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INTUITIONISTIC FUZZY PRE SEMI CONNECTEDNESS

D. Amsaveni*, M. K. Uma and E. Roja

Department of Mathematics, Sri Sarada College for Women, Salem 636016, Tamilnadu, India
E-mail : d_amsaveni@rediffmail.com

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ABSTRACT

In this paper various types of intuitionistic fuzzy pre semi connectedness together with the preservation properities under the concept of intuitionistic fuzzy pre semi open set and intuitionistic fuzzy pre semi continuous functions are introduced and studied.

Keywords: *Intuitionistic fuzzy pre semi super connectedness, intuitionistic fuzzy pre semi strongly connectedness, intuitionistic fuzzy pre semi C_i-connectedness (i =1,2,3,4), intuitionistic fuzzy pre semi C_M - connecteness, intuitionistic fuzzy pre semi C_s - Connectedness, intuitionistic fuzzy pre semi C_5-connectedness; intuistionistic fuzzy pre semi compactness; intuitionistic fuzzy pre semi normal spaces.*

1. INTRODUCTION

After the introduction of the concept of fuzzy sets by Zadeh [14] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2 - 4]. Later this concept was generalized to "intuitionistic L-fuzzy sets" by Atanassov and stoeva [5]. An introduction to intuitionistic fuzzy topological spaces was introduced by Dogan Coker [8]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Coker (1997). The construction is based on the idea of intuitionistic fuzzy set developed by Atanassov (1983,1986; Atanassov and Stoeva, 1983). The concept of fuzzy pre open set was introduced by [7] and the concept of fuzzy pre connectedness and fuzzy pre disconnectedness in fuzzy topological spaces was introduced by [6]. In this paper intuitionistic fuzzy pre semi connectedness, intuitionistic fuzzy pre semi super connectedness, intuitionistic fuzzy pre semi strongly connectedness, intuitionistic fuzzy pre semi C_i-connectedness (i =1,2,3,4), intuitionistic fuzzy pre semi C_M - connecteness, intuitionistic fuzzy pre semi C_s - Connectedness, intuitionistic fuzzy pre semi C_5-connectedness, intuistionistic fuzzy pre semi compactness, intuitionistic fuzzy pre semi normal spaces are studied as in [6,11,12,13]. Some interesting properties and characterizations are studied.

2. PRELIMINARIES

Definition 2.1: [3] Let X be a non empty fixed set. An intuitionistic fuzzy set (*IFS* for short) A is an object having the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ where the functions $\mu_A : X\to I$ and $\gamma_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

Remark 2.1: For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$.

Definition 2.2: [3] Let X be a non empty set and the *IFSs* A and B be in the form
$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}, B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$.Then

(a) $A\subseteq B$ iff $\mu_A(x)\le \mu_B(x)$ and $\gamma_A(x)\ge \gamma_B(x)$ for all $x\in X$;

(b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$;

(c) $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$;

Corresponding author: D. Amsaveni, *E-mail: d_amsaveni@rediffmail.com

(d) $A \cap B = \left\{ \left\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \right\rangle : x \in X \right\}$;
(e) $A \cup B = \left\{ \left\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \right\rangle : x \in X \right\}$;
(f) $[\ ]A = \left\{ \left\langle x, \mu_A(x), 1-\mu_A(x) \right\rangle : x \in X \right\}$;
(g) $\langle\ \rangle A = \left\{ \left\langle x, 1-\gamma_A(x), \gamma_A(x) \right\rangle : x \in X \right\}$.

Definition 2.3: [8] Let X be a non empty set and let $\{A_i : i \in J\}$ be an arbitrary family of *IFSs* in X. Then
(a) $\bigcap A_i = \left\{ \left\langle x, \wedge \mu_{A_i}(x), \vee \gamma_{A_i}(x) \right\rangle : x \in X \right\}$;
(b) $\bigcup A_i = \left\{ \left\langle x, \vee \mu_{A_i}(x), \wedge \gamma_{A_i}(x) \right\rangle : x \in X \right\}$.

Definition 2.4: [8] Let X be a non empty fixed set. Then $0_\sim = \left\{ \left\langle x, 0, 1 \right\rangle : x \in X \right\}$ and $1_\sim = \left\{ \left\langle x, 1, 0 \right\rangle : x \in X \right\}$.

Definition 2.5: [8] Let X and Y be two non empty fixed sets and $f : X \to Y$ be a function. Then
(a) If $B = \left\{ \left\langle y, \mu_B(y), \gamma_B(y) \right\rangle : y \in Y \right\}$ is an *IFS* in *Y*, then the pre image of B under f, denoted by $f^{-1}(B)$, is the *IFS* in X defined by $f^{-1}(B) = \left\{ \left\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \right\rangle : x \in X \right\}$.

(b) If $A = \left\{ \left\langle x, \lambda_A(x), \nu_A(x) \right\rangle : x \in X \right\}$ is an *IFS* in X, then the image of A under f, denoted by $f(A)$, is the *IFS* in *Y* defined by $f(A) = \left\{ \left\langle y, f(\lambda_A)(y), (1 - f(1-\nu_A))(y) \right\rangle : y \in Y \right\}$ where,

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda_A(x) & \text{if} \quad f^{-1}(y) \neq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1-\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & \text{if} \quad f^{-1}(y) \neq 0 \\ 1, & \text{otherwise.} \end{cases}$$

for the *IFS* $A = \left\{ \left\langle x, \mu_A(x), \gamma_A(x) \right\rangle : x \in X \right\}$.

Definition 2.6: [8] Let X be a non empty set. An intuitionistic fuzzy topology (*IFT* for short) on a non empty set X is a family τ of intuitionistic fuzzy sets (*IFSs* for short) in X satisfying the following axioms: (T_1) $0_\sim, 1_\sim \in \tau$, (T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$, (T_3) $\bigcup\limits_{\subseteq \tau} G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\}$. In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (*IFTS* for short) and any *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS* for short) in X.

Proposition 2.1: [8] Let (X, τ) be an *IFTS* on X. Then, we can also construct several *IFTSs* on X in the following way: (a) $\tau_{0,1} = \left\{ [\ ]G : G \in \tau \right\}$, (b) $\tau_{0,2} = \left\{ \langle\ \rangle G : G \in \tau \right\}$.

Definition 2.7: [8] Let X be a non empty set. The complement $\overline{A}$ of an *IFOS* A in an *IFTS* (X, τ) is called an intuitionistic fuzzy closed set (*IFCS* for short) in X.

Definition 2.8: [8] Let (X, τ) be an *IFTS* and $A = \langle x, \mu_A, \gamma_A \rangle$ be an *IFS* in X. Then the fuzzy interior and fuzzy closure of A are defined by $cl(A) = \bigcap \left\{ K : K \text{ is an } IFCS \text{ in } X \text{ and } A \subseteq K \right\}$, $\text{int}(A) = \bigcup \left\{ G : G \text{ is an } IFOS \text{ in } X \text{ and } G \subseteq A \right\}$.

Remark 2.2: [8] Let (X, τ) be an *IFTS*. $cl(A)$ is an *IFCS* and $\text{int}(A)$ is an *IFOS* in X, and
(a) A is an *IFCS* in X iff $cl(A) = A$; (b) A is an *IFOS* in X iff $\text{int}(A) = A$.

Definition2.9: [7] Let X , *Y be* non empty sets and $U = \langle x, \mu_U(x), \gamma_U(x) \rangle$, $V = \langle y, \mu_V(y), \gamma_V(y) \rangle$, are *IFS's* of X and *Y*, respectively. Then $U \times V$ is an *IFS* of $X \times Y$ defined by

$$(U \times V)(x, y) = \langle (x, y), \min(\mu_U(x), \mu_V(y)), \max(\gamma_U(x), \gamma_V(y)) \rangle .$$

Proposition 2.2: [8] Let (X, τ) be an *IFTS*. For any *IFS* A in (X, τ), we have

(a) $cl(\overline{A}) = \overline{\mathrm{int}(A)},$ (b) $\mathrm{int}(\overline{A}) = \overline{cl(A)}.$

Definition 2.10: [8] Let (X, τ) and (Y, ϕ) be two *IFTSs* and let $f : X \to Y$ be a function. Then f is said to be fuzzy continuous iff the pre image of each *IFS* in ϕ is an *IFS* in τ.

Definition 2.11:[8] Let (X, τ) and (Y, ϕ) be two *IFTSs* and let $f : X \to Y$ be a function. Then f is said to be fuzzy open (resp. closed) iff the image of each *IFS* in τ (resp.(1- τ)) is an *IFS* in ϕ (resp.(1-ϕ)).

A space (X, T) represent intutionistic fuzzy topological spaces and for a subset *A* of a space (X, T), *IFcl(A), IFint(A), IFPScl(A), IFPSint(A),* and $\overline{A}$ denote an intutionistic fuzzy closure of *A*, an intutionistic fuzzy interior of *A*, intutionistic fuzzy pre semi closure of *A*, an intutionistic fuzzy pre semi interior of *A* and the complement of *A* in X respectively.

Definition 2.12:[10] A subset *A* of a space (X, T) is called

(i) An *IF* semi open set if $A \subseteq IFcl\,(IF\,\mathrm{int}(A))$ and an *IF* semi closed set if $IF\,\mathrm{int}(IFcl(A)) \subseteq A$;

(ii) An *IF* semi pre open set if $A \subseteq IFcl(IF\,\mathrm{int}(IFcl(A)))$ and an IF semi pre closed set if; An *IF* semi closure (resp. *IF* semi pre closure) of a subset A of (X, T) is the intersection of all *IF* semi closed (resp. *IF* semi pre closed) sets that contain A and is denoted by $IFs\,cl(A)$ (resp. $IFsp\,cl(A)$).

Lemma 2.1: [11] If $A \cap B = 0_\sim$, then $A \subseteq \overline{B}$.

Corollary 2.1: [11] If $A \not\subseteq \overline{B}$, then $A \cap B \neq 0_\sim$.

Definition 2.13: [9] Let $A = \langle x, \mu_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \gamma_B \rangle$ be two IFSs in X . *A* and *B* are said to be q-coincident (AqB for short) iff there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

Proposition2.3: [9] Let *A* and B be two *IFSs* in X . Then

(a) $A\not{q}B \Leftrightarrow A \subseteq \overline{B}$,

(b) $AqB \Leftrightarrow A \not\subseteq \overline{B}$.

Remark 2.3: Let (X, T) be a *IFTS* and $A \subset X$ be a subset of *X*. An intuitionistic fuzzy characteristic function of *A* is defined as $\chi_A(x) = \begin{cases} (1,0) & if \quad x \in A \\ (0,1) & if \quad x \notin A \end{cases}$.

NOTATION

IF denotes intuitionistic fuzzy.
IF pre semi open denotes intuitionistic fuzzy pre semi open.
IF pre semi closed denotes intutioinistic fuzzy pre semi closed.
IF PSOS denotes intuitionistic fuzzy pre semi open set.
IF PSCS denotes intuitionistic fuzzy pre semi closed set.
IF pre semi regular open denotes intuitionistic fuzzy pre semi regular open.
IF pre semi C_i -connectedness (i =1,2,3,4) denotes intuitionistic fuzzy pre semi C_i -connectedness (i =1,2,3,4).
IF pre semi weakly separated denotes intuitionistic fuzzy pre semi weakly separated.
IF pre semi q separated denotes intuitionistic fuzzy pre semi q separated.

IF pre semi C_M - connecteness denotes intuitionistic fuzzy pre semi C_M - connecteness.
IF pre semi C_s - Connectedness denotes intuitionistic fuzzy pre semi C_s - Connectedness.
IF pre semi C_5 -connected denotes intuitionistic fuzzy pre semi C_5 - connected. *IF* pre semi strongly connectedness denotes intuitionistic fuzzy pre semi strongly connectedness
IF pre semi compactness denotes intuitionistic fuzzy pre semi compactness
IF pre semi normal spaces denotes intuitionistic fuzzy pre semi normal spaces.

3. INTUITIONISTIC FUZZY PRE SEMI SUPER CONNECTEDNESS

Definition 3.1: A subset *A* of *IFTS* (X,T) is called an *IF* pre semi regular open set if $IFPS\,\mathrm{int}(IFPScl(A)) = A$ and an *IF* pre semi regular closed set if $IFPScl(IFPS\,\mathrm{int}(A)) = A$.

Definition 3.2: A subset *A* of *IFTS* (X,T) is called *IF* pre semi closed if $IF\ spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *IF* g-open in (X,T).

Definition 3.3: A subset *A* of *IFTS* (X,T) is called *IF* pre semi open if $\overline{A}$ is *IF* pre semi closed.

Definition 3.4 :(a) Let (X,T) be an *IFTS*. If there exists an *IF* pre semi regular open set A in X such that $0_\sim \neq A \neq 1_\sim$, then X is called *IF* pre semi super disconnected. (b) *X* is called *IF* pre semi super connected, if X is not *IF* pre semi super disconnected.

Proposition 3.1: Let *(X, T)* be an *IFTS*. Then the following assertions are equivalent.
(a) *X* is *IF* pre semi super connected.
(b) For each *IFPSOS* $A \neq 0_\sim$ in *X* we have $IFPScl(A) = 1_\sim$.
(c) For each *IFPSCS* $A \neq 1_\sim$ in *X* we have $IFPS\,\mathrm{int}(A) = 0_\sim$.
(d) There exist no *IFPSOSs A* and *B* in *X* such that $A \neq 0_\sim \neq B$ and $A \subseteq \overline{B}$.
(e) There exist no *IFPSOSs A* and *B* in *X* such that $A \neq 0_\sim \neq B$, $B = \overline{IFPScl(A)}$, $A = \overline{IFPScl(B)}$.
(f) There exist no *IFPSCSs* A and B in X such that $A \neq 1_\sim \neq B$, $B = \overline{IFPS\,\mathrm{int}(A)}$, $A = \overline{IFPS\,\mathrm{int}(B)}$.

Proof: (a) $\Rightarrow$ (b) : Assume that there exists an *IFPSOS* $A \neq 0_\sim$ such that $IFPScl(A) \neq 1_\sim$. Now take $B = IFPS\,\mathrm{int}(IFPScl(A))$. Then *B* is a proper *IF* pre semi regular open set in *X*, and this in contradiction with the *IF* pre semi super connectedness of *X*.

(b) $\Rightarrow$ (c) : Let $A \neq 1_\sim$ be an *IFPSCS* in *X*. If we take $B = \overline{A}$, then *B* is an *IFPSOS* in *X* and $B \neq 0_\sim$. Hence, by (b) $IFPScl(B) = 1_\sim \Rightarrow \overline{IFPScl(B)} = 0_\sim \Rightarrow IFPS\,\mathrm{int}(\overline{B}) = 0_\sim \Rightarrow IFPS\,\mathrm{int}(A) = 0_\sim$.

(c) $\Rightarrow$ (d): Let *A* and *B* be *IFPSOSs* in *X* such that $A \neq 0_\sim \neq B$ and $A \subseteq \overline{B}$. Since $\overline{B}$ is an *IFPSCS* in *X* and $B \neq 0_\sim \Rightarrow \overline{B} = 1_\sim$, Hence by (c) $IFPS\,\mathrm{int}(\overline{B}) = 0_\sim$. But from $A \subseteq \overline{B}$, it follows that $0_\sim \neq A = IFPS\,\mathrm{int}(A) \subseteq IFPS\,\mathrm{int}(\overline{B}) = 0_\sim$, which is a contradiction.

(d) $\Rightarrow$ (a) : Let $0_\sim \neq A \neq 1_\sim$ be an IF pre semi regular open set in *X*. If we take $B = \overline{IFPScl(A)}$, we get $B \neq 0_\sim$. (Because, otherwise we have

$$B = 0_\sim \Rightarrow \overline{IFPScl(A)} = 0_\sim \Rightarrow IFPScl(A) = 1_\sim$$
$$\Rightarrow A = IFPS\,\mathrm{int}(IFPScl(A)) = IFPS\,\mathrm{int}(1_\sim) = 1_\sim,$$

But the last result contradicts the fact that $A \neq 1_\sim$) We also have $A \subseteq \overline{B}$, and this is a contradiction too.

(a) $\Rightarrow$ (e) : Let *A* and *B* be *IFPSOSs* in *X* such that $A \neq 0_\sim \neq B$ and $B = \overline{IFPScl(A)}$, $A = \overline{IFPScl(B)}$. Now, we have $IFPS\,\mathrm{int}(IFPScl(A)) = IFPS\,\mathrm{int}(\overline{B}) = \overline{IFPScl(B)} = A$ and $A \neq 0_\sim$, $A \neq 1_\sim$.

(If not, i.e. if $A = 1_\sim$, then $1_\sim = \overline{IFPScl(B)} \Rightarrow 0_\sim = IFPScl(B) \Rightarrow B = 0_\sim$) But this is a contradiction.

(e)$\Rightarrow$ (a) : Let A be an *IFPSOS* in X such that $A = IFPS\,\text{int}(IFPScl(A))$, $0_\sim \neq A \neq 1_\sim$. Now take $B = \overline{IFPScl(A)}$. In this case we get $B \neq 0_\sim$ and B is an *IFPSOS* in X and $\overline{IFPScl(B)} = \overline{IFPScl(\overline{IFPScl(A)})} = \overline{\overline{IFPS\,\text{int}(IFPScl(A))}} = IFPS\,\text{int}(IFPScl(A))$ which is a contradiction.

(e)$\Rightarrow$ (f) : Let A and B be *IFPSCSs* in X such that $A \neq 1_\sim \neq B$, $B = \overline{IFPS\,\text{int}(A)}$, $A = \overline{IFPS\,\text{int}(B)}$. Taking $C = \overline{A}$ and $D = \overline{B}$, C and D becomes *IFPSOSs* in X and $C \neq 0_\sim \neq D$, $\overline{IFPScl(C)} = \overline{IFPScl(\overline{A})}$ $= \overline{\overline{IFPS\,\text{int}(A)}} = IFPS\,\text{int}(A) = \overline{B} = D$ and similarly $\overline{IFPScl(D)} = C$ **.** But this an obvious contradiction.

(f)$\Rightarrow$ (e) : We can use a similar technique as in (e) $\Rightarrow$ (f).

Definition 3.5: A subset of an *IFTS* (X,T) is called an *IF* pre semi super connected subset of X if it is an *IF* pre semi super connected topological space as a *IF* subspace of X.

Proposition 3.2: Let (X,T) be an *IFTS.* Let *A* be an *IF* pre semi super connected subset of (X,T). If there exist *IF* pre semi closed sets *f* and *k* in *X* such that $\bar{f} + k = f + \bar{k} = 1$, then $f / A = 1$ or $k / A = 1$.

Proposition 3.3: Let (X,T) be an *IFTS* and $A \subset X$ be an *IF* pre semi super connected subset of X such that χ_A is an *IF* pre semi open set in *X*. If A is a *IF* pre semi regular open set, then either $\chi_A \subseteq A$ or $\chi_A \subseteq \overline{A}$.

Proposition 3.4: Let $\{O_\alpha\}_{\alpha \in A}$ be an family of subsets of an *IFTS* (X,T) such that χ_{O_α} is *IF* pre semi open. If $\bigcap_{\alpha \in A} O_\alpha \neq \phi$ and each O_α is an *IF* pre semi super connected subset of *X*, then $\bigcup_{\alpha \in A} O_\alpha$ is also an *IF* pre semi super connected of *X*.

Proof: Let $Y = \bigcup_{\alpha \in A} O_\alpha$ and let $A = \{ x, \mu_A(x), \gamma_A(x) \}$ and $G = \{ x, \chi_{O_\alpha}, 1 - \chi_{O_\alpha} \}$.

Suppose that *Y* is not an *IF* pre semi super connected subset of *X*. Then there exists a proper *IF* pre semi regular open set A_Y in the *IF* subspace *Y* of *X*. Each χ_{O_α} is *IF* pre semi opens in *X*. So each χ_{O_α} / Y is *IF* pre semi open in *Y*. Also each O_α is an *IF* pre semi super connected subset of the subspace *Y* as it is so in *X*. Therefore, by Proposition 3.3, for each $\alpha \in A$, either $\chi_{O_\alpha} / Y \subseteq A_Y$ or $\chi_{O_\alpha} / Y \subseteq \overline{A}_Y$ .Suppose $x_0 \in \bigcup_{\alpha \in A} O_\alpha$. Then either $A_Y(x_0) = 1$ or $A_Y(x_0) = 0$ .If $A_Y(x_0) = 1$, then $\chi_{O_\alpha} / Y \subseteq A_Y$, for every $\alpha \in A$. Hence, $\chi_A / Y = \bigcup_{\alpha \in A} (\chi_{O_\alpha} / Y) \subseteq A_Y$. But $A_Y \subseteq \chi_Y / Y$; so $A_Y = 1$, which is a contradiction since $A_Y \neq 1$. Similarly, if $A_Y(x_0) = 0$, then $A_Y = 0$, which is also a contradiction.

Proposition 3.5: If *A* and *B* are *IF* pre semi super connected subsets of an *IFTS* (X,T) and $IFPS\,\text{int}\,\chi_B / A \neq 0$ or $IFPS\,\text{int}\,\chi_A / B \neq 0$, then $A \bigcup B$ is an *IF* pre semi super connected subset of *X*.

Proposition 3.6: If $\{A_\alpha\}_{\alpha \in \Gamma}$ is a family of *IF* pre semi super connected subsets of an *IFTS* (X,T) such that $\bigcap_{\alpha \in \Gamma} \chi_{A_\alpha} \neq 0$, then $\bigcup_{\alpha \in \Gamma} A_\alpha$ is an *IF* pre semi super connected subset of *X*.

Proposition 3.7: Let (X,T) be an *IFTS.* Suppose that (X,T) is *IF* pre semi super connected and *C* is an *IF* pre semi connected subset of *X*. Further, suppose that $X \setminus C$ contains a set *V* such that $\chi_V / X \setminus C$ is an *IF* pre semi open set in the *IF* subspace $X \setminus C$ of X. Then $C \bigcup V$ is an *IF* pre semi super connected subset of *X*.

Proposition 3.8: If *A* and *B* are subsets of an *IFTS* (X,T) and $\chi_A \subseteq \chi_B \subseteq IFPS\,cl\,\chi_A$ and *A* is an *IF* pre semi super connected subset of *X*, then so is *B*.

4. INTUITIONISTIC FUZZY PRE SEMI STRONGLY CONNECTEDNESS

Definition 4.1: An *IFTS* (X,T) is said to be *IF* pre semi strongly connected, if there exists no non - zero *IFPSCSs A* and *B* in *X* such that $\mu_A + \mu_B \subseteq 1$ and $\gamma_A + \gamma_B \supseteq 1$.

Proposition 4.1: Let (X,T) be an *IFTS.* (X,T) is IF pre semi strongly connected, iff there exists no *IFPSOSs A* and *B* in *X* such that $A \neq 1_\sim \neq B$ and $\mu_A + \mu_B \supseteq 1$ and $\gamma_A + \gamma_B \subseteq 1$.

Proposition 4.2: If $A \subset X$ and (X,T) is an *IFTS* then *A* is an *IF* pre semi strongly connected subset of X $\Leftrightarrow$ for any *IFPSOSs* A_1 and A_2 in X, $\chi_A \subseteq A_1 + A_2 \Leftrightarrow$ either $\chi_A \subseteq A_1$ or $\chi_A \subseteq A_2$.

Proposition 4.3: If *F* is a subset of *IFTS* (X,T) such that χ_F is *IF* pre semi closed in *X*, then *X* is *IF* pre semi strongly connected $\Rightarrow$ *F* is an *IF* pre semi strongly connected subset of *X*.

Proposition 4.4: Let (X,T) and (Y,S) be *IF* pre semi strongly connected spaces. Assume *X* and *Y* are product related. Then $X \times Y$ is an IF pre semi strongly connected space.

Proposition 4.5: Let (X,T) be an *IFTS.* And let $f:(X,T) \to (Y,S)$ be an *IF* pre semi continuous and surjective function. If *X* is *IF* pre semi strongly connected, then so is *Y.*

5. INTUITIONISTIC FUZZY PRE SEMI C_i - CONNECTEDNESS.

Definition 5.1: Let *N* be an *IFS* in *IFTS* (X,T).

(a) If there exist *IFPSOSs M* and *W* in *X* satisfying the following properties, then *N* is called *IF* pre semi C_i - disconnected $(i = 1,2,3,4)$:

$$C_1: N \subseteq M \cup W,\ M \cap W \subseteq \bar{N},\ N \cap M \neq 0_\sim\ \ N \cap W \neq 0_\sim,$$
$$C_2: N \subseteq M \cup W,\ N \cap M \cap W = 0_\sim,\ N \cap M \neq 0_\sim\ \ N \cap W \neq 0_\sim,$$
$$C_3: N \subseteq M \cup W,\ M \cap W \subseteq \bar{N},\ \ M \not\subseteq \bar{N}\ \ W \not\subseteq \bar{N},$$
$$C_4: N \subseteq M \cup W,\ N \cap M \cap W = 0_\sim,\ \ M \not\subseteq \bar{N}\ \ W \not\subseteq \bar{N},$$

(b) *N* is said to be *IF* pre semi C_i - connected $(i = 1,2,3,4)$ if *N* is not *IF* pre semi C_i - disconnected $(i = 1,2,3,4)$.

Obviously, we can obtain the following implications between several types of *IF* pre semi C_i - connectedness $(i = 1,2,3,4)$:

IF pre semi C_1 - connectedness $\Rightarrow$ *IF* pre semi C_2 - connectedness
$\Downarrow$ $\Downarrow$
IF pre semi C_3 - connectedness $\Rightarrow$ *IF* pre semi C_4 - connectedness

Example 5.1: Let (X,T) be an *IFTS.* Let $X = \{a,b,c\}$ and

$$M = \left\langle x, \left(\frac{a}{.01}, \frac{b}{0}, \frac{c}{.25}\right), \left(\frac{a}{.99}, \frac{b}{1}, \frac{c}{.75}\right)\right\rangle,$$
$$W = \left\langle x, \left(\frac{a}{.02}, \frac{b}{0}, \frac{c}{.25}\right), \left(\frac{a}{.98}, \frac{b}{1}, \frac{c}{.75}\right)\right\rangle, \quad .$$

Then $T = \{0_{\sim}, 1_{\sim}, M, W\}$ is an *IFT* on *X*.

Consider the *IFS* $N = \left\langle x, \left(\frac{a}{.01}, \frac{b}{0}, \frac{c}{.23}\right), \left(\frac{a}{.99}, \frac{b}{1}, \frac{c}{.77}\right) \right\rangle$ in *X*.

Now, *N* is *IF* pre semi C_1 - disconnected and *IF* pre semi C_2 - connected, *IF* pre semi C_3 - connected.

Definition 5.2: Let *A* and *B* be two *IFSs* in *IFTS* (X, T). *A* and *B* are said to be

(a) *IF* pre semi weakly separated, if $IFPS\,cl(A) \subseteq \overline{B}$ and $IFPS\,cl(B) \subseteq \overline{A}$;

(b) *IF* pre semi q separated, $IFPS\,cl(A) \cap B = 0_{\sim} = A \cap IFPS\,cl(B)$.

Here, we generalize the concepts of *IF* pre semi C_s - connectedness and *IF* pre semi C_M - connectedness.

Definition 5.3: (a) An *IFTS* (X, T) is said to be *IF* pre semi C_s - disconnected, if there exist *IF* pre semi weakly separated non zero *IFSs A* and *B* in (X, T) such that $A \cup B = 1_{\sim}$.

(b) (X, T) is called *IF* pre semi C_s - connected, if (X, T) is not *IF* pre semi C_s - disconnected.

Example 5.2: Let (X, T) be an *IFTS*. Let $X = \{a, b, c\}$ and $A = \left\langle x, \left(\frac{a}{0}, \frac{b}{0}, \frac{c}{.2}\right), \left(\frac{a}{1}, \frac{b}{1}, \frac{c}{.8}\right) \right\rangle$, Then $T = \{0_{\sim}, 1_{\sim}, A\}$ is an *IFT* on *X*, and *X* is *IF* pre semi C_s - connected.

Definition 5.4:

(a) Let (X, T) be an *IFTS*. *X* is said to be *IF* pre semi C_M - disconnected if there exist *IF* pre semi q separated non zero *IFSs A* and *B* in *X* such that $A \cup B = 1_{\sim}$.

(b) *X* is called *IF* pre semi C_M - connected, if *X* is not *IF* pre semi C_M disconnected.

Let us give the relation between these two types of *IF* pre semi connectedness in *IFTSs*.

Proposition 5.1: Let (X, T) be an *IFTS*. (X, T) is *IF* pre semi C_s - connected iff (X, T) is *IF* pre semi C_M - connected.

Remark 5.1: Since the *IF* pre semi C_s - connectedness and *IF* pre semi C_M - connectedness of *IFTS* (X, T) are identical, we may define *IF* pre semi C_s - connectedness and *IF* pre semi C_M - connectedness of an *IFS*.

Definition 5.5: Let (X, T) be an *IFTS*. Then

(a) (X, T) is called *IF* pre semi C_5 - disconnected, if there exist an *IFS A* which is both *IF* pre semi open and *IF* pre semi closed such that $0_{\sim} \neq A \neq 1_{\sim}$.

(b) *X* is called *IF* pre semi disconnected, if there exist *IFPSOSs* $A \neq 0_{\sim}$ and $B \neq 0_{\sim}$ such that $A \cup B = 1_{\sim}$ and $A \cap B = 0_{\sim}$.

(c) *X* is called *IF* pre semi C_5 - connected, if *X* is not *IF* pre semi C_5 - disconnected.

(d) *X* is called *IF* pre semi connected, if *X* is not *IF* pre semi disconnected.

Proposition 5.2: *IF* pre semi C_5 - connected implies *IF* pre semi connectedness

Proposition 5.3: Let (X,T) and (Y,S) be an *IFTSs*. Let $f:(X,T)\to(Y,S)$ be an *IF* pre semi continuous and surjective function. If *X* is *IF* pre semi connected, then so is *Y*.

Proposition 5.4: Let (X,T) be an *IFTS*. If (X,T) is *IF* pre semi disconnected, then so are the *IFTSs* $(X,T_{0,1})$ and $(X,T_{0,2})$.

Proposition 5.5: Let (X,T) be an *IFTS*. (X,T) is *IF* pre semi C_5 - connected iff there exist no nonzero *IFPSOSs A* and *B* in *X* such that $A=\overline{B}$.

Proposition 5.6: Let (X,T) be an *IFTS*. (X,T) is *IF* pre semi C_5 - connected iff there exist no non - zero *IFSs* in *X* such that $B=\overline{A}$, $B=\overline{IFPScl(A)}$, $A=\overline{IFPS\,cl(B)}$.

6. INTUISTIONISTIC FUZZY PRE SEMI COMPACTNESS

Definition 6.1: Let (X,T) be an *IFTS*. Then

(a) If a family $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of *IFPSOSs* in *X* satisfies the condition $\bigcup\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}=1_\sim$, then it is called a *IFPSO* cover of *X*. A finite subfamily of a *IFPSO* cover $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of *X*, which is also a *IFPSO* cover of *X*, is called a finite sub cover of $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$.

(b) A family $\left\{\left\langle x,\mu_{K_i},\gamma_{K_i}\right\rangle : i\in J\right\}$ of *IFPSCSs* in *X* satisfies the finite intersection property (*FIP* for short) iff every finite subfamily $\left\{\left\langle x,\mu_{K_i},\gamma_{K_i}\right\rangle : i=1,2,.......n\right\}$ of the family satisfies the condition $\bigcap_{i=1}^{n}\left\{\left\langle x,\mu_{K_i},\gamma_{K_i}\right\rangle\right\}\neq 0_\sim$.

Definition 6.2: An *IFTS* (X,T) is called *IF* pre semi compact iff every *IFPSO* cover of *X* has a finite sub cover.

Proposition 6.1: Let (X,T) be an *IFTS*. Then (X,T) is *IF* pre semi compact iff the *IFTS* $(X,T_{0,1})$ is *IF* compact.

Definition 6.3: Let (X,T) be an *IFTS* and *A* an *IFS* in *X*. Then

(a) If a family $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of *IFPSOS* in *X* satisfies the condition $A\subseteq\bigcup\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$. Then it is called a *IFPSO* cover of *A*. A finite sub family of the *IFPSO* cover $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of *A*, which is also a *IFPSO* cover of *A*, is called a finite sub cover of $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$.

(b) An *IFS* $A=\left\langle x,\mu_A,\gamma_A\right\rangle$ in an *IFTS* (X,T) is called *IF* pre semi compact iff every *IFPSO* cover of *A* has a finite sub cover.

Proposition 6.2: Let (X,T) and (Y,S) be *IFTSs* and $f:(X,T)\to(Y,S)$ be a *IF* pre semi continuous function. If *A* is *IF* pre semi compact in (X,T), then so is $f(A)$ in (Y,S).

Proposition 6.3: Let (X,T) and (Y,S) be *IFTS*. Let $f:(X,T)\to(Y,S)$ be a *IF* pre semi irresolute and onto mapping. If (X,T) is *IF* pre semi compact, then (Y,S) is *IF* pre semi compact.

7. INTUITIONISTIC FUZZY PRE SEMI NORMAL SPACES

Definition 7.1: An *IFTS* (X,T) is said to be *IF* pre semi normal if for every *IFPSCS A* and *IFPSOS B* in (X,T) such that $A\subseteq B$, there exists a *IFS C* such that $A\subseteq IF\,PS\,\mathrm{int}(C)\subseteq IF\,PScl(C)\subseteq B$.

Proposition 7.1: Let (X,T) and (Y,S) be any two *IFTSs*. If $f:(X,T)\to(Y,S)$ is *IF* pre semi homeomorphism and (Y,S) is *IF* pre semi normal, then (X,T) is *IF* pre semi normal.

Proposition 7.2: Let $f:(X,T)\to(Y,S)$ be a *IF* pre semi homeomorphism from a *IF* pre semi normal space (X,T) onto a *IFTS* (Y,S). Then (Y,S) is *IF* pre semi normal.

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