



On pg-Separation Axioms

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ABSTRACT

In this paper by using pg-open sets we define almost pg-normality and mild pg-normality also we continue the study of further properties of pg-normality. We show that these three axioms are regular open hereditary. We also define the class of almost pg-irresolute mappings and show that pg-normality is invariant under almost pg-irresolute M-pg-open continuous surjection.

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1. INTRODUCTION

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T₁ and T₂ spaces, namely, S₁ and S₂. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P. Aruna Swathi Vyjayanthi studied ν -Normal Almost- ν -Normal, Mildly- ν -Normal and ν -US spaces. Inspired with these we introduce pg-Normal Almost- pg-Normal, Mildly- pg-Normal, pg-US, pg-S₁ and pg-S₂. Also we examine pg-convergence, sequentially pg-compact, sequentially pg-continuous maps, and sequentially sub pg-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. PRELIMINARIES

Definition 2.1: $A \subseteq X$ is called

- (i) g-closed if $\text{cl } A \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (ii) pg-closed if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen in X .

Definition 2.2: A space X is said to be

- (i) T₁ (T₂) if for any $x \neq y$ in X , there exist (disjoint) open sets U, V in X such that $x \in U$ and $y \in V$.
- (ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X .
- (iii) normal [resp: mildly normal] if for any pair of disjoint [resp: regular-closed] closed sets F_1 and F_2 , there exist disjoint open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.
- (iv) almost normal if for each closed set A and each regular closed set B such that $A \cap B = \emptyset$, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.
- (v) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \emptyset$, there exist disjoint open sets U and V such that $x \in U$ and $A \subseteq V$.
- (vi) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A .

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- (vii) R_0 if for any point x and a closed set F with $x \notin F$ in X , there exists an open set G containing F but not x .
- (viii) R_1 iff for $x, y \in X$ with $cl\{x\} \neq cl\{y\}$, there exist disjoint open sets U and V such that $cl\{x\} \subset U, cl\{y\} \subset V$.
- (ix) US-space if every convergent sequence has exactly one limit point to which it converges.
- (x) pre-US space if every pre-convergent sequence has exactly one limit point to which it converges.
- (xi) pre- S_1 if it is pre-US and every sequence $\langle x_n \rangle$ pre-converges with subsequence of $\langle x_n \rangle$ pre-side points.
- (xii) pre- S_2 if it is pre-US and every sequence $\langle x_n \rangle$ in X pre-converges which has no pre-side point.
- (xiii) is weakly countable compact if every infinite subset of X has a limit point in X .
- (xiv) Baire space if for any countable collection of closed sets with empty interior in X , their union also has empty interior in X .

Definition 2.3: Let $A \subset X$. Then a point x is said to be a

- (i) limit point of A if each open set containing x contains some point y of A such that $x \neq y$.
- (ii) T_0 -limit point of A if each open set containing x contains some point y of A such that $cl\{x\} \neq cl\{y\}$, or equivalently, such that they are topologically distinct.
- (iii) pre- T_0 -limit point of A if each open set containing x contains some point y of A such that $pcl\{x\} \neq pcl\{y\}$, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0 -axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let $X = \{a, b, c, d\}$ and $\tau = \{\{a\}, \{b, c\}, \{a, b, c\}, X, \emptyset\}$. Then b and c are the limit points but not the T_0 -limit points of the set $\{b, c\}$. Further d is a T_0 -limit point of $\{b, c\}$.

Example 2: Let $X = (0, 1)$ and $\tau = \{\emptyset, X, \text{ and } U_n = (0, 1 - 1/n), n = 2, 3, 4, \dots\}$. Then every point of X is a limit point of X . Every point of $X \sim U_2$ is a T_0 -limit point of X , but no point of U_2 is a T_0 -limit point of X .

Definition 2.4: A set A together with all its T_0 -limit points will be denoted by $T_0\text{-cl}A$.

Note 2: i. Every T_0 -limit point of a set A is a limit point of the set but the converse is not true in general.
ii. In T_0 -space both are same.

Note 3: R_0 -axiom is weaker than T_1 -axiom. It is independent of the T_0 -axiom. However $T_1 = R_0 + T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1 -space is weakly countable compact iff it is countable compact.

3. pg- T_0 LIMIT POINT:

Definition 3.01: In X , a point x is said to be a pg- T_0 -limit point of A if each pg-open set containing x contains some point y of A such that $pgcl\{x\} \neq pgcl\{y\}$, or equivalently; such that they are topologically distinct with respect to pg-open sets.

Note 5: regular open set \Rightarrow open set \Rightarrow pre-open set \Rightarrow pg-open set we have
 $r\text{-}T_0\text{-limit point} \Rightarrow T_0\text{-limit point} \Rightarrow \text{pre-}T_0\text{-limit point} \Rightarrow \text{pg-}T_0\text{-limit point}$

Example 3: Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. For $A = \{a, b, d\}$, a is pg- T_0 -limit point.

Definition 3.02: A set A together with all its pg- T_0 -limit points is denoted by $T_0\text{-pgcl}(A)$

Lemma 3.01: If x is a pg- T_0 -limit point of a set A then x is pg-limit point of A .

Lemma 3.02:

- (i) If X is pg- T_0 -space then every pg- T_0 -limit point and every pg-limit point are equivalent.
- (ii) If X is $r\text{-}T_0$ -space then every pg- T_0 -limit point and every pg-limit point are equivalent.

Theorem 3.03: For $x \neq y \in X$,

- (i) x is a pg- T_0 -limit point of $\{y\}$ iff $x \notin pgcl\{y\}$ and $y \in pgcl\{x\}$.
- (ii) x is not a pg- T_0 -limit point of $\{y\}$ iff either $x \in pgcl\{y\}$ or $pgcl\{x\} = pgcl\{y\}$.
- (iii) x is not a pg- T_0 -limit point of $\{y\}$ iff either $x \in pgcl\{y\}$ or $y \in pgcl\{x\}$.

Corollary 3.04:

- (i) If x is a $pg-T_0$ -limit point of $\{y\}$, then y cannot be a pg -limit point of $\{x\}$.
- (ii) If $pgcl\{x\} = pgcl\{y\}$, then neither x is a $pg-T_0$ -limit point of $\{y\}$ nor y is a $pg-T_0$ -limit point of $\{x\}$.
- (iii) If a singleton set A has no $pg-T_0$ -limit point in X , then $pgclA = pgcl\{x\}$ for all $x \in pgcl\{A\}$.

Lemma 3.05: In X , if x is a pg -limit point of a set A , then in each of the following cases x becomes $pg-T_0$ -limit point of A ($\{x\} \neq A$).

- (i) $pgcl\{x\} \neq pgcl\{y\}$ for $y \in A, x \neq y$.
- (ii) $pgcl\{x\} = \{x\}$
- (iii) X is a $pg-T_0$ -space.
- (iv) $A \sim \{x\}$ is pg -open

4. $pg-T_0$ AND $pg-R_i$ AXIOMS, $i = 0, 1$:

In view of Lemma 3.6(iii), $pg-T_0$ -axiom implies the equivalence of the concept of limit point of a set with that of $pg-T_0$ -limit point of the set. But for the converse, if $x \in pgcl\{y\}$ then $pgcl\{x\} \neq pgcl\{y\}$ in general, but if x is a $pg-T_0$ -limit point of $\{y\}$, then $pgcl\{x\} = pgcl\{y\}$

Lemma 4.01: In a space X , a limit point x of $\{y\}$ is a $pg-T_0$ -limit point of $\{y\}$ iff $pgcl\{x\} \neq pgcl\{y\}$.

This lemma leads to characterize the equivalence of $pg-T_0$ -limit point and pg -limit point of a set as the $pg-T_0$ -axiom.

Theorem 4.02: The following conditions are equivalent:

- (i) X is a $pg-T_0$ space
- (ii) Every pg -limit point of a set A is a $pg-T_0$ -limit point of A
- (iii) Every r -limit point of a singleton set $\{x\}$ is a $pg-T_0$ -limit point of $\{x\}$
- (iv) For any x, y in $X, x \neq y$ if $x \in pgcl\{y\}$, then x is a $pg-T_0$ -limit point of $\{y\}$

Note 6: In a $pg-T_0$ -space X if every point of X is a r -limit point of X , then every point of X is $pg-T_0$ -limit point of X . But a space X in which each point is a $pg-T_0$ -limit point of X is not necessarily a $pg-T_0$ -space

Theorem 4.03: The following conditions are equivalent:

- (i) X is a $pg-R_0$ space
- (ii) For any x, y in X , if $x \in pgcl\{y\}$, then x is not a $pg-T_0$ -limit point of $\{y\}$
- (iii) A point pg -closure set has no $pg-T_0$ -limit point in X
- (iv) A singleton set has no $pg-T_0$ -limit point in X .

Theorem 4.04: In a $pg-R_0$ space X , a point x is $pg-T_0$ -limit point of A iff every pg -open set containing x contains infinitely many points of A with each of which x is topologically distinct

Theorem 4.05: X is $pg-R_0$ space iff a set A of the form $A = \cup pgcl\{x_i, i=1 \text{ to } n\}$ a finite union of point closure sets has no $pg-T_0$ -limit point.

If $pg-R_0$ space is replaced by rR_0 space in the above theorem, we have the following corollaries:

Corollary 4.06: The following conditions are equivalent:

- (i) X is a $r-R_0$ space
- (ii) For any x, y in X , if $x \in pgcl\{y\}$, then x is not a $pg-T_0$ -limit point of $\{y\}$
- (iii) A point pg -closure set has no $pg-T_0$ -limit point in X
- (iv) A singleton set has no $pg-T_0$ -limit point in X .

Corollary 4.07: In an rR_0 -space X ,

- (i) If a point x is rT_0 -limit point of a set then every pg -open set containing x contains infinitely many points of A with each of which x is topologically distinct.
- (ii) If a point x is $pg-T_0$ -limit point of a set then every pg -open set containing x contains infinitely many points of A with each of which x is topologically distinct.
- (iii) If $A = \cup pgcl\{x_i, i=1 \text{ to } n\}$ a finite union of point closure sets has no $pg-T_0$ -limit point.
- (iv) If $X = \cup pgcl\{x_i, i=1 \text{ to } n\}$ then X has no $pg-T_0$ -limit point.

Various characteristic properties of $pg-T_0$ -limit points studied so far is enlisted in the following theorem.

Theorem 4.08: In a $pg-R_0$ -space, we have the following:

- (i) A singleton set has no $pg-T_0$ -limit point in X .
- (ii) A finite set has no $pg-T_0$ -limit point in X .
- (iii) A point pg -closure has no set $pg-T_0$ -limit point in X .
- (iv) A finite union point pg -closure sets have no set $pg-T_0$ -limit point in X .
- (v) For $x, y \in X, x \in T_0- pgcl\{y\}$ iff $x = y$.
- (vi) For any $x, y \in X, x \neq y$ iff neither x is $pg-T_0$ -limit point of $\{y\}$ nor y is $pg-T_0$ -limit point of $\{x\}$.
- (vii) For any $x, y \in X, x \neq y$ iff $T_0- pgcl\{x\} \cap T_0- pgcl\{y\} = \phi$.
- (viii) Any point $x \in X$ is a $pg-T_0$ -limit point of a set A in X iff every pg -open set containing x contains infinitely many points of A with each which x is topologically distinct.

Theorem 4.09: X is $pg-R_1$ iff for any pg -open set U in X and points x, y such that $x \in X \sim U, y \in U$, there exists a pg -open set V in X such that $y \in V \subset U, x \notin V$.

Lemma 4.10: In $pg-R_1$ space X , if x is a $pg-T_0$ -limit point of X , then for any non empty pg -open set U , there exists a non empty pg -open set V such that $V \subset U, x \notin pgcl(V)$.

Lemma 4.11: In a pg -regular space X , if x is a $pg-T_0$ -limit point of X , then for any non empty pg -open set U , there exists a non empty pg -open set V such that $pgcl(V) \subset U, x \notin pgcl(V)$.

Corollary 4.12: In a regular space X ,

- (i) If x is a $pg-T_0$ -limit point of X , then for any non empty pg -open set U , there exists a non empty pg -open set V such that $pgcl(V) \subset U, x \notin pgcl(V)$.
- (ii) If x is a T_0 -limit point of X , then for any non empty pg -open set U , there exists a non empty pg -open set V such that $pgcl(V) \subset U, x \notin pgcl(V)$.

Theorem 4.13: If X is a pg -compact $pg-R_1$ -space, then X is a Baire Space.

Proof: Let $\{A_n\}$ be a countable collection of pg -closed sets of X , each A_n having empty interior in X . Take A_1 , since A_1 has empty interior, A_1 does not contain any pg -open set say U_0 . Therefore we can choose a point $y \in U_0$ such that $y \notin A_1$.

For X is pg -regular, and $y \in (X \sim A_1) \cap U_0$, a pg -open set, we can find a pg -open set U_1 in X such that $y \in U_1, pgcl(U_1) \subset (X \sim A_1) \cap U_0$. Hence U_1 is a non empty pg -open set in X such that $pgcl(U_1) \subset U_0$ and $pgcl(U_1) \cap A_1 = \phi$. Continuing this process, in general, for given non empty pg -open set U_{n-1} , we can choose a point of U_{n-1} which is not in the pg -closed set A_n and a pg -open set U_n containing this point such that $pgcl(U_n) \subset U_{n-1}$ and $pgcl(U_n) \cap A_n = \phi$. Thus we get a sequence of nested non empty pg -closed sets which satisfies the finite intersection property. Therefore $\bigcap pgcl(U_n) \neq \phi$.

Then some $x \in \bigcap pgcl(U_n)$ which in turn implies that $x \in U_{n-1}$ as $pgcl(U_n) \subset U_{n-1}$ and $x \notin A_n$ for each n .

Corollary 4.14: If X is a compact $pg-R_1$ -space, then X is a Baire Space.

Corollary 4.15: Let X be a pg -compact $pg-R_1$ -space. If $\{A_n\}$ is a countable collection of pg -closed sets in X , each A_n having non-empty pg -interior in X , then there is a point of X which is not in any of the A_n .

Corollary 4.16: Let X be a pg -compact R_1 -space. If $\{A_n\}$ is a countable collection of pg -closed sets in X , each A_n having non-empty pg -interior in X , then there is a point of X which is not in any of the A_n .

Theorem 4.17: Let X be a non empty compact $pg-R_1$ -space. If every point of X is a $pg-T_0$ -limit point of X then X is uncountable.

Proof: Since X is non empty and every point is a $pg-T_0$ -limit point of X , X must be infinite. If X is countable, we construct a sequence of pg -open sets $\{V_n\}$ in X as follows:

Let $x \in V_1$, then for x_1 is a $pg-T_0$ -limit point of X , we can choose a non empty pg -open set V_2 in X such that $V_2 \subset V_1$ and $x_1 \notin pgcl V_2$. Next for x_2 and non empty pg -open set V_2 , we can choose a non empty pg -open set V_3 in X such that $V_3 \subset V_2$ and $x_2 \notin pgcl V_3$. Continuing this process for each x_n and a non empty pg -open set V_n , we can choose a non empty pg -open set V_{n+1} in X such that $V_{n+1} \subset V_n$ and $x_n \notin pgcl V_{n+1}$.

Now consider the nested sequence of pg -closed sets $pgcl V_1 \supset pgcl V_2 \supset pgcl V_3 \supset \dots \supset pgcl V_n \supset \dots$. Since X is pg -compact and $\{pgcl V_n\}$ the sequence of pg -closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x \in pgcl V_n$. Further $x \in X$ and $x \in V_1$, which is not equal to any of the points of X . Hence X is uncountable.

Corollary 4.18: Let X be a non empty pg-compact pg- R_1 -space. If every point of X is a pg- T_0 -limit point of X then X is uncountable

5. pg- T_0 -IDENTIFICATION SPACES AND pg-SEPARATION AXIOMS

Definition 5.01: Let (X, τ) be a topological space and let \mathfrak{R} be the equivalence relation on X defined by $x\mathfrak{R}y$ iff $pgcl\{x\} = pgcl\{y\}$

Problem 5.02: show that $x\mathfrak{R}y$ iff $pgcl\{x\} = pgcl\{y\}$ is an equivalence relation

Definition 5.03: The space $(X_0, Q(X_0))$ is called the pg- T_0 -identification space of (X, τ) , where X_0 is the set of equivalence classes of \mathfrak{R} and $Q(X_0)$ is the decomposition topology on X_0 .

Let $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$ denote the natural map

Lemma 5.04: If $x \in X$ and $A \subset X$, then $x \in pgclA$ iff every pg-open set containing x intersects A .

Theorem 5.05: The natural map $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$ is closed, open and $P_X^{-1}(P_X(O)) = O$ for all $O \in PO(X, \tau)$ and $(X_0, Q(X_0))$ is pg- T_0

Proof: Let $O \in PO(X, \tau)$ and let $C \in P_X(O)$. Then there exists $x \in O$ such that $P_X(x) = C$. If $y \in C$, then $pgcl\{y\} = pgcl\{x\}$, which, by lemma, implies $y \in O$. Since $\tau \subset PO(X, \tau)$, then $P_X^{-1}(P_X(U)) = U$ for all $U \in \tau$, which implies P_X is closed and open.

Let $G, H \in X_0$ such that $G \neq H$; let $x \in G$ and $y \in H$. Then $pgcl\{x\} \neq pgcl\{y\}$, which implies $x \notin pgcl\{y\}$ or $y \notin pgcl\{x\}$, say $x \notin pgcl\{y\}$. Since P_X is continuous and open, then $G \in A = P_X\{X \sim pgcl\{y\}\} \notin PO(X_0, Q(X_0))$ and $H \notin A$

Theorem 5.06: The following are equivalent:

(i) X is pg- R_0 (ii) $X_0 = \{pgcl\{x\}: x \in X\}$ and (iii) $(X_0, Q(X_0))$ is pg- T_1

Proof: (i) \Rightarrow (ii) Let $C \in X_0$, and let $x \in C$. If $y \in C$, then $y \in pgcl\{y\} = pgcl\{x\}$, which implies $C \in pgcl\{x\}$. If $y \in pgcl\{x\}$, then $x \in pgcl\{y\}$, since, otherwise, $x \in X \sim pgcl\{y\} \in PO(X, \tau)$ which implies $pgcl\{x\} \subset X \sim pgcl\{y\}$, which is a contradiction. Thus, if $y \in pgcl\{x\}$, then $x \in pgcl\{y\}$, which implies $pgcl\{y\} = pgcl\{x\}$ and $y \in C$. Hence $X_0 = \{pgcl\{x\}: x \in X\}$

(ii) \Rightarrow (iii) Let $A \neq B \in X_0$. Then there exists $x, y \in X$ such that $A = pgcl\{x\}$; $B = pgcl\{y\}$, and $pgcl\{x\} \cap pgcl\{y\} = \emptyset$. Then $A \in C = P_X\{X \sim pgcl\{y\}\} \in PO(X_0, Q(X_0))$ and $B \notin C$. Thus $(X_0, Q(X_0))$ is pg- T_1

(iii) \Rightarrow (i) Let $x \in U \in \alpha GO(X)$. Let $y \notin U$ and $C_x, C_y \in X_0$ containing x and y respectively. Then $x \notin pgcl\{y\}$, which implies $C_x \neq C_y$ and there exists pg-open set A such that $C_x \in A$ and $C_y \notin A$. Since P_X is continuous and open, then $y \in B = P_X^{-1}(A) \in x \in PGO(X)$ and $x \notin B$, which implies $y \notin pgcl\{x\}$. Thus $pgcl\{x\} \subset U$. This is true for all $pgcl\{x\}$ implies $\cap pgcl\{x\} \subset U$. Hence X is pg- R_0

Theorem 5.07: (X, τ) is pg- R_1 iff $(X_0, Q(X_0))$ is pg- T_2

The proof is straight forward from theorems 5.05 and 5.06 and is omitted

Theorem 5.08: X is pg- T_i ; $i = 0, 1, 2$. iff there exists a pg-continuous, almost-open, 1-1 function from (X, τ) into a pg- T_i space; $i = 0, 1, 2$. respectively.

Theorem 5.09: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is pg-continuous, pg-open, and $x, y \in X$ such that $pgcl\{x\} = pgcl\{y\}$, then $pgcl\{f(x)\} = pgcl\{f(y)\}$.

Theorem 5.10: The following are equivalent

(i) (X, τ) is pg- T_0

(ii) Elements of X_0 are singleton sets and

(iii) There exists a pg-continuous, pg-open, 1-1 function $f: (X, \tau) \rightarrow (Y, \sigma)$, where (Y, σ) is pg- T_0

Proof: (i) is equivalent to (ii) and (i) \Rightarrow (iii) are straight forward and is omitted.

(iii) \Rightarrow (i) Let $x, y \in X$ such that $f(x) \neq f(y)$, which implies $pgcl\{f(x)\} \neq pgcl\{f(y)\}$. Then by theorem 5.09, $pgcl\{x\} \neq pgcl\{y\}$. Hence (X, τ) is pg- T_0

Corollary 5.11: A space (X, τ) is $pg-T_i$; $i = 1, 2$ iff (X, τ) is $pg-T_{i-1}$; $i = 1, 2$, respectively, and there exists a pg -continuous, pg -open, $1-1$ function $f: (X, \tau)$ into a $pg-T_0$ space.

Definition 5.04: $f: X \rightarrow Y$ is point- pg -closure $1-1$ iff for $x, y \in X$ such that $pgcl\{x\} \neq pgcl\{y\}$, $pgcl\{f(x)\} \neq pgcl\{f(y)\}$.

Theorem 5.12:

(i) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is point- pg -closure $1-1$ and (X, τ) is $pg-T_0$, then f is $1-1$

(ii) If $f: (X, \tau) \rightarrow (Y, \sigma)$, where (X, τ) and (Y, σ) are $pg-T_0$ then f is point- pg -closure $1-1$ iff f is $1-1$

The following result can be obtained by combining results for $pg-T_0$ -identification spaces, pg -induced functions and $pg-T_i$ spaces; $i = 1, 2$.

Theorem 5.13: X is $pg-R_i$; $i = 0, 1$ iff there exists a pg -continuous, almost-open point- pg -closure $1-1$ function $f: (X, \tau)$ into a $pg-R_i$ space; $i = 0, 1$ respectively.

6. pg -Normal; Almost pg -normal and Mildly pg -normal spaces

Definition 6.1: A space X is said to be pg -normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint pg -open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 4: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. Then X is pg -normal.

Example 5: Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then X is not pg -normal and is not normal.

Example 6: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$ is pg -normal, normal and almost normal.

We have the following characterization of pg -normality.

Theorem 6.1: For a space X the following are equivalent:

(i) X is pg -normal.

(ii) For every pair of open sets U and V whose union is X , there exist pg -closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(iii) For every closed set F and every open set G containing F , there exists a pg -open set U such that $F \subset U \subset pgcl(U) \subset G$.

Proof: (i) \Rightarrow (ii): Let U and V be a pair of open sets in a pg -normal space X such that $X = U \cup V$. Then $X-U$, $X-V$ are disjoint closed sets. Since X is pg -normal there exist disjoint pg -open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$.

Let $A = X-U_1$, $B = X-V_1$. Then A and B are pg -closed sets such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(b) \Rightarrow (c): Let F be a closed set and G be an open set containing F . Then $X-F$ and G are open sets whose union is X . Then by (b), there exist pg -closed sets W_1 and W_2 such that $W_1 \subset X-F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X-W_1$, $X-G \subset X-W_2$ and $(X-W_1) \cap (X-W_2) = \emptyset$. Let $U = X-W_1$ and $V = X-W_2$. Then U and V are disjoint pg -open sets such that $F \subset U \subset X-V \subset G$. As $X-V$ is pg -closed set, we have $pgcl(U) \subset X-V$ and $F \subset U \subset pgcl(U) \subset G$.

(c) \Rightarrow (a): Let F_1 and F_2 be any two disjoint closed sets of X . Put $G = X-F_2$, then $F_1 \cap G = \emptyset$. $F_1 \subset G$ where G is an open set. Then by (c), there exists a pg -open set U of X such that $F_1 \subset U \subset pgcl(U) \subset G$. It follows that $F_2 \subset X-pgcl(U) = V$, say, then V is pg -open and $U \cap V = \emptyset$. Hence F_1 and F_2 are separated by pg -open sets U and V . Therefore X is pg -normal.

Theorem 6.2: A regular open subspace of a pg -normal space is pg -normal.

Example 7: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$ is pg -normal and pg -regular.

However we observe that every pg -normal $pg-R_0$ space is pg -regular.

Definition 6.2: A function $f: X \rightarrow Y$ is said to be almost- pg -irresolute if for each x in X and each pg -neighborhood V of $f(x)$, $pgcl(f^{-1}(V))$ is a pg -neighborhood of x .

Clearly every pg -irresolute map is almost- pg -irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 6.1: f is almost pg-irresolute iff $f^{-1}(V) \subset \text{pg-int}(\text{pgcl}(f^{-1}(V)))$ for every $V \in \text{PGO}(Y)$.

Lemma 6.2: f is almost pg-irresolute iff $f(\text{pgcl}(U)) \subset \text{pgcl}(f(U))$ for every $U \in \text{PGO}(X)$.

Proof: Let $U \in \text{PGO}(X)$. Suppose $y \notin \text{pgcl}(f(U))$. Then there exists $V \in \text{PGO}(Y)$ such that $V \cap f(U) = \emptyset$. Hence $f^{-1}(V) \cap U = \emptyset$. Since $U \in \text{PGO}(X)$, we have $\text{pg-int}(\text{pgcl}(f^{-1}(V))) \cap \text{pgcl}(U) = \emptyset$. Then by lemma 6.1, $f^{-1}(V) \cap \text{pgcl}(U) = \emptyset$ and hence $V \cap f(\text{pgcl}(U)) = \emptyset$. This implies that $y \notin f(\text{pgcl}(U))$.

Conversely, if $V \in \text{PGO}(Y)$, then $W = X - \text{pgcl}(f^{-1}(V)) \in \text{PGO}(X)$. By hypothesis, $f(\text{pgcl}(W)) \subset \text{pgcl}(f(W))$ and hence $X - \text{pg-int}(\text{pgcl}(f^{-1}(V))) = \text{pgcl}(W) \subset f^{-1}(\text{pgcl}(f(W))) \subset f^{-1}(\text{pgcl}(f(X - f^{-1}(V)))) \subset f^{-1}[\text{pgcl}(Y - V)] = f^{-1}(Y - V) = X - f^{-1}(V)$.

Therefore, $f^{-1}(V) \subset \text{pg-int}(\text{pgcl}(f^{-1}(V)))$. By lemma 6.1, f is almost pg-irresolute.

Now we prove the following result on the invariance of pg-normality.

Theorem 6.3: If f is an M-pg-open continuous almost pg-irresolute function from a pg-normal space X onto a space Y , then Y is pg-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A . Then by continuity of f , $f^{-1}(A)$ is closed and $f^{-1}(B)$ is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is pg-normal, there exists a pg-open set U in X such that $f^{-1}(A) \subset U \subset \text{pgcl}(U) \subset f^{-1}(B)$. Then $f(f^{-1}(A)) \subset f(U) \subset f(\text{pgcl}(U)) \subset f(f^{-1}(B))$. Since f is M-pg-open almost pg-irresolute surjection, we obtain $A \subset f(U) \subset \text{pgcl}(f(U)) \subset B$. Then again by Theorem 6.1 the space Y is pg-normal.

Lemma 6.3: A mapping f is M-pg-closed if and only if for each subset B in Y and for each pg-open set U in X containing $f^{-1}(B)$, there exists a pg-open set V containing B such that $f^{-1}(V) \subset U$.

Theorem 6.4: If f is an M-pg-closed continuous function from a pg-normal space onto a space Y , then Y is pg-normal.

Proof of the theorem is routine and hence omitted.

Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

Theorem 6.5: If f is an M-pg-closed map from a weakly Hausdorff pg-normal space X onto a space Y such that $f^{-1}(y)$ is S-closed relative to X for each $y \in Y$, then Y is pg- T_2 .

Proof: Let y_1 and y_2 be any two distinct points of Y . Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is pg-normal, there exist disjoint pg-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for $i = 1, 2$. Since f is M-pg-closed, there exist pg-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$ for $i = 1, 2$. Then it follows that $U_1 \cap U_2 = \emptyset$. Hence Y is pg- T_2 .

Theorem 6.6: For a space X we have the following:

- (a) If X is normal then for any disjoint closed sets A and B , there exist disjoint pg-open sets U, V such that $A \subset U$ and $B \subset V$;
- (b) If X is normal then for any closed set A and any open set V containing A , there exists an pg-open set U of X such that $A \subset U \subset \text{pgcl}(U) \subset V$.

Definition 6.2: X is said to be almost pg-normal if for each closed set A and each regular closed set B such that $A \cap B = \emptyset$, there exist disjoint pg-open sets U and V such that $A \subset U$ and $B \subset V$.

Clearly, every pg-normal space is almost pg-normal, but not conversely in general.

Now, we have characterization of almost pg-normality in the following.

Theorem 6.7: For a space X the following statements are equivalent:

- (i) X is almost pg-normal
- (ii) For every pair of sets U and V , one of which is open and the other is regular open whose union is X , there exist pg-closed sets G and H such that $G \subset U, H \subset V$ and $G \cup H = X$.
- (iii) For every closed set A and every regular open set B containing A , there is a pg-open set V such that $A \subset V \subset \text{pgcl}(V) \subset B$.

Proof: (a) \Rightarrow (b) Let U be an open set and V be a regular open set in an almost pg-normal space X such that $U \cup V = X$. Then $(X - U)$ is closed set and $(X - V)$ is regular closed set with $(X - U) \cap (X - V) = \emptyset$. By almost pg-normality of X , there

exist disjoint pg-open sets U_1 and V_1 such that $X-U \subset U_1$ and $X-V \subset V_1$. Let $G = X-U_1$ and $H = X-V_1$. Then G and H are pg-closed sets such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(b) \Rightarrow (c) and (c) \Rightarrow (a) are obvious.

One can prove that almost pg-normality is also regular open hereditary.

Almost pg-normality does not imply almost pg-regularity in general. However, we observe that every almost pg-normal $pg-R_0$ space is almost pg-regular.

Theorem 6.8: Every almost regular, pg -compact space X is almost pg -normal.

Recall that a function $f: X \rightarrow Y$ is called rc-continuous if inverse image of regular closed set is regular closed.

Now, we state the invariance of almost pg -normality in the following.

Theorem 6.9: If f is continuous M - pg -open rc-continuous and almost pg -irresolute surjection from an almost pg -normal space X onto a space Y , then Y is almost pg -normal.

Definition 6.3: A space X is said to be mildly pg -normal if for every pair of disjoint regular closed sets F_1 and F_2 of X , there exist disjoint pg -open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 8: Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$ is Mildly pg -normal.

We have the following characterization of mild pg -normality.

Theorem 6.10: For a space X the following are equivalent.

- (i) X is mildly pg -normal.
- (ii) For every pair of regular open sets U and V whose union is X , there exist pg -closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.
- (iii) For any regular closed set A and every regular open set B containing A , there exists a pg -open set U such that $A \subset U \subset pgcl(U) \subset B$.
- (iv) For every pair of disjoint regular closed sets, there exist pg -open sets U and V such that $A \subset U$, $B \subset V$ and $pgcl(U) \cap pgcl(V) = \phi$.

This theorem may be proved by using the arguments similar to those of Theorem 6.7.

Also, we observe that mild pg -normality is regular open hereditary.

Definition 6.4: A space X is weakly pg -regular if for each point x and a regular open set U containing $\{x\}$, there is a pg -open set V such that $x \in V \subset clV \subset U$.

Example 9: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then X is weakly pg -regular.

Example 10: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then X is not weakly pg -regular.

Theorem 6.11: If $f: X \rightarrow Y$ is an M - pg -open rc-continuous and almost pg -irresolute function from a mildly pg -normal space X onto a space Y , then Y is mildly pg -normal.

Proof: Let A be a regular closed set and B be a regular open set containing A . Then by rc-continuity of f , $f^{-1}(A)$ is a regular closed set contained in the regular open set $f^{-1}(B)$. Since X is mildly pg -normal, there exists a pg -open set V such that $f^{-1}(A) \subset V \subset pgcl(V) \subset f^{-1}(B)$ by Theorem 6.10. As f is M - pg -open and almost pg -irresolute surjection, it follows that $f(V) \in PGO(Y)$ and $A \subset f(V) \subset pgcl(f(V)) \subset B$. Hence Y is mildly pg -normal.

Theorem 6.12: If $f: X \rightarrow Y$ is rc-continuous, M - pg -closed map from a mildly pg -normal space X onto a space Y , then Y is mildly pg -normal.

7. pg -US SPACES:

Definition 7.1: A sequence $\langle x_n \rangle$ is said to be pg -converges to a point x of X , written as $\langle x_n \rangle \rightarrow^{pg} x$ if $\langle x_n \rangle$ is eventually in every pg -open set containing x .

Clearly, if a sequence $\langle x_n \rangle$ *r*-converges to a point x of X , then $\langle x_n \rangle$ *pg*-converges to x .

Definition 7.2: X is said to be *pg*-US if every sequence $\langle x_n \rangle$ in X *pg*-converges to a unique point.

Definition 7.3: A set F is sequentially *pg*-closed if every sequence in F *pg*-converges to a point in F .

Definition 7.4: A subset G of a space X is said to be sequentially *pg*-compact if every sequence in G has a subsequence which *pg*-converges to a point in G .

Definition 7.5: A point y is a *pg*-cluster point of sequence $\langle x_n \rangle$ iff $\langle x_n \rangle$ is frequently in every *pg*-open set containing x . The set of all *pg*-cluster points of $\langle x_n \rangle$ will be denoted by $pg-cl(x_n)$.

Definition 7.6: A point y is *pg*-side point of a sequence $\langle x_n \rangle$ if y is a *pg*-cluster point of $\langle x_n \rangle$ but no subsequence of $\langle x_n \rangle$ *pg*-converges to y .

Definition 7.7: A space X is said to be

- (i) *pg*-S₁ if it is *pg*-US and every sequence $\langle x_n \rangle$ *pg*-converges with subsequence of $\langle x_n \rangle$ *pg*-side points.
- (ii) *pg*-S₂ if it is *pg*-US and every sequence $\langle x_n \rangle$ in X *pg*-converges which has no *pg*-side point.

Using sequentially continuous functions, we define sequentially *pg*-continuous functions.

Definition 7.8: A function f is said to be sequentially *pg*-continuous at $x \in X$ if $f(x_n) \rightarrow^{pg} f(x)$ whenever $\langle x_n \rangle \rightarrow^{pg} x$. If f is sequentially *pg*-continuous at all $x \in X$, then f is said to be sequentially *pg*-continuous.

Theorem 7.1: We have the following:

- (i) Every *pg*-T₂ space is *pg*-US.
- (ii) Every *pg*-US space is *pg*-T₁.
- (iii) X is *pg*-US iff the diagonal set is a sequentially *pg*-closed subset of $X \times X$.
- (iv) X is *pg*-T₂ iff it is both *pg*-R₁ and *pg*-US.
- (v) Every regular open subset of a *pg*-US space is *pg*-US.
- (vi) Product of arbitrary family of *pg*-US spaces is *pg*-US.
- (vii) Every *pg*-S₂ space is *pg*-S₁ and Every *pg*-S₁ space is *pg*-US.

Theorem 7.2: In a *pg*-US space every sequentially *pg*-compact set is sequentially *pg*-closed.

Proof: Let X be *pg*-US space. Let Y be a sequentially *pg*-compact subset of X . Let $\langle x_n \rangle$ be a sequence in Y . Suppose that $\langle x_n \rangle$ *pg*-converges to a point in $X-Y$. Let $\langle x_{np} \rangle$ be subsequence of $\langle x_n \rangle$ that *pg*-converges to a point $y \in Y$ since Y is sequentially *pg*-compact. Also, let a subsequence $\langle x_{np} \rangle$ of $\langle x_n \rangle$ *pg*-converge to $x \in X-Y$. Since $\langle x_{np} \rangle$ is a sequence in the *pg*-US space X , $x = y$. Thus, Y is sequentially *pg*-closed set.

Theorem 7.3: Let f and g be two sequentially *pg*-continuous functions. If Y is *pg*-US, then the set $A = \{x \mid f(x) = g(x)\}$ is sequentially *pg*-closed.

Proof: Let Y be *pg*-US and suppose that there is a sequence $\langle x_n \rangle$ in A *pg*-converging to $x \in X$. Since f and g are sequentially *pg*-continuous functions, $f(x_n) \rightarrow^{pg} f(x)$ and $g(x_n) \rightarrow^{pg} g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, A is sequentially *pg*-closed.

8. SEQUENTIALLY sub-*pg*-CONTINUITY:

In this section we introduce and study the concepts of sequentially sub-*pg*-continuity, sequentially nearly *pg*-continuity and sequentially *pg*-compact preserving functions and study their relations and the property of *pg*-US spaces.

Definition 8.1: A function f is said to be

- (i) sequentially nearly *pg*-continuous if for each point $x \in X$ and each sequence $\langle x_n \rangle \rightarrow^{pg} x$ in X , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ such that $\langle f(x_{nk}) \rangle \rightarrow^{pg} f(x)$.
- (ii) sequentially sub-*pg*-continuous if for each point $x \in X$ and each sequence $\langle x_n \rangle \rightarrow^{pg} x$ in X , there exists a subsequence $\langle x_{nk} \rangle$ of $\langle x_n \rangle$ and a point $y \in Y$ such that $\langle f(x_{nk}) \rangle \rightarrow^{pg} y$.
- (iii) sequentially *pg*-compact preserving if $f(K)$ is sequentially *pg*-compact in Y for every sequentially *pg*-compact set K of X .

Lemma 8.1: Every function f is sequentially sub-*pg*-continuous if Y is a sequentially *pg*-compact.

Proof: Let $\langle x_n \rangle \rightarrow^{pg} x$ in X . Since Y is sequentially pg-compact, there exists a subsequence $\{f(x_{n_k})\}$ of $\{f(x_n)\}$ pg-converging to a point $y \in Y$. Hence f is sequentially sub-pg-continuous.

Theorem 8.1: Every sequentially nearly pg-continuous function is sequentially pg-compact preserving.

Proof: Assume f is sequentially nearly pg-continuous and K any sequentially pg-compact subset of X . Let $\langle y_n \rangle$ be any sequence in $f(K)$. Then for each positive integer n , there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $\langle x_n \rangle$ is a sequence in the sequentially pg-compact set K , there exists a subsequence $\langle x_{n_k} \rangle$ of $\langle x_n \rangle$ pg-converging to a point $x \in K$. By hypothesis, f is sequentially nearly pg-continuous and hence there exists a subsequence $\langle x_{j_i} \rangle$ of $\langle x_{n_k} \rangle$ such that $f(x_{j_i}) \rightarrow^{pg} f(x)$. Thus, there exists a subsequence $\langle y_{j_i} \rangle$ of $\langle y_n \rangle$ pg-converging to $f(x) \in f(K)$. This shows that $f(K)$ is sequentially pg-compact set in Y .

Theorem 8.2: Every sequentially pre-continuous function is sequentially pg-continuous.

Proof: Let f be a sequentially pre-continuous and $\langle x_n \rangle \rightarrow^p x \in X$. Then $\langle x_n \rangle \rightarrow^p x$. Since f is sequentially pre-continuous, $f(x_n) \rightarrow^p f(x)$. But we know that $\langle x_n \rangle \rightarrow^p x$ implies $\langle x_n \rangle \rightarrow^{pg} x$ and hence $f(x_n) \rightarrow^{pg} f(x)$ implies f is sequentially pg-continuous.

Theorem 8.3: Every sequentially pg-compact preserving function is sequentially sub-pg-continuous.

Proof: Suppose f is a sequentially pg-compact preserving function. Let x be any point of X and $\langle x_n \rangle$ any sequence in X pg-converging to x . We shall denote the set $\{x_n \mid n = 1, 2, 3, \dots\}$ by A and $K = A \cup \{x\}$. Then K is sequentially pg-compact since $\langle x_n \rangle \rightarrow^{pg} x$. By hypothesis, f is sequentially pg-compact preserving and hence $f(K)$ is a sequentially pg-compact set of Y . Since $\{f(x_n)\}$ is a sequence in $f(K)$, there exists a subsequence $\{f(x_{n_k})\}$ of $\{f(x_n)\}$ pg-converging to a point $y \in f(K)$. This implies that f is sequentially sub-pg-continuous.

Theorem 8.4: A function $f: X \rightarrow Y$ is sequentially pg-compact preserving iff $f|_K: K \rightarrow f(K)$ is sequentially sub-pg-continuous for each sequentially pg-compact subset K of X .

Proof: Suppose f is a sequentially pg-compact preserving function. Then $f(K)$ is sequentially pg-compact set in Y for each sequentially pg-compact set K of X . Therefore, by Lemma 8.1 above, $f|_K: K \rightarrow f(K)$ is sequentially pg-continuous function.

Conversely, let K be any sequentially pg-compact set of X . Let $\langle y_n \rangle$ be any sequence in $f(K)$. Then for each positive integer n , there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $\langle x_n \rangle$ is a sequence in the sequentially pg-compact set K , there exists a subsequence $\langle x_{n_k} \rangle$ of $\langle x_n \rangle$ pg-converging to a point $x \in K$. By hypothesis, $f|_K: K \rightarrow f(K)$ is sequentially sub-pg-continuous and hence there exists a subsequence $\langle y_{n_k} \rangle$ of $\langle y_n \rangle$ pg-converging to a point $y \in f(K)$. This implies that $f(K)$ is sequentially pg-compact set in Y . Thus, f is sequentially pg-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub-pg-continuous function to be sequentially pg-compact preserving.

Corollary 8.1: If f is sequentially sub-pg-continuous and $f(K)$ is sequentially pg-closed set in Y for each sequentially pg-compact set K of X , then f is sequentially pg-compact preserving function.

REFERENCES

- [1] S.P. Arya and M.P. Bhamini, A note on semi-US spaces, Ranchi Uni. Math. J. Vol. 13 (1982), 60-68.
- [2] Ashish Kar and P. Bhattacharyya, Some weak separation axioms, Bull. Cal. Math. Soc., 82 (1990), 415-422.
- [3] C.E. Aull, Sequences in topological spaces, Comm. Math. (1968), 329-36.
- [4] S. Balasubramanian and P. Aruna Swathi Vyjayanthi, On v -separation axioms Inter. J. Math. Archive, Vol 2, No. 8(2011) 1464-1473.
- [5] S. Balasubramanian and M. Lakshmi Sarada, pgr -separation axioms, Bull. Kerala Math. Association, Vol 8. No.1 (2011) 157 – 173.
- [6] H.F. Cullen, Unique sequential limits, Boll. UMI, 20 (1965) 123-127.
- [7] Charles Dorsett, semi- T_1 , semi- T_2 and semi- R_1 spaces, Ann. Soc. Sci. Bruxelles, 92 (1978) 143-158.

- [8] K. K. Dube and B.L. namdeo, T_0 -Limit point of a set and its implications, J. Tripura Math. Soc, Vol.9 (2007)85-96.
- [9] G. L. Garg and D.Sivaraj, presemiclosed mappings, Periodica Math. Hung., 19(2) (1988), 97-106.
- [10] S. R. Malghan and G. B. Navalagi, Almost $-p$ -regular, p -completely regular and almost $-p$ -completely regular spaces, Bull. Math. Soc. Sci. Math., R.S.R. Tome 34(82), nr.4 (1990), 317-326.
- [11] S. N. Maheshwari and R. Prasad, Some new separation axioms, Ann. Soc. Sci., Bruxelles, 89 (1975), 395-402.
- [12] G. B. Navalagi, Further properties of preseparation axioms, (Unpublished)
- [13] G. B. Navalagi, P-Normal Almost-P-Normal, Mildly-P-Normal, Topology Atlas.
- [14] G. B. Navalagi, Pre-US Spaces, Topology Atlas.
- [15] T. Noiri, Almost continuity and some separation axioms, Glasnik Mat., 9(29) (1974), 131-135.
- [16] T. Noiri, Sequentially subcontinuous functions, Accad. Naz. Dei. Lincei. Estratto dei. Rendiconti. Della Classe di Sci. Fis. Mat. Nat. Series. VIII, Vol. LVIII, fase. 3 (1975), 370-376.
- [17] Paul and Bhattacharyya, On p -normal spaces, Soochow Jour. Math., Vol.21. No.3, (1995), 273-289
- [18] M. K. Singal and S. P. Arya, On almost normal and almost completely regular spaces, Glasnik Mat., 5(25) (1970), 141-152.
- [19] M. K. Singal and A. R. Singal, Mildly normal spaces, Kyungpook Math. J., 13(1) (1973)27-31.
- [20] T. Thompson, S-closed spaces, Proc. Amer. Math. Soc., 60(1976)335-338.
- [21] A. Wilansky, Between T_1 and T_2 , Amer.Math. Monthly. 74 (1967), 261-266.
