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On $\mathcal{M}_X\alpha\delta$-closed sets in $\mathcal{M}$-Structures

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ABSTRACT

We introduce a new set called $\mathcal{M}_X\alpha\delta$-closed which are defined on a family of sets satisfying some minimal conditions. Further we studied the properties of $\mathcal{M}_X\alpha\delta$-closed sets.

Keywords: *$\mathcal{M}_X\alpha\delta$-closed set.*

1. INTRODUCTION

In 1950, H. Maki, J. Umehara and T. Noiri [3] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-cl and m_X-int operators respectively. Further they introduced m-continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [4–11]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [4], [3] and [7].

In this paper we introduce $\mathcal{M}_X\alpha\delta$-closed set. Further, we obtain some characterizations and some properties.

2. PRELIMINARIES

In this section, we introduce the $\mathcal{M}$-structure and define some important subsets associated to the $\mathcal{M}$-structure and the relation between them.

Definition 2.1: [3] Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. Where m_X is an $\mathcal{M}$-structure (or a minimal structure) on X, if φ and X belong to m_X.

The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of m_X-open set is said to be m_X-closed.

Definition 2.2: [3] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For a subset A of X, m_X-closure of A and m_X-interior of A are defined as follows:
$m_X\text{-}cl(A) = \bigcap\{F: A \subseteq F,\ \ X - F \in m_X\}$
$m_X\text{-}int(A) = \bigcup\{F: U \subseteq A,\ \ U \in m_X\}$

Lemma 2.3: [3] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For subsets A and B of X, the following properties hold:

(a) $m_X\text{-}cl(X - A) = X - m_X\text{-}int(A)$ and $m_X\text{-}int(X - A) = X - m_X\text{-}cl(A)$.
(b) If $X - A \in m_X$, then $m_X\text{-}cl(A) = A$ and if $A \in m_X$ then $m_X\text{-}int(A) = A$ $m_X\text{-}int(A) = A$.
(c) $m_X\text{-}cl(\varphi) = \varphi$, $m_X\text{-}cl(X) = X$, $m_X\text{-}int(\varphi) = \varphi$ and $m_X\text{-}int(X) = X$.
(d) If $A \subseteq B$ then $m_X\text{-}cl(A) \subseteq m_X\text{-}cl(B)$ and $m_X\text{-}int(A) \subseteq m_X\text{-}int(B)$.
(e) $A \subseteq m_X\text{-}cl(A)$ and $m_X\text{-}int(A) \subseteq A$.
(f) $m_X\text{-}cl(m_X\text{-}cl(A)) = m_X\text{-}cl(A)$ and $m_X\text{-}int(m_X\text{-}int(A)) = m_X\text{-}int(A)$.
(g) $m_X\text{-}int(A \cap B) = (m_X\text{-}int(A)) \cap (m_X\text{-}int(B))$ and $(m_X\text{-}int(A)) \cup (m_X\text{-}int(B)) \subseteq m_X\text{-}int(A \cup B)$.
(h) $m_X\text{-}cl(A \cup B) = (m_X\text{-}cl(A)) \cup (m_X\text{-}cl(B))$ and $m_X\text{-}cl(A \cap B) \subseteq (m_X\text{-}cl(A)) \cap (m_X\text{-}cl(B))$.

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Lemma 2.4: [7] Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X\text{-}cl(A)$ if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x.

Definition 2.5: [10] A minimal structure m_X on a nonempty set X is said to have the property $\mathcal{B}$ if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 2.6: A minimal structure m_X with the property $\mathcal{B}$ coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7: [5] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X satisfying the property $\mathcal{B}$. For a subset A of X, the following property hold:

(a) $A \in m_X$ iff $m_X\text{-}int(A) = A$
(b) $A \in m_X$ iff $m_X\text{-}cl(A) = A$
(c) $m_X\text{-}int(A) \in m_X$ and $m_X\text{-}cl(A) \in m_X$.

3. $\mathcal{M}_X\alpha\delta$-CLOSED SETS

Definition 3.1: A subset A of an m-space $(X,\ m_X)$ is called

(a) $m_X\alpha$-open set if $A \subseteq m_X int\left(m_X cl\left(m_X int(A)\right)\right)$ and an $m_X\alpha$-closed set if $m_X cl\left(m_X int\left(m_X cl(A)\right)\right) \subseteq A$.
(b) m_X-regular open set if $A = m_X int\left(m_X cl(A)\right)$.

The $m_X\delta$-interior of a subset is the union of all m_X-regular open set of X contained in A and is denoted by $m_X\text{-}int_\delta(A)$.

The subset A is called m_X-δ-open if $A = m_X\text{-}int_\delta(A)$, i.e. a set is $m_X\delta$-open if it is the union of regular open sets. the complement of a $m_X\delta$-open is called $m_X\delta$-closed. Alternatively, a set $A \subseteq (X,\ m_X)$ is called $m_X\delta$-closed if $A = m_X\text{-}cl_\delta(A)$, Where $m_X cl_\delta(A) = \{x \,/\, x \in U \in m_X \Rightarrow m_X - int(m_X - cl(A)) \cap A \neq \varphi\}$

Definition 3.2: A subset A of an m-space $(X,\ m_X)$ is called an

(a) $m_X\alpha g$-closed set if $m_X\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha$-open in $(X,\ m_X)$.
(b) $\mathcal{M}_X\alpha\delta$-closed set if $m_X\text{-}cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha g$-open in $(X,\ m_X)$.

Example 3.3: Let $X = \{a,\ b,\ c\}$. Define $\mathcal{M}$-structure on X as follows: $m_X = \{\varphi, X, \{a\}\}$. Then $m_X\alpha$-open $= \{\varphi, X, \{a\}, \{a,b\}, \{a,c\}\}$, $m_X\delta$-open $= \{\varphi,\ X\}$, $m_X\alpha g$-open$= \{\varphi, X, \{a\}, \{a,b\}, \{a,c\}\}$ and $\mathcal{M}_X\alpha\delta$-open$= \{\varphi, X, \{a\}\}$.

Example 3.4: Let $X = \{a,\ b,\ c\}$. Define $\mathcal{M}$-structure on X as follows: $m_X = \{\varphi, X, \{a\}, \{a,b\}\}$.

Then $m_X\alpha$-open$= \{\varphi, X, \{a\}, \{a,b\}, \{a,c\}\}$, $m_X\alpha g$-open $= \{\varphi, X, \{a\}, \{a,b\}, \{a,c\}\}$ and $\mathcal{M}_X\alpha\delta$-open $= \{\varphi, X, \{a\}\}$.

Definition 3.5: The intersection of all $m_X\alpha g$-open subsets of $(X,\ m_X)$ containing A is called the $m_X\alpha g$-kernel of A (briefly, $m_X\alpha g\ ker(A)$). *i.e.*, $m_X\alpha g\ ker(A) = \cap\{G \in m_X\alpha gO(X) : A \subseteq G\}$.

Theorem 3.6: Let A be a subset of $(X,\ m_X)$, then A is $\mathcal{M}_X\alpha\delta$-closed if and only if $m_X cl_\delta(A) \subseteq m_X\alpha g\ ker(A)$.

Proof: Suppose that A is $\mathcal{M}_X\alpha\delta$-closed and let $D = \{S : S \subseteq X, A \subseteq S : S\ is\ an\ m_X\alpha g\ open\}$.

Then $m_X\alpha g\ ker(A) = \cap_{S\in D} S$.Observe that $S \in D$ implies that $A \subseteq S$ follows $m_X cl_\delta(A) \subseteq S$ for all $S \in D$.

Conversely, if $m_X cl_\delta(A) \subseteq m_X\alpha g\ ker(A)$, take $S \in \alpha gO(X,\ m_X)$ such that $A \subseteq S$ then by hypothesis, $m_X cl_\delta(A) \subseteq m_X\alpha g\ ker(A) \subseteq S$. This shows that A is $\mathcal{M}_X\alpha\delta$-closed.

Theorem 3.7: For subsets A and B of $(X,\ m_X)$, the following properties hold:

(a) If A is m_X-δ-closed, then A is $\mathcal{M}_X\alpha\delta$-closed.
(b) If m_X has the property $\mathfrak{B}$ and A is $\mathcal{M}_X\alpha\delta$-closed and $m_X\alpha g$-open then A is m_X-δ-closed.
(c) If A is $\mathcal{M}_X\alpha\delta$-closed and $A \subseteq B \subseteq cl_\delta(A)$ then B is $\mathcal{M}_X\alpha\delta$-closed.

Proof: (a) Let A be an m_X-δ-closed set in $(X,\ m_X)$. Let $A \subseteq U$, where U is $m_X\alpha g$-open in $(X,\ m_X)$. Since A is m_X-δ-closed, $m_X cl_\delta(A) = $ A , $m_X cl_\delta(A) \subseteq U$. Therefore, A is $\mathcal{M}_X\alpha\delta$-closed.

(b) Since A is $m_X\alpha g$-open and $\mathcal{M}_X\alpha\delta$-closed, we have $m_X cl_\delta(A) \subseteq A$. Therefore, A is m_X-δ-closed

(c) Let U be an $m_X\alpha g$-open set of $(X,\ m_X)$ such that $B \subseteq U$, then $A \subseteq U$. Since A is $\mathcal{M}_X\alpha\delta$-closed, $m_X cl_\delta(A) \subseteq U$.

Now $m_X cl_\delta(B) \subseteq m_X cl_\delta(m_X cl_\delta(A)) \subseteq U$. Therefore, B is also an $\mathcal{M}_X\alpha\delta$-closed set of $(X,\ m_X)$.

Theorem 3.8: Union of two $\mathcal{M}_X\alpha\delta$-closed sets is $\mathcal{M}_X\alpha\delta$-closed.

Proof: Let A and B be two $\mathcal{M}_X\alpha\delta$-closed sets in $(X,\ m_X)$. Let $A \cup B \subseteq U$, U is $m_X\alpha g$-open. Since A and B are $\mathcal{M}_X\alpha\delta$-closed sets, $m_X cl_\delta(A) \subseteq U$ and $m_X cl_\delta(B) \subseteq U$. This implies that $m_X cl_\delta(A \cup B) \subseteq m_X cl_\delta(A) \cup m_X cl_\delta(\mathrm{B}) \subseteq U$ and so $m_X cl_\delta(A \cup B) \subseteq U$. Therefore $A \cup B$ is $\mathcal{M}_X\alpha\delta$-closed.

Theorem 3.9: Let m_X be an $\mathcal{M}$-structure on X satisfying the property $\mathfrak{B}$ and $A \subseteq X$. Then A is an $\mathcal{M}_X\alpha\delta$-closed set if and only if there does not exist a nonempty $m_X\alpha g$-closed set F such that $F \neq \varphi$ and $F \subseteq m_X cl_\delta(A) - \mathrm{A}$.

Proof: Suppose that A is an $\mathcal{M}_X\alpha\delta$-closed set and let $F \subseteq X$ be an $m_X\alpha g$-closed set such that $F \subseteq m_X cl_\delta(A) - \mathrm{A}$. It follows that, $A \subseteq X - F$ and $X - F$ is an $m_X\alpha g$-open set. Since A is an $\mathcal{M}_X\alpha\delta$-closed set,

we have that $m_X cl_\delta(A) \subseteq X - F$ and $F \subseteq X - m_X cl_\delta(A)$. Follows that, $F \subseteq \left(X - m_X cl_\delta(A)\right) \cap \left(X - m_X cl_\delta(A)\right) = \varphi$, implying that $F = \varphi$.

Conversely, if $A \subseteq U$ and U is an $m_X\alpha g$-open set, then $m_X cl_\delta(A) \cap (X - U) \subseteq m_X cl_\delta(A) \cap (X - A) = m_X cl_\delta(A) - \mathrm{A}$. Since $m_X cl_\delta(A) - \mathrm{A}$ does not contain subsets $m_X\alpha g$-closed sets different from the empty set, we obtain that $m_X cl_\delta(A) \cap (X - U) = \varphi$ and this implies that $m_X cl_\delta(A) \subseteq U$, in consequence A is $m_X\alpha g$-closed.

Theorem 3.10: Let $(X,\ m_X)$ be an m-space and $A \subseteq X$, then A is $\mathcal{M}_X\alpha\delta$-open if and only if $F \subset m_X int_\delta(A)$ where F is $m_X\alpha g$-closed and $F \subset A$.

Proof: Let A be an $\mathcal{M}_X\alpha\delta$-open, F be $m_X\alpha g$-closed set such that $F \subset A$. Then $X - A \subset X - F$, but $X - F$ is $m_X\alpha g$-closed and $X - A$ is $\mathcal{M}_X\alpha\delta$-closed implies that $m_X cl_\delta(X - A) \subset X - F$. Follows that $X - m_X int_\delta(A) \subset X - F$. in consequence $F \subset m_X int_\delta(A)$.

Conversely, if F is $m_X\alpha g$-closed, $F \subset A$ and $F \subset m_X int_\delta(A)$. Let $X - A \subset U$ where U is $m_X\alpha g$-open, then $X - U \subset A$ and $X - U$ is $m_X\alpha g$-closed. By hypothesis, $X - U \subset m_X int_\delta(A)$. Follows $X - m_X int_\delta(A) \subset U$ but it is equivalent to $m_X cl_\delta(X - A) \subset U$. Therefore, $X - A$ is $\mathcal{M}_X\alpha\delta$-closed and hence A is $\mathcal{M}_X\alpha\delta$-open.

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