



On α g-Separation Axioms

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Abstract

In this paper by using α g-open sets we define almost α g-normality and mild α g-normality also we continue the study of further properties of α g-normality. We show that these three axioms are regular open hereditary. We also define the class of almost α g-irresolute mappings and show that α g-normality is invariant under almost α g-irresolute M - α g-open continuous surjection.

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1. Introduction

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2 . Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s -convergence, sequentially semi-closed sets, sequentially s -compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P. Aruna Swathi Vyjayanthi studied ν -Normal Almost- ν -Normal, Mildly- ν -Normal and ν -US spaces. Inspired with these we introduce α g-Normal Almost- α g-Normal, Mildly- α g-Normal, α g-US, α g- S_1 and α g- S_2 . Also we examine α g-convergence, sequentially α g-compact, sequentially α g-continuous maps, and sequentially sub α g-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. Preliminaries

Definition 2.1: $A \subseteq X$ is called

- (i) g -closed if $\text{cl } A \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (ii) α g-closed if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X .

Definition 2.2: A function f is said to be almost-pre-irresolute if for each x in X and each pre-neighborhood V of $f(x)$, $\text{pcl}(f^{-1}(V))$ is a pre-neighborhood of x .

Definition 2.3: A space X is said to be

- (i) T_1 (T_2) if for any $x \neq y$ in X , there exist (disjoint) open sets U, V in X such that $x \in U$ and $y \in V$.
- (ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X .
- (iii) normal [resp: mildly normal] if for any pair of disjoint [resp: regular-closed] closed sets F_1 and F_2 , there exist disjoint open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.
- (iv) almost normal if for each closed set A and each regular closed set B such that $A \cap B = \emptyset$, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.
- (v) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \emptyset$, there exist disjoint open sets U and V such that $x \in U$ and $A \subseteq V$.

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- (vi) A subset A of a space X is S -closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A .
- (vii) R_0 if for any point x and a closed set F with $x \notin F$ in X , there exists an open set G containing F but not x .
- (viii) R_1 iff for $x, y \in X$ with $cl\{x\} \neq cl\{y\}$, there exist disjoint open sets U and V such that $cl\{x\} \subset U, cl\{y\} \subset V$.
- (ix) US -space if every convergent sequence has exactly one limit point to which it converges.
- (x) pre- US space if every pre-convergent sequence has exactly one limit point to which it converges.
- (xi) pre- S_1 if it is pre- US and every sequence $\langle x_n \rangle$ pre-converges with subsequence of $\langle x_n \rangle$ pre-side points.
- (xii) pre- S_2 if it is pre- US and every sequence $\langle x_n \rangle$ in X pre-converges which has no pre-side point.
- (xiii) is weakly countable compact if every infinite subset of X has a limit point in X .
- (xiv) Baire space if for any countable collection of closed sets with empty interior in X , their union also has empty interior in X .

Definition 2.4: Let $A \subset X$. Then a point x is said to be a

- (i) limit point of A if each open set containing x contains some point y of A such that $x \neq y$.
- (ii) T_0 -limit point of A if each open set containing x contains some point y of A such that $cl\{x\} \neq cl\{y\}$, or equivalently, such that they are topologically distinct.
- (iii) pre- T_0 -limit point of A if each open set containing x contains some point y of A such that $pcl\{x\} \neq pcl\{y\}$, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0 -axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let $X = \{a, b, c, d\}$ and $\tau = \{\{a\}, \{b, c\}, \{a, b, c\}, X, \phi\}$. Then b and c are the limit points but not the T_0 -limit points of the set $\{b, c\}$. Further d is a T_0 -limit point of $\{b, c\}$.

Example 2: Let $X = (0, 1)$ and $\tau = \{\phi, X, \text{and } U_n = (0, 1 - 1/n), n = 2, 3, 4, \dots\}$. Then every point of X is a limit point of X . Every point of $X \sim U_2$ is a T_0 -limit point of X , but no point of U_2 is a T_0 -limit point of X .

Definition 2.5: A set A together with all its T_0 -limit points will be denoted by $T_0\text{-cl}A$.

Note 2: i. Every T_0 -limit point of a set A is a limit point of the set but the converse is not true in general.
ii. In T_0 -space both are same.

Note 3: R_0 -axiom is weaker than T_1 -axiom. It is independent of the T_0 -axiom. However $T_1 = R_0 + T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1 -space is weakly countable compact iff it is countable compact.

3. αg - T_0 LIMIT POINT:

Definition 3.01: In X , a point x is said to be a αg - T_0 -limit point of A if each αg -open set containing x contains some point y of A such that $\alpha gcl\{x\} \neq \alpha gcl\{y\}$, or equivalently; such that they are topologically distinct with respect to αg -open sets.

Example 3: regular open set \Rightarrow open set \Rightarrow α -open set \Rightarrow αg -open set we have
 r - T_0 -limit point \Rightarrow T_0 -limit point \Rightarrow α - T_0 -limit point \Rightarrow αg - T_0 -limit point

Definition 3.02: A set A together with all its αg - T_0 -limit points is denoted by $T_0\text{-}\alpha gcl(A)$

Lemma 3.01: If x is a αg - T_0 -limit point of a set A then x is αg -limit point of A .

Lemma 3.02: If X is αg - T_0 -space then every αg - T_0 -limit point and every αg -limit point are equivalent.

Corollary 3.03: If X is r - T_0 -space then every αg - T_0 -limit point and every αg -limit point are equivalent.

Theorem 3.04: For $x \neq y \in X$,

- (i) x is a αg - T_0 -limit point of $\{y\}$ iff $x \notin \alpha gcl\{y\}$ and $y \in \alpha gcl\{x\}$.
- (ii) x is not a αg - T_0 -limit point of $\{y\}$ iff either $x \in \alpha gcl\{y\}$ or $\alpha gcl\{x\} = \alpha gcl\{y\}$.
- (iii) x is not a αg - T_0 -limit point of $\{y\}$ iff either $x \in \alpha gcl\{y\}$ or $y \in \alpha gcl\{x\}$.

Corollary 3.05:

- (i) If x is a α g- T_0 -limit point of $\{y\}$, then y cannot be a α g-limit point of $\{x\}$.
- (ii) If $\alpha gcl\{x\} = \alpha gcl\{y\}$, then neither x is a α g- T_0 -limit point of $\{y\}$ nor y is a α g- T_0 -limit point of $\{x\}$.
- (iii) If a singleton set A has no α g- T_0 -limit point in X , then $\alpha gcl A = \alpha gcl\{x\}$ for all $x \in \alpha gcl\{A\}$.

Lemma 3.06: In X , if x is a α g-limit point of a set A , then in each of the following cases x becomes α g- T_0 -limit point of A ($\{x\} \neq A$).

- (i) $\alpha gcl\{x\} \neq \alpha gcl\{y\}$ for $y \in A, x \neq y$.
- (ii) $\alpha gcl\{x\} = \{x\}$
- (iii) X is a α g- T_0 -space.
- (iv) $A \sim \{x\}$ is α g-open

Corollary 3.07: In X , if x is a limit point of a set A , then in each of the following cases x becomes α g- T_0 -limit point of A ($\{x\} \neq A$).

- (i) $\alpha gcl\{x\} \neq \alpha gcl\{y\}$ for $y \in A, x \neq y$.
- (ii) $\alpha gcl\{x\} = \{x\}$
- (iii) X is a α g- T_0 -space.
- (iv) $A \sim \{x\}$ is α g-open

4. α g- T_0 AND α g- R_i AXIOMS, $i = 0, 1$:

In view of Lemma 3.6(iii), α g- T_0 -axiom implies the equivalence of the concept of limit point of a set with that of α g- T_0 -limit point of the set. But for the converse, if $x \in \alpha gcl\{y\}$ then $\alpha gcl\{x\} \neq \alpha gcl\{y\}$ in general, but if x is a α g- T_0 -limit point of $\{y\}$, then $\alpha gcl\{x\} = \alpha gcl\{y\}$

Lemma 4.01: In a space X , a limit point x of $\{y\}$ is a α g- T_0 -limit point of $\{y\}$ iff $\alpha gcl\{x\} \neq \alpha gcl\{y\}$.

This lemma leads to characterize the equivalence of α g- T_0 -limit point and α g-limit point of a set as the α g- T_0 -axiom.

Theorem 4.02: The following conditions are equivalent:

- (i) X is a α g- T_0 space
- (ii) Every α g-limit point of a set A is a α g- T_0 -limit point of A
- (iii) Every r -limit point of a singleton set $\{x\}$ is a α g- T_0 -limit point of $\{x\}$
- (iv) For any x, y in $X, x \neq y$ if $x \in \alpha gcl\{y\}$, then x is a α g- T_0 -limit point of $\{y\}$

Note 5: In a α g- T_0 -space X if every point of X is a r -limit point of X , then every point of X is α g- T_0 -limit point of X . But a space X in which each point is a α g- T_0 -limit point of X is not necessarily a α g- T_0 -space

Theorem 4.03: The following conditions are equivalent:

- (i) X is a α g- R_0 space
- (ii) For any x, y in X , if $x \in \alpha gcl\{y\}$, then x is not a α g- T_0 -limit point of $\{y\}$
- (iii) A point α g-closure set has no α g- T_0 -limit point in X
- (iv) A singleton set has no α g- T_0 -limit point in X .

Since every r - R_0 -space is α g- R_0 -space, we have the following corollary

Corollary 4.04: The following conditions are equivalent:

- (i) X is a r - R_0 space
- (ii) For any x, y in X , if $x \in \alpha gcl\{y\}$, then x is not a α g- T_0 -limit point of $\{y\}$
- (iii) A point α g-closure set has no α g- T_0 -limit point in X
- (iv) A singleton set has no α g- T_0 -limit point in X .

Theorem 4.05: In a α g- R_0 space X , a point x is α g- T_0 -limit point of A iff every α g-open set containing x contains infinitely many points of A with each of which x is topologically distinct

If α g- R_0 space is replaced by rR_0 space in the above theorem, we have the following corollaries:

Corollary 4.06: In an rR_0 -space X ,

- (i) If a point x is rT_0 -limit point of a set then every α g-open set containing x contains infinitely many points of A with each of which x is topologically distinct.
- (ii) If a point x is α g- T_0 -limit point of a set then every α g-open set containing x contains infinitely many points of A with each of which x is topologically distinct.

Theorem 4.07: X is αg - R_0 space iff a set A of the form $A = \cup \alpha gcl\{x_i \mid i=1 \text{ to } n\}$ a finite union of point closure sets has no αg - T_0 -limit point.

Corollary 4.08: If X is rR_0 space and

- (i) If $A = \cup \alpha gcl\{x_i \mid i=1 \text{ to } n\}$, a finite union of point closure sets has no αg - T_0 -limit point.
- (ii) If $X = \cup \alpha gcl\{x_i \mid i=1 \text{ to } n\}$, then X has no αg - T_0 -limit point.

Theorem 4.09: The following conditions are equivalent:

- (i) X is αg - R_0 -space
- (ii) For any x and a set in X , x is a αg - T_0 -limit point of A iff every αg -open set containing x contains infinitely many points of A with each of which x is topologically distinct.

Various characteristic properties of αg - T_0 -limit points studied so far is enlisted in the following theorem for a ready reference.

Theorem 4.10: In a αg - R_0 -space, we have the following:

- (i) A singleton set has no αg - T_0 -limit point in X .
- (ii) A finite set has no αg - T_0 -limit point in X .
- (iii) A point αg -closure has no set αg - T_0 -limit point in X .
- (iv) A finite union point αg -closure sets have no set αg - T_0 -limit point in X .
- (v) For $x, y \in X$, $x \in T_0-\alpha gcl\{y\}$ iff $x = y$.
- (vi) For any $x, y \in X$, $x \neq y$ iff neither x is αg - T_0 -limit point of $\{y\}$ nor y is αg - T_0 -limit point of $\{x\}$.
- (vii) For any $x, y \in X$, $x \neq y$ iff $T_0-\alpha gcl\{x\} \cap T_0-\alpha gcl\{y\} = \phi$.
- (viii) Any point $x \in X$ is a αg - T_0 -limit point of a set A in X iff every αg -open set containing x contains infinitely many points of A with each which x is topologically distinct.

Theorem 4.11: X is αg - R_1 iff for any αg -open set U in X and points x, y such that $x \in X \sim U$, $y \in U$, there exists a αg -open set V in X such that $y \in V \subset U$, $x \notin V$.

Lemma 4.12: In αg - R_1 space X , if x is a αg - T_0 -limit point of X , then for any non empty αg -open set U , there exists a non empty αg -open set V such that $V \subset U$, $x \notin \alpha gcl(V)$.

Lemma 4.13: In a αg -regular space X , if x is a αg - T_0 -limit point of X , then for any non empty αg -open set U , there exists a non empty αg -open set V such that $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.

Corollary 4.14: In a regular space X ,

- (i) If x is a αg - T_0 -limit point of X , then for any non empty αg -open set U , there exists a non empty αg -open set V such that $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.
- (ii) If x is a T_0 -limit point of X , then for any non empty αg -open set U , there exists a non empty αg -open set V such that $\alpha gcl(V) \subset U$, $x \notin \alpha gcl(V)$.

Theorem 4.15: If X is a αg -compact αg - R_1 -space, then X is a Baire Space.

Proof: Let $\{A_n\}$ be a countable collection of αg -closed sets of X , each A_n having empty interior in X . Take A_1 , since A_1 has empty interior, A_1 does not contain any αg -open set say U_0 . Therefore we can choose a point $y \in U_0$ such that $y \notin A_1$. For X is αg -regular, and $y \in (X \sim A_1) \cap U_0$, a αg -open set, we can find a αg -open set U_1 in X such that $y \in U_1$, $\alpha gcl(U_1) \subset (X \sim A_1) \cap U_0$. Hence U_1 is a non empty αg -open set in X such that $\alpha gcl(U_1) \subset U_0$ and $vcl(U_1) \cap A_1 = \phi$. Continuing this process, in general, for given non empty αg -open set U_{n-1} , we can choose a point of U_{n-1} which is not in the αg -closed set A_n and a αg -open set U_n containing this point such that $\alpha gcl(U_n) \subset U_{n-1}$ and $\alpha gcl(U_n) \cap A_n = \phi$. Thus we get a sequence of nested non empty αg -closed sets which satisfies the finite intersection property. Therefore $\cap \alpha gcl(U_n) \neq \phi$. Then some $x \in \cap \alpha gcl(U_n)$ which in turn implies that $x \in U_{n-1}$ as $\alpha gcl(U_n) \subset U_{n-1}$ and $x \notin A_n$ for each n .

Corollary 4.16: If X is a compact αg - R_1 -space, then X is a Baire Space.

Corollary 4.17: Let X be a αg -compact αg - R_1 -space. If $\{A_n\}$ is a countable collection of αg -closed sets in X , each A_n having non-empty αg -interior in X , then there is a point of X which is not in any of the A_n .

Corollary 4.18: Let X be a αg -compact R_1 -space. If $\{A_n\}$ is a countable collection of αg -closed sets in X , each A_n having non-empty αg -interior in X , then there is a point of X which is not in any of the A_n .

Theorem 4.19: Let X be a non empty compact αg - R_1 -space. If every point of X is a αg - T_0 -limit point of X then X is uncountable.

Proof: Since X is non empty and every point is a αg - T_0 -limit point of X , X must be infinite. If X is countable, we construct a sequence of αg - open sets $\{V_n\}$ in X as follows:

Let $X = V_1$, then for x_1 is a αg - T_0 -limit point of X , we can choose a non empty αg -open set V_2 in X such that $V_2 \subset V_1$ and $x_1 \notin \alpha gcl V_2$. Next for x_2 and non empty αg -open set V_2 , we can choose a non empty αg -open set V_3 in X such that $V_3 \subset V_2$ and $x_2 \notin \alpha gcl V_3$. Continuing this process for each x_n and a non empty αg -open set V_n , we can choose a non empty αg -open set V_{n+1} in X such that $V_{n+1} \subset V_n$ and $x_n \notin \alpha gcl V_{n+1}$.

Now consider the nested sequence of αg -closed sets $\alpha gcl V_1 \supset \alpha gcl V_2 \supset \alpha gcl V_3 \supset \dots \supset \alpha gcl V_n \supset \dots$. Since X is αg -compact and $\{\alpha gcl V_n\}$ the sequence of αg -closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x \in \alpha gcl V_n$. Further $x \in X$ and $x \in V_1$, which is not equal to any of the points of X . Hence X is uncountable.

Corollary 4.20: Let X be a non empty αg -compact αg - R_1 -space. If every point of X is a αg - T_0 -limit point of X then X is uncountable

5. αg - T_0 -IDENTIFICATION SPACES AND αg -SEPARATION AXIOMS

Definition 5.01: Let (X, τ) be a topological space and let \mathfrak{R} be the equivalence relation on X defined by $x\mathfrak{R}y$ iff $\alpha gcl\{x\} = \alpha gcl\{y\}$

Problem 5.02: show that $x\mathfrak{R}y$ iff $\alpha gcl\{x\} = \alpha gcl\{y\}$ is an equivalence relation

Definition 5.03: The space $(X_0, Q(X_0))$ is called the αg - T_0 -identification space of (X, τ) , where X_0 is the set of equivalence classes of \mathfrak{R} and $Q(X_0)$ is the decomposition topology on X_0 .

Let $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$ denote the natural map

Lemma 5.04: If $x \in X$ and $A \subset X$, then $x \in \alpha gcl A$ iff every αg -open set containing x intersects A .

Theorem 5.05: The natural map $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$ is closed, open and $P_X^{-1}(P_X(O)) = O$ for all $O \in PO(X, \tau)$ and $(X_0, Q(X_0))$ is αg - T_0

Proof: Let $O \in PO(X, \tau)$ and let $C \in P_X(O)$. Then there exists $x \in O$ such that $P_X(x) = C$. If $y \in C$, then $\alpha gcl\{y\} = \alpha gcl\{x\}$, which, by lemma, implies $y \in O$. Since $\tau \subset PO(X, \tau)$, then $P_X^{-1}(P_X(U)) = U$ for all $U \in \tau$, which implies P_X is closed and open.

Let $G, H \in X_0$ such that $G \neq H$; let $x \in G$ and $y \in H$. Then $\alpha gcl\{x\} \neq \alpha gcl\{y\}$, which implies $x \notin \alpha gcl\{y\}$ or $y \notin \alpha gcl\{x\}$, say $x \notin \alpha gcl\{y\}$. Since P_X is continuous and open, then $G \in A = P_X\{X \sim \alpha gcl\{y\}\} \notin PO(X_0, Q(X_0))$ and $H \notin A$

Theorem 5.06: The following are equivalent:

(i) X is αg R_0 (ii) $X_0 = \{\alpha gcl\{x\}: x \in X\}$ and (iii) $(X_0, Q(X_0))$ is αg T_1

Proof:

(i) \Rightarrow (ii) Let $C \in X_0$, and let $x \in C$. If $y \in C$, then $y \in \alpha gcl\{y\} = \alpha gcl\{x\}$, which implies $C \in \alpha gcl\{x\}$. If $y \in \alpha gcl\{x\}$, then $x \in \alpha gcl\{y\}$, since, otherwise, $x \in X \sim \alpha gcl\{y\} \in PO(X, \tau)$ which implies $\alpha gcl\{x\} \subset X \sim \alpha gcl\{y\}$, which is a contradiction. Thus, if $y \in \alpha gcl\{x\}$, then $x \in \alpha gcl\{y\}$, which implies $\alpha gcl\{y\} = \alpha gcl\{x\}$ and $y \in C$. Hence $X_0 = \{\alpha gcl\{x\}: x \in X\}$

(ii) \Rightarrow (iii) Let $A \neq B \in X_0$. Then there exists $x, y \in X$ such that $A = \alpha gcl\{x\}$; $B = \alpha gcl\{y\}$, and $\alpha gcl\{x\} \cap \alpha gcl\{y\} = \phi$. Then $A \in C = P_X(X \sim \alpha gcl\{y\}) \in PO(X_0, Q(X_0))$ and $B \notin C$. Thus $(X_0, Q(X_0))$ is αg - T_1

(iii) \Rightarrow (i) Let $x \in U \in \alpha GO(X)$. Let $y \notin U$ and $C_x, C_y \in X_0$ containing x and y respectively. Then $x \notin \alpha gcl\{y\}$, which implies $C_x \neq C_y$ and there exists αg -open set A such that $C_x \in A$ and $C_y \notin A$. Since P_X is continuous and open, then $y \in B = P_X^{-1}(A) \in x \in \alpha g O(X)$ and $x \notin B$, which implies $y \notin \alpha gcl\{x\}$. Thus $\alpha gcl\{x\} \subset U$. This is true for all $\alpha gcl\{x\}$ implies $\cap \alpha gcl\{x\} \subset U$. Hence X is αg - R_0

Theorem 5.07: (X, τ) is αg - R_1 iff $(X_0, Q(X_0))$ is αg - T_2

The proof is straight forward from using theorems 5.05 and 5.06 and is omitted

Theorem 5.08: X is αg - T_i ; $i = 0, 1, 2$. iff there exists a αg -continuous, almost-open, 1-1 function from (X, τ) into a αg - T_i space; $i = 0, 1, 2$. respectively.

Proof: If X is α g- T_i ; $i = 0,1,2$., then the identity function on X satisfies the desired properties. The converse is (ii) part of Theorem 2.13.

The following example shows that if $f: (X, \tau) \rightarrow (Y, \sigma)$ is continuous, α g-open, bijective, $A \in PO(Y, \sigma)$, and (Y, σ) α g- T_i ; $i = 0,1,2$, then $f^{-1}(A)$ need not be α g-open and (X, τ) need not be α g- T_i ; $i = 0,1,2$

Theorem 5.09: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is α g-continuous, α g-open, and $x, y \in X$ such that $\alpha gcl\{x\} = \alpha gcl\{y\}$, then $\alpha gcl\{f(x)\} = \alpha gcl\{f(y)\}$.

Theorem 5.10: The following are equivalent

- (i) (X, τ) is α g- T_0
- (ii) Elements of X_0 are singleton sets and
- (iii) There exists a α g-continuous, α g-open, 1-1 function $f: (X, \tau) \rightarrow (Y, \sigma)$, where (Y, σ) is α g- T_0

Proof: (i) is equivalent to (ii) and (i) \Rightarrow (iii) are straight forward and is omitted.

(iii) \Rightarrow (i) Let $x, y \in X$ such that $f(x) \neq f(y)$, which implies $\alpha gcl\{f(x)\} \neq \alpha gcl\{f(y)\}$. Then by theorem 5.09, $\alpha gcl\{x\} \neq \alpha gcl\{y\}$. Hence (X, τ) is α g- T_0

Corollary 5.11: A space (X, τ) is α g- T_i ; $i = 1,2$ iff (X, τ) is α g- T_{i-1} ; $i = 1,2$, respectively, and there exists a α g-continuous, α g-open, 1-1 function $f: (X, \tau)$ into a α g- T_0 space.

Definition 5.04: $f: X \rightarrow Y$ is point- α g-closure 1-1 iff for $x, y \in X$ such that $\alpha gcl\{x\} \neq \alpha gcl\{y\}$, $\alpha gcl\{f(x)\} \neq \alpha gcl\{f(y)\}$.

Theorem 5.12:

- (i) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is point- α g-closure 1-1 and (X, τ) is α g- T_0 , then f is 1-1
- (ii) If $f: (X, \tau) \rightarrow (Y, \sigma)$, where (X, τ) and (Y, σ) are α g- T_0 then f is point- α g-closure 1-1 iff f is 1-1

Proof: omitted

The following result can be obtained by combining results for α g- T_0 - identification spaces, α g-induced functions and α g- T_i spaces; $i = 1,2$.

Theorem 5.13: X is α g- R_i ; $i = 0,1$ iff there exists a α g-continuous, almost-open point- α g-closure 1-1 function $f: (X, \tau)$ into a α g- R_i space; $i = 0,1$ respectively.

6. α g-Normal; Almost α g-normal and Mildly α g-normal spaces

Definition 6.1: A space X is said to be α g-normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint α g-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 4: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. Then X is α g-normal.

Example 5: Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then X is not α g-normal and is not normal.

We have the following characterization of α g-normality.

Theorem 6.1: For a space X the following are equivalent:

- (i) X is α g-normal.
- (ii) For every pair of open sets U and V whose union is X , there exist α g-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.
- (iii) For every closed set F and every open set G containing F , there exists a α g-open set U such that $F \subset U \subset \alpha gcl(U) \subset G$.

Proof: (i) \Rightarrow (ii): Let U and V be a pair of open sets in a α g-normal space X such that $X = U \cup V$. Then $X-U$, $X-V$ are disjoint closed sets. Since X is α g-normal there exist disjoint α g-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$. Let $A = X-U_1$, $B = X-V_1$. Then A and B are α g-closed sets such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(b) \Rightarrow (c): Let F be a closed set and G be an open set containing F . Then $X-F$ and G are open sets whose union is X .

Then by (b), there exist αg -closed sets W_1 and W_2 such that $W_1 \subset X-F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X-W_1$, $X-G \subset X-W_2$ and $(X-W_1) \cap (X-W_2) = \phi$. Let $U = X-W_1$ and $V = X-W_2$. Then U and V are disjoint αg -open sets such that $F \subset U \subset X-V \subset G$. As $X-V$ is αg -closed set, we have $\alpha gcl(U) \subset X-V$ and $F \subset U \subset \alpha gcl(U) \subset G$.

(c) \Rightarrow (a): Let F_1 and F_2 be any two disjoint closed sets of X . Put $G = X-F_2$, then $F_1 \cap G = \phi$. $F_1 \subset G$ where G is an open set. Then by (c), there exists a αg -open set U of X such that $F_1 \subset U \subset \alpha gcl(U) \subset G$. It follows that $F_2 \subset X-\alpha gcl(U) = V$, say, then V is αg -open and $U \cap V = \phi$. Hence F_1 and F_2 are separated by αg -open sets U and V . Therefore X is αg -normal.

Theorem 6.2: A regular open subspace of a αg -normal space is αg -normal.

Proof: Let Y be a regular open subspace of a αg -normal space X . Let A and B be disjoint closed subsets of Y . As Y is regular open, A, B are closed sets of X . By αg -normality of X , there exist disjoint αg -open sets U and V in X such that $A \subset U$ and $B \subset V$, $U \cap Y$ and $V \cap Y$ are αg -open in Y such that $A \subset U \cap Y$ and $B \subset V \cap Y$. Hence Y is αg -normal.

Example 6: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ is αg -normal and αg -regular.

However we observe that every αg -normal $\alpha g-R_0$ space is αg -regular. Now, we define the following.

Definition 6.2: A function $f: X \rightarrow Y$ is said to be almost- αg -irresolute if for each x in X and each αg -neighborhood V of $f(x)$, $\alpha gcl(f^{-1}(V))$ is a αg -neighborhood of x .

Clearly every αg -irresolute map is almost αg -irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 6.1: f is almost αg -irresolute iff $f^{-1}(V) \subset \alpha g\text{-int}(\alpha gcl(f^{-1}(V)))$ for every $V \in \alpha gO(Y)$.

Now we prove the following.

Lemma 6.2: f is almost αg -irresolute iff $f(\alpha gcl(U)) \subset \alpha gcl(f(U))$ for every $U \in \alpha gO(X)$.

Proof: Let $U \in \alpha gO(X)$. Suppose $y \notin \alpha gcl(f(U))$. Then there exists $V \in \alpha gO(y)$ such that $V \cap f(U) = \phi$. Hence $f^{-1}(V) \cap U = \phi$. Since $U \in \alpha gO(X)$, we have $\alpha g\text{-int}(\alpha gcl(f^{-1}(V))) \cap \alpha gcl(U) = \phi$. Then by lemma 6.1, $f^{-1}(V) \cap \alpha gcl(U) = \phi$ and hence $V \cap f(\alpha gcl(U)) = \phi$. This implies that $y \notin f(\alpha gcl(U))$.

Conversely, if $V \in \alpha gO(Y)$, then $W = X - \alpha gcl(f^{-1}(V)) \in \alpha gO(X)$. By hypothesis, $f(\alpha gcl(W)) \subset \alpha gcl(f(W))$ and hence $X - \alpha g\text{-int}(\alpha gcl(f^{-1}(V))) = \alpha gcl(W) \subset f^{-1}(\alpha gcl(f(W))) \subset f^{-1}(\alpha gcl[f(X-f^{-1}(V))]) \subset f^{-1}[\alpha gcl(Y-V)] = f^{-1}(Y-V) = X-f^{-1}(V)$. Therefore, $f^{-1}(V) \subset \alpha g\text{-int}(\alpha gcl(f^{-1}(V)))$. By lemma 6.1, f is almost αg -irresolute.

Now we prove the following result on the invariance of αg -normality.

Theorem 6.3: If f is an M - αg -open continuous almost αg -irresolute function from a αg -normal space X onto a space Y , then Y is αg -normal.

Proof: Let A be a closed subset of Y and B be an open set containing A . Then by continuity of f , $f^{-1}(A)$ is closed and $f^{-1}(B)$ is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is αg -normal, there exists a αg -open set U in X such that $f^{-1}(A) \subset U \subset \alpha gcl(U) \subset f^{-1}(B)$. Then $f(f^{-1}(A)) \subset f(U) \subset f(\alpha gcl(U)) \subset f(f^{-1}(B))$. Since f is M - αg -open almost αg -irresolute surjection, we obtain $A \subset f(U) \subset \alpha gcl(f(U)) \subset B$. Then again by Theorem 6.1 the space Y is αg -normal.

Lemma 6.3: A mapping f is M - αg -closed if and only if for each subset B in Y and for each αg -open set U in X containing $f^{-1}(B)$, there exists a αg -open set V containing B such that $f^{-1}(V) \subset U$.

Now we prove the following:

Theorem 6.4: If f is an M - αg -closed continuous function from a αg -normal space onto a space Y , then Y is αg -normal.

Proof of the theorem is routine and hence omitted.

Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

Theorem 6.5: If f is an M - α g-closed map from a weakly Hausdorff α g-normal space X onto a space Y such that $f^{-1}(y)$ is S -closed relative to X for each $y \in Y$, then Y is α g- T_2 .

Proof: Let y_1 and y_2 be any two distinct points of Y . Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is α g-normal, there exist disjoint α g-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for $i = 1, 2$. Since f is M - α g-closed, there exist α g-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$ for $i = 1, 2$. Then it follows that $U_1 \cap U_2 = \emptyset$. Hence Y is α g- T_2 .

Theorem 6.6: For a space X we have the following:

- (a) If X is normal then for any disjoint closed sets A and B , there exist disjoint α g-open sets U, V such that $A \subset U$ and $B \subset V$;
- (b) If X is normal then for any closed set A and any open set V containing A , there exists an α g-open set U of X such that $A \subset U \subset \alpha gcl(U) \subset V$.

Definition 6.2: X is said to be almost α g-normal if for each closed set A and each regular closed set B such that $A \cap B = \emptyset$, there exist disjoint α g-open sets U and V such that $A \subset U$ and $B \subset V$.

Clearly, every α g-normal space is almost α g-normal, but not conversely in general.

Example 7: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then X is almost α g-normal and not α g-normal.

Now, we have characterization of almost α g-normality in the following.

Theorem 6.7: For a space X the following statements are equivalent:

- (i) X is almost α g-normal
- (ii) For every pair of sets U and V , one of which is open and the other is regular open whose union is X , there exist α g-closed sets G and H such that $G \subset U, H \subset V$ and $G \cup H = X$.
- (iii) For every closed set A and every regular open set B containing A , there is a α g-open set V such that $A \subset V \subset \alpha gcl(V) \subset B$.

Proof:

(a) \Rightarrow (b) Let U be an open set and V be a regular open set in an almost α g-normal space X such that $U \cup V = X$. Then $(X-U)$ is closed set and $(X-V)$ is regular closed set with $(X-U) \cap (X-V) = \emptyset$. By almost α g-normality of X , there exist disjoint α g-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$. Let $G = X - U_1$ and $H = X - V_1$. Then G and H are α g-closed sets such that $G \subset U, H \subset V$ and $G \cup H = X$.

(b) \Rightarrow (c) and **(c) \Rightarrow (a)** are obvious.

One can prove that almost α g-normality is also regular open hereditary.

Almost α g-normality does not imply almost α g-regularity in general. However, we observe that every almost α g-normal α g- R_0 space is almost α g-regular.

Next, we prove the following.

Theorem 6.8: Every almost regular, ν -compact space X is almost α g-normal.

Recall that a function $f: X \rightarrow Y$ is called rc-continuous if inverse image of regular closed set is regular closed.

Now, we state the invariance of almost α g-normality in the following.

Theorem 6.9: If f is continuous M - α g-open rc-continuous and almost α g-irresolute surjection from an almost α g-normal space X onto a space Y , then Y is almost α g-normal.

Definition 6.3: A space X is said to be mildly α g-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X , there exist disjoint α g-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 8: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then X is mildly α g-normal.

We have the following characterization of mild α g-normality.

Theorem 6.10: For a space X the following are equivalent.

- (i) X is mildly αg -normal.
- (ii) For every pair of regular open sets U and V whose union is X , there exist αg -closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.
- (iii) For any regular closed set A and every regular open set B containing A , there exists a αg -open set U such that $A \subset U \subset \alpha gcl(U) \subset B$.
- (iv) For every pair of disjoint regular closed sets, there exist αg -open sets U and V such that $A \subset U$, $B \subset V$ and $\alpha gcl(U) \cap \alpha gcl(V) = \phi$.

This theorem may be proved by using the arguments similar to those of Theorem 6.7.

Also, we observe that mild αg -normality is regular open hereditary.

We define the following

Definition 6.4: A space X is weakly αg -regular if for each point x and a regular open set U containing $\{x\}$, there is a αg -open set V such that $x \in V \subset cV \subset U$.

Example 9: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then X is weakly αg -regular.

Example 10: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then X is not weakly αg -regular.

Theorem 6.11: If $f : X \rightarrow Y$ is an M - αg -open rc -continuous and almost αg -irresolute function from a mildly αg -normal space X onto a space Y , then Y is mildly αg -normal.

Proof: Let A be a regular closed set and B be a regular open set containing A . Then by rc -continuity of f , $f^{-1}(A)$ is a regular closed set contained in the regular open set $f^{-1}(B)$. Since X is mildly αg -normal, there exists a αg -open set V such that $f^{-1}(A) \subset V \subset \alpha gcl(V) \subset f^{-1}(B)$ by Theorem 6.10. As f is M - αg -open and almost αg -irresolute surjection, it follows that $f(V) \in \alpha g O(Y)$ and $A \subset f(V) \subset \alpha gcl(f(V)) \subset B$. Hence Y is mildly αg -normal.

Theorem 6.12: If $f : X \rightarrow Y$ is rc -continuous, M - αg -closed map from a mildly αg -normal space X onto a space Y , then Y is mildly αg -normal.

7. αg -US spaces:

Definition 7.1: A sequence $\langle x_n \rangle$ is said to be αg -converges to a point x of X , written as $\langle x_n \rangle \rightarrow^{\alpha g} x$ if $\langle x_n \rangle$ is eventually in every αg -open set containing x .

Clearly, if a sequence $\langle x_n \rangle$ r -converges to a point x of X , then $\langle x_n \rangle$ αg -converges to x .

Definition 7.2: X is said to be αg -US if every sequence $\langle x_n \rangle$ in X αg -converges to a unique point.

Theorem 7.1: Every αg -US space is αg - T_1 .

Proof: Let X be αg -US space. Let x and y be two distinct points of X . Consider the sequence $\langle x_n \rangle$ where $x_n = x$ for every n . Clearly, $\langle x_n \rangle \rightarrow^{\alpha g} x$. Also, since $x \neq y$ and X is αg -US, $\langle x_n \rangle$ cannot αg -converge to y , i.e, there exists a αg -open set V containing y but not x . Similarly, for the sequence $\langle y_n \rangle$ where $y_n = y$ for all n , and proceeding as above we get a αg -open set U containing x but not y . Thus, the space X is αg - T_1 .

Theorem 7.2: Every αg - T_2 space is αg -US.

Proof: Let X be αg - T_2 space and $\langle x_n \rangle$ be a sequence in X . If possible suppose that $\langle x_n \rangle$ αg -converge to two distinct points x and y . That is, $\langle x_n \rangle$ is eventually in every αg -open set containing x and also in every αg -open set containing y .

This is contradiction since X is αg - T_2 space. Hence the space X is αg -US.

Definition 7.3: A set F is sequentially αg -closed if every sequence in F αg -converges to a point in F .

Theorem 7.3: X is αg -US iff the diagonal set is a sequentially αg -closed subset of $X \times X$.

Proof: Let X be αg -US. Let $\langle x_n, x_n \rangle$ be a sequence in Δ . Then $\langle x_n \rangle$ is a sequence in X . As X is αg -US, $\langle x_n \rangle \rightarrow^{\alpha g} x$ for a unique $x \in X$. i.e., if $\langle x_n \rangle \rightarrow^{\alpha g} x$ and y . Thus, $x = y$. Hence Δ is sequentially αg -closed.

Conversely, let Δ be sequentially αg -closed and let $\langle x_n \rangle \rightarrow^{\alpha g} x$ and y . Hence $\langle x_n, x_n \rangle \rightarrow^{\alpha g} (x, y)$. Since Δ is sequentially αg -closed, $(x, y) \in \Delta$ which means that $x = y$ implies space X is αg -US.

Definition 7.4: A subset G of a space X is said to be sequentially αg -compact if every sequence in G has a subsequence which αg -converges to a point in G .

Theorem 7.4: In a αg -US space every sequentially αg -compact set is sequentially αg -closed.

Proof: Let X be αg -US space. Let Y be a sequentially αg -compact subset of X . Let $\langle x_n \rangle$ be a sequence in Y . Suppose that $\langle x_n \rangle$ αg -converges to a point in $X - Y$. Let $\langle x_{np} \rangle$ be subsequence of $\langle x_n \rangle$ that αg -converges to a point $y \in Y$ since Y is sequentially αg -compact. Also, let a subsequence $\langle x_{np} \rangle$ of $\langle x_n \rangle$ αg -converge to $x \in X - Y$. Since $\langle x_{np} \rangle$ is a sequence in the αg -US space X , $x = y$. Thus, Y is sequentially αg -closed set.

Next, we give a hereditary property of αg -US spaces.

Theorem 7.5: Every regular open subset of a αg -US space is αg -US.

Proof: Let X be a αg -US space and $Y \subset X$ be an regular open set. Let $\langle x_n \rangle$ be a sequence in Y . Suppose that $\langle x_n \rangle$ αg -converges to x and y in Y . We shall prove that $\langle x_n \rangle$ αg -converges to x and y in X . Let U be any αg -open subset of X containing x and V be any αg -open set of X containing y . Then, $U \cap Y$ and $V \cap Y$ are αg -open sets in Y . Therefore, $\langle x_n \rangle$ is eventually in $U \cap Y$ and $V \cap Y$ and so in U and V . Since X is αg -US, this implies that $x = y$. Hence the subspace Y is αg -US.

Theorem 7.6: A space X is αg - T_2 iff it is both αg - R_1 and αg -US.

Proof: Let X be αg - T_2 space. Then X is αg - R_1 and αg -US by Theorem 7.2.

Conversely, let X be both αg - R_1 and αg -US space. By Theorem 7.1, X is both αg - T_1 and αg - R_1 and, it follows that space X is αg - T_2 .

Definition 7.5: A point y is a αg -cluster point of sequence $\langle x_n \rangle$ iff $\langle x_n \rangle$ is frequently in every αg -open set containing x . The set of all αg -cluster points of $\langle x_n \rangle$ will be denoted by αg -cl(x_n).

Definition 7.6: A point y is αg -side point of a sequence $\langle x_n \rangle$ if y is a αg -cluster point of $\langle x_n \rangle$ but no subsequence of $\langle x_n \rangle$ αg -converges to y .

Now, we define the following.

Definition 7.7: A space X is said to be

- (i) αg - S_1 if it is αg -US and every sequence $\langle x_n \rangle$ αg -converges with subsequence of $\langle x_n \rangle$ αg -side points.
- (ii) αg - S_2 if it is αg -US and every sequence $\langle x_n \rangle$ in X αg -converges which has no αg -side point.

Lemma 7.1: Every αg - S_2 space is αg - S_1 and Every αg - S_1 space is αg -US.

Now using the notion of sequentially continuous functions, we define the notion of sequentially αg -continuous functions.

Definition 7.8: A function f is said to be sequentially αg -continuous at $x \in X$ if $f(x_n) \rightarrow^{\alpha g} f(x)$ whenever $\langle x_n \rangle \rightarrow^{\alpha g} x$. If f is sequentially αg -continuous at all $x \in X$, then f is said to be sequentially αg -continuous.

Theorem 7.7: Let f and g be two sequentially αg -continuous functions. If Y is αg -US, then the set $A = \{x \mid f(x) = g(x)\}$ is sequentially αg -closed.

Proof: Let Y be αg -US and suppose that there is a sequence $\langle x_n \rangle$ in A αg -converging to $x \in X$. Since f and g are sequentially αg -continuous functions, $f(x_n) \rightarrow^{\alpha g} f(x)$ and $g(x_n) \rightarrow^{\alpha g} g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, A is sequentially αg -closed.

Next, we prove the product theorem for αg -US spaces.

Theorem 7.8: Product of arbitrary family of αg -US spaces is αg -US.

Proof: Let $X = \prod_{\lambda \in \Lambda} X_\lambda$ where X_λ is α g-US. Let a sequence $\langle x_n \rangle$ in X α g-converges to $x (= x_\lambda)$ and $y (= y_\lambda)$. Then $\langle x_{n\lambda} \rangle \rightarrow^{\alpha g} x_\lambda$ and y_λ for all $\lambda \in \Lambda$. For suppose there exists a $\mu \in \Lambda$ such that $\langle x_{n\mu} \rangle$ does not α g-converges to x_μ .

Then there exists a τ_μ - α g-open set U_μ containing x_μ such that $\langle x_{n\mu} \rangle$ is not eventually in U_μ . Consider the set $U = \prod_{\lambda \in \Lambda} X_\lambda \times U_\mu$. Then U is a α g-open subset of X and $x \in U$. Also, $\langle x_n \rangle$ is not eventually in U , which contradicts the fact that $\langle x_n \rangle \rightarrow^{\alpha g} x$. Thus we get $\langle x_{n\lambda} \rangle \rightarrow^{\alpha g} x_\lambda$ and y_λ for all $\lambda \in \Lambda$. Since X_λ is α g-US for each $\lambda \in \Lambda$. Thus $x = y$. Hence X is α g-US.

8. Sequentially sub- α g-continuity:

Definition 8.1: A function f is said to be

- (i) sequentially nearly α g-continuous if for each point $x \in X$ and each sequence $\langle x_n \rangle \rightarrow^{\alpha g} x$ in X , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ such that $\langle f(x_{nk}) \rangle \rightarrow^{\alpha g} f(x)$.
- (ii) sequentially sub- α g-continuous if for each point $x \in X$ and each sequence $\langle x_n \rangle \rightarrow^{\alpha g} x$ in X , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ and a point $y \in Y$ such that $\langle f(x_{nk}) \rangle \rightarrow^{\alpha g} y$.
- (iii) sequentially α g-compact preserving if $f(K)$ is sequentially α g-compact in Y for every sequentially α g-compact set K of X .

Lemma 8.1: Every function f is sequentially sub- α g-continuous if Y is a sequentially α g-compact.

Proof: Let $\langle x_n \rangle \rightarrow^{\alpha g} x$ in X . Since Y is sequentially α g-compact, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ α g-converging to a point $y \in Y$. Hence f is sequentially sub- α g-continuous.

Theorem 8.1: Every sequentially nearly α g-continuous function is sequentially α g-compact preserving.

Proof: Assume f is sequentially nearly α g-continuous and K any sequentially α g-compact subset of X . Let $\langle y_n \rangle$ be any sequence in $f(K)$. Then for each positive integer n , there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $\langle x_n \rangle$ is a sequence in the sequentially α g-compact set K , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ α g-converging to a point $x \in K$. By hypothesis, f is sequentially nearly α g-continuous and hence there exists a subsequence $\langle x_j \rangle$ of $\langle x_{nk} \rangle$ such that $f(x_j) \rightarrow^{\alpha g} f(x)$. Thus, there exists a subsequence $\langle y_j \rangle$ of $\langle y_n \rangle$ α g-converging to $f(x) \in f(K)$. This shows that $f(K)$ is sequentially α g-compact set in Y .

Theorem 8.2: Every sequentially α -continuous function is sequentially α g-continuous.

Proof: Let f be a sequentially α -continuous and $\langle x_n \rangle \rightarrow^\alpha x \in X$. Then $\langle x_n \rangle \rightarrow^{\alpha g} x$. Since f is sequentially α -continuous, $f(x_n) \rightarrow^\alpha f(x)$. But we know that $\langle x_n \rangle \rightarrow^\alpha x$ implies $\langle x_n \rangle \rightarrow^{\alpha g} x$ and hence $f(x_n) \rightarrow^{\alpha g} f(x)$ implies f is sequentially α g-continuous.

Theorem 8.3: Every sequentially α g-compact preserving function is sequentially sub- α g-continuous.

Proof: Suppose f is a sequentially α g-compact preserving function. Let x be any point of X and $\langle x_n \rangle$ any sequence in X α g-converging to x . We shall denote the set $\{x_n | n = 1, 2, 3 \dots\}$ by A and $K = A \cup \{x\}$. Then K is sequentially α g-compact since $(x_n) \rightarrow^{\alpha g} x$. By hypothesis, f is sequentially α g-compact preserving and hence $f(K)$ is a sequentially α g-compact set of Y . Since $\{f(x_n)\}$ is a sequence in $f(K)$, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ α g-converging to a point $y \in f(K)$. This implies that f is sequentially sub- α g-continuous.

Theorem 8.4: A function $f: X \rightarrow Y$ is sequentially α g-compact preserving iff $f|_K: K \rightarrow f(K)$ is sequentially sub- α g-continuous for each sequentially α g-compact subset K of X .

Proof: Suppose f is a sequentially α g-compact preserving function. Then $f(K)$ is sequentially α g-compact set in Y for each sequentially α g-compact set K of X . Therefore, by Lemma 8.1 above, $f|_K: K \rightarrow f(K)$ is sequentially α g-continuous function.

Conversely, let K be any sequentially α g-compact set of X . Let $\langle y_n \rangle$ be any sequence in $f(K)$. Then for each positive integer n , there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $\langle x_n \rangle$ is a sequence in the sequentially α g-compact set K , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ α g-converging to a point $x \in K$. By hypothesis, $f|_K: K \rightarrow f(K)$ is sequentially sub- α g-continuous and hence there exists a subsequence $\langle y_{nk} \rangle$ of $\langle y_n \rangle$ α g-converging to a point $y \in f(K)$. This implies that $f(K)$ is sequentially α g-compact set in Y . Thus, f is sequentially α g-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub- α -continuous function to be sequentially α -compact preserving.

Corollary 8.1: If f is sequentially sub- α -continuous and $f(K)$ is sequentially α -closed set in Y for each sequentially α -compact set K of X , then f is sequentially α -compact preserving function.

Proof: Omitted.

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