

(ii) For every pair of regular open sets U and V whose union is X, there exist αg-closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(iii) For any regular closed set A and every regular open set B containing A, there exists a αg-open set U such that $A \subset U \subset \alpha gcl(U) \subset B$.

(iv) For every pair of disjoint regular closed sets, there exist αg-open sets U and V such that $A \subset U$, $B \subset V$ and $\alpha gcl(U) \cap \alpha gcl(V) = \phi$.

This theorem may be proved by using the arguments similar to those of Theorem 6.7.

Also, we observe that mild αg-normality is regular open hereditary.

We define the following

Definition 6.4: A space X is weakly αg-regular if for each point x and a regular open set U containing {x}, there is a αg-open set V such that $x \in V \subset clV \subset U$.

Example 9: Let X = {a, b, c} and τ = {ϕ,{b},{a, b},{b, c}, X}. Then X is weakly αg-regular.

Example 10: Let X = {a, b, c} and τ = {ϕ,{a},{b},{a, b}, X}. Then X is not weakly αg-regular.

Theorem 6.11: If $f : X \to Y$ is an M-αg-open rc-continuous and almost αg-irresolute function from a mildly αg-normal space X onto a space Y, then Y is mildly αg-normal.

Proof: Let A be a regular closed set and B be a regular open set containing A. Then by rc-continuity of *f*, $f^{-1}(A)$ is a regular closed set contained in the regular open set $f^{-1}(B)$. Since X is mildly αg-normal, there exists a αg-open set V such that $f^{-1}(A) \subset V \subset \alpha gcl(V) \subset f^{-1}(B)$ by Theorem 6.10. As *f* is M-αg-open and almost αg-irresolute surjection, it follows that $f(V) \in \alpha g\, O(Y)$ and $A \subset f(V) \subset \alpha gcl(f(V)) \subset B$. Hence Y is mildly αg-normal.

Theorem 6.12: If $f: X \to Y$ is rc-continuous, M-αg-closed map from a mildly αg-normal space X onto a space Y, then Y is mildly αg-normal.

7. αg-US spaces:

Definition 7.1: A sequence $<x_n>$ is said to be αg-converges to a point x of X, written as $<x_n> \to^{\alpha g} x$ if $<x_n>$ is eventually in every αg-open set containing x.

Clearly, if a sequence $<x_n>$ *r*-converges to a point x of X, then $<x_n>$ αg-converges to x.

Definition 7.2: X is said to be αg-US if every sequence $<x_n>$ in X αg-converges to a unique point.

Theorem 7.1: Every αg-US space is αg-T_1.

Proof: Let X be αg-US space. Let x and y be two distinct points of X. Consider the sequence $<x_n>$ where $x_n = x$ for every n. Cleary, $<x_n> \to^{\alpha g} x$. Also, since $x \neq y$ and X is αg-US, $<x_n>$ cannot αg-converge to y, i.e, there exists a αg-open set V containing y but not x. Similarly, for the sequence $<y_n>$ where $y_n = y$ for all n, and proceeding as above we get a αg-open set U containing x but not y. Thus, the space X is αg-T_1.

Theorem 7.2: Every αg-T_2 space is αg-US.

Proof: Let X be αg-T_2 space and $<x_n>$ be a sequence in X. If possible suppose that $<x_n>$ αg-converge to two distinct points x and y. That is, $<x_n>$ is eventually in every αg-open set containing x and also in every αg-open set containing y.

This is contradiction since X is αg-T_2 space. Hence the space X is αg-US.

Definition 7.3: A set F is sequentially αg-closed if every sequence in F αg-converges to a point in F.

Theorem 7.3: X is αg-US iff the diagonal set is a sequentially αg-closed subset of X x X.

Proof: Let X be αg-US. Let $<x_n, x_n>$ be a sequence in Δ. Then $<x_n>$ is a sequence in X. As X is αg-US, $<x_n> \to^{\alpha g} x$ for a unique $x \in X$. i.e., if $<x_n> \to^{\alpha g}$ x and y. Thus, x = y. Hence Δ is sequentially αg-closed.

Conversely, let Δ be sequentially αg-closed and let $<x_n> \rightarrow^{\alpha g}$ x and y. Hence $<x_n, x_n> \rightarrow^{\alpha g}$ (x,y). Since Δ is sequentially αg-closed, $(x,y) \in \Delta$ which means that x = y implies space X is αg-US.

Definition 7.4: A subset G of a space X is said to be sequentially αg-compact if every sequence in G has a subsequence which αg-converges to a point in G.

Theorem 7.4: In a αg-US space every sequentially αg-compact set is sequentially αg-closed.

Proof: Let X be αg-US space. Let Y be a sequentially αg-compact subset of X. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ αg-converges to a point in X-Y. Let $<x_{np}>$ be subsequence of $<x_n>$ that αg-converges to a point $y \in Y$ since Y is sequentially αg-compact. Also, let a subsequence $<x_{np}>$ of $<x_n>$ αg-converge to $x \in X-Y$. Since $<x_{np}>$ is a sequence in the αg-US space X, x = y. Thus, Y is sequentially αg-closed set.

Next, we give a hereditary property of αg-US spaces.

Theorem 7.5: Every regular open subset of a αg-US space is αg-US.

Proof: Let X be a αg-US space and $Y \subset X$ be an regular open set. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ αg-converges to x and y in Y. We shall prove that $<x_n>$ αg-converges to x and y in X. Let U be any αg-open subset of X containing x and V be any αg-open set of X containing y. Then, $U\cap Y$ and $V\cap Y$ are αg-open sets in Y. Therefore, $<x_n>$ is eventually in $U\cap Y$ and $V\cap Y$ and so in U and V. Since X is αg-US, this implies that x = y. Hence the subspace Y is αg-US.

Theorem 7.6: A space X is αg-T_2 iff it is both αg-R_1 and αg-US.

Proof: Let X be αg-T_2 space. Then X is αg-R_1 and αg-US by Theorem 7.2.

Conversely, let X be both αg-R_1 and αg-US space. By Theorem 7.1, X is both αg-T_1 and αg-R_1 and, it follows that space X is αg-T_2.

Definition 7.5: A point y is a αg-cluster point of sequence $<x_n>$ iff $<x_n>$ is frequently in every αg-open set containing x. The set of all αg-cluster points of $<x_n>$ will be denoted by αg-cl(x_n).

Definition 7.6: A point y is αg-side point of a sequence $<x_n>$ if y is a αg-cluster point of $<x_n>$ but no subsequence of $<x_n>$ αg-converges to y.

Now, we define the following.

Definition 7.7: A space X is said to be
(i) αg-S_1 if it is αg-US and every sequence $<x_n>$ αg-converges with subsequence of $<x_n>$ αg-side points.
(ii) αg-S_2 if it is αg-US and every sequence $<x_n>$ in X αg-converges which has no αg-side point.

Lemma 7.1: Every αg-S_2 space is αg-S_1 and Every αg-S_1 space is αg-US.

Now using the notion of sequentially continuous functions, we define the notion of sequentially αg-continuous functions.

Definition 7.8: A function f is said to be sequentially αg-continuous at $x \in X$ if $f(x_n) \rightarrow^{\alpha g} f(x)$ whenever $<x_n> \rightarrow^{\alpha g}$ x. If f is sequentially αg-continuous at all $x\in X$, then f is said to be sequentially αg-continuous.

Theorem 7.7: Let f and g be two sequentially αg-continuous functions. If Y is αg-US, then the set $A = \{x \mid f(x) = g(x)\}$ is sequentially αg-closed.

Proof: Let Y be αg-US and suppose that there is a sequence $<x_n>$ in A αg-converging to $x \in X$. Since f and g are sequentially αg-continuous functions, $f(x_n) \rightarrow^{\alpha g} f(x)$ and $g(x_n) \rightarrow^{\alpha g} g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, A is sequentially αg-closed.

Next, we prove the product theorem for αg-US spaces.

Theorem 7.8: Product of arbitrary family of αg-US spaces is αg-US.

Proof: Let $X = \prod_{\lambda\in\wedge} X_\lambda$ where X_λ is αg-US. Let a sequence $<x_n>$ in X αg-converges to x (= x_λ) and y (= y_λ). Then $<x_{n\lambda}> \to^{\alpha g} x_\lambda$ and y_λ for all $\lambda \in \wedge$. For suppose there exists a $\mu\in\wedge$ such that $<x_{n\mu}>$ does not αg-converges to x_μ.

Then there exists a τ_μ-αg-open set U_μ containing x_μ such that $<x_{n\mu}>$ is not eventually in U_μ. Consider the set $U = \prod_{\lambda\in\wedge} X_\lambda \times U_\mu$. Then U is a αg-open subset of X and $x \in U$. Also, $<x_n>$ is not eventually in U, which contradicts the fact that $<x_n> \to^{\alpha g} x$. Thus we get $<x_{n\lambda}> \to^{\alpha g} x_\lambda$ and y_λ for all $\lambda \in \wedge$. Since X_λ is αg-US for each $\lambda\in\wedge$. Thus x = y. Hence X is αg-US.

8. Sequentially sub-αg-continuity:

Definition 8.1: A function *f* is said to be

(i) sequentially nearly αg-continuous if for each point $x\in X$ and each sequence $<x_n> \to^{\alpha g} x$ in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $<f(x_{nk})> \to^{\alpha g} f(x)$.

(ii) sequentially sub-αg-continuous if for each point $x\in X$ and each sequence $<x_n> \to^{\alpha g} x$ in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point $y\in Y$ such that $<f(x_{nk})> \to^{\alpha g} y$.

(iii) sequentially αg-compact preserving if *f*(K) is sequentially αg-compact in Y for every sequentially αg-compact set K of X.

Lemma 8.1: Every function *f* is sequentially sub-αg-continuous if Y is a sequentially αg-compact.

Proof: Let $<x_n> \to^{\alpha g} x$ in X. Since Y is sequentially αg-compact, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ αg-converging to a point $y\in Y$. Hence *f* is sequentially sub-αg-continuous.

Theorem 8.1: Every sequentially nearly αg-continuous function is sequentially αg-compact preserving.

Proof: Assume *f* is sequentially nearly αg-continuous and K any sequentially αg-compact subset of X. Let $<y_n>$ be any sequence in *f* (K). Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially αg-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ αg-converging to a point $x \in K$. By hypothesis, *f* is sequentially nearly αg-continuous and hence there exists a subsequence $<x_j>$ of $<x_{nk}>$ such that $f(x_j) \to^{\alpha g} f(x)$. Thus, there exists a subsequence $<y_j>$ of $<y_n>$ αg-converging to $f(x)\in f(K)$. This shows that *f*(K) is sequentially αg-compact set in Y.

Theorem 8.2: Every sequentially α-continuous function is sequentially αg-continuous.

Proof: Let *f* be a sequentially α-continuous and $<x_n> \to^{\alpha} x\in X$. Then $<x_n> \to^{\alpha} x$. Since *f* is sequentiallyα-continuous, $f(x_n)\to^{\alpha} f(x)$. But we know that $<x_n>\to^{\alpha} x$ implies $<x_n> \to^{\alpha g} x$ and hence $f(x_n) \to^{\alpha g} f(x)$ implies *f* is sequentially αg-continuous.

Theorem 8.3: Every sequentially αg-compact preserving function is sequentially sub-αg-continuous.

Proof: Suppose *f* is a sequentially αg-compact preserving function. Let x be any point of X and $<x_n>$ any sequence in X αg-converging to x. We shall denote the set $\{x_n \mid n= 1, 2, 3 \ldots\}$ by A and $K = A \cup \{x\}$. Then K is sequentially αg-compact since $(x_n) \to^{\alpha g} x$. By hypothesis, *f* is sequentially αg-compact preserving and hence *f*(K) is a sequentially αg-compact set of Y. Since $\{f(x_n)\}$ is a sequence in *f*(K), there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ αg-converging to a point $y\in f(K)$. This implies that *f* is sequentially sub-αg-continuous.

Theorem 8.4: A function $f: X\to Y$ is sequentially αg-compact preserving iff $f_{/K}: K \to f(K)$ is sequentially sub-αg-continuous for each sequentially αg-compact subset K of X.

Proof: Suppose *f* is a sequentially αg-compact preserving function. Then *f* (K) is sequentially αg-compact set in Y for each sequentially αg-compact set K of X. Therefore, by Lemma 8.1 above, $f_{/K}: K\to f(K)$ is sequentially αg-continuous function.

Conversely, let K be any sequentially αg-compact set of X. Let $<y_n>$ be any sequence in *f*(K). Then for each positive integer n, there exists a point $x_n\in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially αg-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ αg-converging to a point $x \in K$. By hypothesis, $f_{/K}: K\to f(K)$ is sequentially

sub-αg-continuous and hence there exists a subsequence $<y_{nk}>$ of $<y_n>$ αg-converging to a point $y\in f(K)$.This implies that *f*(K) is sequentially αg-compact set in Y. Thus, *f* is sequentially αg-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub-αg-continuous function to be sequentially αg-compact preserving.

Corollary 8.1: If f is sequentially sub-αg-continuous and f (K) is sequentially αg-closed set in Y for each sequentially αg-compact set K of X, then f is sequentially αg-compact preserving function.

Proof: Omitted.

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