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ON RPS-CONTINUOUS AND RPS-IRRESOLUTE FUNCTIONS

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ABSTRACT

The authors introduced rps-closed sets and rps-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The aim of this paper is to introduce rps-continuous functions and rps-irresolute functions by using rps-closed sets and characterize their basic properties.

Keywords: *rps-open, rps-closed, rps-continuous, rps-irresolute.*
MSC 2010: *54C08*

1. INTRODUCTION:

The concept of continuity is connected with the concept of topology. The concept of g-continuity is discussed with g-open sets and open sets. The concept of irresoluteness is studied by using nearly open sets in topological spaces. The purpose of this paper is to introduce the concepts of rps-continuity and rps-irresoluteness that are characterized and their relationships with weak and generalized continuity available in [2, 3, 5, 6, 7, 8, 11, 13, 14, 15, 17, 22] are investigated.

2. PRELIMINARIES:

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X\A denotes the complement of A in X. We recall the following definitions and results .

Definition: 2.1

A subset A of a space (X, τ) is called

(i) pre-open [13] if A $\subseteq$ *int cl* A and pre-closed if *cl int* A $\subseteq$ A;

(ii) semi-open [9] if A $\subseteq$ *cl int* A and semi-closed if *int cl* A $\subseteq$ A;

(iii) semi-pre-open [1] if A $\subseteq$ *cl int cl* A and semi-pre-closed if *int cl int* A $\subseteq$ A;

(iv) α-open [16] if A $\subseteq$ *int cl int* A and α-closed if *cl int cl A* $\subseteq$ A;

(v) regular open [20] if A = *int cl* A and regular closed if A = *cl int* A.

The semi-pre-closure(resp.semi-closure, resp. pre-closure, resp. α-closure) of a subset A of X is the intersection of all Semi-pre-closed (resp. semi-closed, resp. pre-closed, resp. α-closed) sets containing A and is denoted by *spcl*A (resp. *scl*A, resp.*pcl*A, resp.*αcl*A).

Definition: 2.2

A subset A of a space X is called g-closed[10] (resp. rg-closed[17], resp. αg-closed[11], resp. gs-closed[4], resp. gp-closed[12], resp. gsp-closed[6], resp. gpr-closed[8], resp. rwg-closed[21], resp. pre-semi-closed[22], resp. pgpr-closed[2], resp. rps-closed[18]) if *cl*A $\subseteq$U (resp. *cl*A $\subseteq$U, resp. *αcl*A $\subseteq$ U, resp. *scl*A $\subseteq$ U, resp. *pcl*A$\subseteq$U, resp. *spcl*A$\subseteq$U, resp. *pcl*A$\subseteq$U, resp. *cl int*A$\subseteq$U, resp. *spcl*A$\subseteq$U, resp.*pcl*A$\subseteq$U, resp. *spcl*A$\subseteq$U) whenever A$\subseteq$U and U is open (resp. regular-open, resp.open, resp.open, resp.open, resp.open, resp. regular open, resp.regular open, resp. g-open, resp. rg-open, resp. rg-open)

A subset B of a space X is g-open if and only if X\B is g-closed. The analogous results hold for rg-open, αg- open, gs-open, gp-open, gsp-open, gpr-open, rwg-open, pre-semi-open, pgpr-open, rps-open sets.

Definition: 2.3

A function f:(X, τ)→(X, σ) is called g-continuous[5] (resp. rg-continuous[17], resp.αg-continuous[11], resp.gs-continuous[7], resp.gp-continuous[3], resp.gsp-continuous[6], resp.gpr-continuous[8], resp.rwg-continuous[15], resp.pre-semicontinuous[22], resp. pgpr-continuous[2], resp. regular continuous[17], resp. pre-continuous[13], resp. semi-pre-continuous[22], resp. α-continuous[14]) if $f^{-1}(V)$ is g-closed (resp. rg-closed, resp. αg-closed, resp. gs-closed, resp. gp-closed, resp. gsp-closed, resp. gpr-closed, resp. rwg-closed, resp. pre-semi-closed, resp. pgpr-closed, resp. regular closed, resp. pre-closed, resp. semi-pre-closed, resp. α-closed) in X for every closed sub set V of Y.

Definition: 2.4[19]

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For a subset A of a space X, *rps-cl*A = ∩{ F: A ⊆ F and F is rps-closed in X} is called the rps-closure of A.

Definition: 2.5[19]
Let (X, τ) be a topological space

and τ_{rps} = {V ⊆ X: *rps-cl*(X \ V) = X \ V}.

Lemma: 2.6[19]
Let x∈ X. Then x∈ *rps-cl*A if and only if V∩A ≠ ∅ for every rps-open set V containing x.

Remark: 2.7[19]
(i) rps-closure of a set A is not always rps-closed;
(ii) If A is rps-closed then *rps-cl*A = A.

Lemma: 2.8[19]
Let A and B be subsets of (X, τ). Then

(i) *rps-cl* ∅ = ∅ and *rps-cl* X = X.
(ii) If A ⊆ B, *rps-cl* A ⊆ *rps-cl* B.
(iii) A ⊆ *rps-cl* A.

Remark: 2.9[19]
Suppose τ_{rps} is a topology. If A is rps-closed in(X, τ), then A is closed in (X, τ_{rps}) .

Lemma: 2.10[19]
A set A ⊆ X is rps-open if and only if F ⊆ *spint* A whenever F ⊆ A, F is rg-closed.

Definition: 2.11[19]
A function f: (X, τ) → (Y, σ) is called rg-irresolute if $f^{-1}(V)$ is rg-closed in (X, τ) for every rg-closed sub set V of (Y, σ).

Lemma: 2.12 [18]
(i) Every semi-pre-closed set is rps-closed;
(ii) Every pgpr-pre-closed set is rps-closed;
. **(iii)** Every rps-closed set is pre-semi-closed.

3. RPS-CONTINUOUS FUNCTIONS:
In this section, we introduce and study rps-continuous functions.

Definition: 3.1
A function f: (X, τ) → (Y, σ) is called rps-continuous if $f^{-1}(V)$ is rps-closed in (X, τ) for every closed sub set V of (Y, σ).

Proposition: 3.2
Let f: (X, τ) → (Y, σ) be a function. Then

(i) if f is semi-pre-continuous, then f is rps-continuous;
(ii) if f is pgpr-continuous, then f is rps-continuous;
(iii) if f is pre-continuous, then f is rps-continuous;
(iv) if f is α-continuous, then f is rps-continuous;
(v) if f is regular continuous, then f is rps-continuous.

Proof: Suppose f is semi-pre-continuous (resp. pgpr-continuous). Let V be closed in (Y, σ). Then $f^{-1}(V)$ is semi-pre-closed (resp. pgpr-closed) in (X, τ). Using Lemma 2.12, $f^{-1}(V)$ is rps-closed in (X, τ). Then by using Definition 3.1, f is rps-continuous. This proves (i) and (ii). Now since regular closed ⇒ α-closed ⇒ pre-closed ⇒semi-pre-closed, the proof for (iii), (iv) and (v) follows from (i). This completes the proof.

Examples can be constructed to show that the reverse implications in Proposition 3.2 are not true.

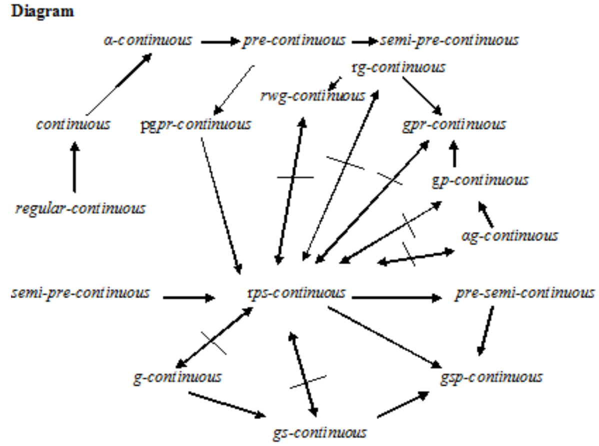
Proposition: 3.3
rps-continuity ⇒ pre-semi-continuity ⇒ gsp-continuity.

Proof: Let f: (X, τ) → (Y, σ) be a function. Suppose f is rps-continuous. Let V be closed in (Y, σ). Then $f^{-1}(V)$ is rps-closed in (X, τ). Using Lemma 2.12(iii), $f^{-1}(V)$ is pre-semi-closed in (X, τ). Then f is pre-semi-continuous. The rest follows from the fact that every pre-semi-closed set is gsp-closed. The proof is completed.

The reverse implications are not true. Examples can be constructed to see that the concepts of gp- continuity, rwg-continuity, αg- continuity, gpr- continuity, rg- continuity and g-continuity are independent with the concept of rps-continuity. The concepts of gs-continuity and rps-continuity are also independent.

Thus the above discussions lead to the following implication diagram. In this diagram, by "A ⟶ B" we mean A implies B but not conversely and "A ◀/▶ B" means that A and B are independent of each other.

Diagram



Theorem: 3.4
Let f: (X, τ) → (Y, σ) be a function. Then the following are equivalent.

(i) f is rps-continuous;
(ii) The inverse image of each open set in (Y, σ) is rps-open in (X, τ) ;
(iii) The inverse image of each closed set in (Y, σ) is rps-closed in (X, τ) .

Proof: Suppose (i) holds. Let G be open in Y. Then Y \ G is closed in Y. By (i) $f^{-1}(Y \setminus G)$ is rps-closed in X. But $f^{-1}(Y \setminus G) = X \setminus f^{-1}(G)$ which is rps-closed in X. Therefore $f^{-1}(G)$ is rps-open in X. This proves (i) ⇒ (ii). The implications **(ii)** ⇒ **(iii)** and **(iii)** ⇒ **(i)** follow easily.

Now we characterize rps-continuous functions by using the various closure and interior operators.

Theorem: 3.5
If a function f: (X, τ) → (Y, σ) is rps-continuous then f(*rps-cl* A) ⊆ *cl*(f(A)) for every subset A of X.

Proof: Let f: (X, τ) → (Y, σ) be rps-continuous. Let A ⊆ X. Then *cl*(f(A)) is closed in Y. Since f is rps-continuous, f^{-1}(*cl*(f(A)) is rps-closed in X.
Suppose y∈ f(*rps-cl* A).
Then

y=f(x), x∈ *rps-cl*A.

Let G be an open set containing y = f(x). Since f is rps-continuous, by Theorem 3.4, f^{-1}(G) is rps-open containing x so that f^{-1}(G) ∩ A ≠ Ø by Lemma 2.6. Therefore f(f^{-1}(G) ∩ A) ≠ Ø which implies

f(f^{-1}(G)) ∩ f(A) ≠ Ø.

Since f(f^{-1}(G))⊆G, G ∩ f(A) ≠ Ø.

This proves that y∈ *cl*(f(A)) that implies

f(*rps-cl* A) ⊆ *cl*(f(A)).

Theorem: 3.6
Let X be a space in which every singleton set is rg-closed. Then f : (X, τ) → (Y, σ) is rps-continuous if and only if x∈ *spint*(f^{-1}(V)) for every open sub set V of Y containing f(x).

Proof: Suppose f : (X, τ) → (Y, σ) is rps-continuous. Fix x∈ X and an open set V in Y such that f(x)∈ V. Then f^{-1}(V) is rps-open. Since x∈ f^{-1}(V) and since {x} is rg-closed, x∈ *spint*(f^{-1}(V)) by Lemma 2.10.

Conversely, assume that x∈ *spint*(f^{-1}(V)) for every open sub set V of Y containing f(x). Let V be an open set in Y. Suppose F ⊆ f^{-1}(V) and F is rg-closed. Let x∈ F. Then f(x) ∈ V so that x∈ *spint*(f^{-1}(V)) that implies F ⊆ *spint*(f^{-1}(V)) . Therefore by Lemma 2.10, f^{-1}(V) is rps-open. This proves that f is rps-continuous.

Theorem: 3.7
f: X → Y be a function. Let (X, τ) and (Y, σ) be any two spaces such that τ_{rps} is a topology on X. Then the following statements are equivalent.

(i) For every subset A of X, f(*rps-cl* A) ⊆ *cl*(f(A)) holds.
(ii) f: (X, τ_{rps}) → (Y, σ) is continuous.

Proof: Suppose (i) holds. Let A be open in (Y, σ).
Then Y \ A is closed in (Y, σ).
Then by (i)

f(*rps-cl*(f^{-1}(Y \ A)) ⊆ *cl*(f (f^{-1}(Y \ A))) ⊆ *cl* (Y \ A) = Y \ A

that implies

rps-cl(f^{-1}(Y \ A) ⊆ f^{-1} (Y \ A).

Using Definition 2.4, f^{-1}(Y \ A) = *rps-cl*(f^{-1}(Y \ A)).
That is X \ f^{-1}(A) = *rps-cl*(X \ f^{-1}(A)).

Then by Definition 2.5 , f^{-1}(A) is open in (X, τ_{rps}) and so f : (X, τ_{rps}) → (Y, σ) is continuous. This proves (ii).

Conversely suppose (ii) holds. Let A⊆X. Then *cl*(f(A)) is closed in (Y, σ). Since f : (X, τ_{rps}) → (Y, σ) is continuous, f^{-1}(*cl*(f(A)) is closed in (X, τ_{rps})
that implies by Definition 2.5,

rps-cl(f^{-1}(*cl*(f(A))) = f^{-1}(*cl*(f(A))).

Now we have

A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(*cl*(f(A))) and by Lemma 2.8 (ii),

rps-cl A ⊆ *rps-cl*(f^{-1}(*cl*(f(A))) = f^{-1}(*cl*(f(A))).

Therefore f(*rps-cl* A) ⊆ *cl*(f(A)).

The composition of two rps-continuous functions need not be rps-continuous. However the following proposition is true on composition of functions.

Proposition: 3.8
Let (X, τ), (Y, σ) and (Z, μ) be topological spaces such that σ_{rps} = σ. Let f:(X, τ)→(Y, σ) and g:(Y, σ) → (Z, μ) be rps-continuous functions. Then the composition g∘f: (X, τ) → (Z, μ) is a rps-continuous.

Proof: Let V be closed in (Z, μ). Since g is rps-continuous, g^{-1}(V) is rps-closed in (Y, σ).

Since σ_{rps} = σ, by Remark 2.9, g^{-1}(V) is closed in (Y, σ).

Since f is rps-continuous, f^{-1}(g^{-1}(V)) is rps-closed in (X, τ).

That is $(g∘f)^{-1}$(V) is rps-closed in (X, τ). Therefore g∘f is rps-continuous.

4. RPS-IRRESOLUTE FUNCTIONS:
In this section, rps-irresolute functions are introduced and their basic properties are discussed.

Definition: 4.1
A function f: (X, τ) → (Y, σ) is called rps-irresolute if f^{-1}(V) is rps-closed in (X, τ) for every rps-closed subset V of (Y, σ).

Theorem: 4.2
Every rps-irresolute function is rps-continuous.

Proof: Suppose f: (X, τ) → (Y, σ) is rps-irresolute. Let V be any closed subset of Y. Then V is semi-pre-closed in Y. Then using Lemma 2.12(i), V is rps-closed in Y. Since f is rps-irresolute, f^{1}(V) is rps-closed in X. This proves the theorem.

Theorem: 4.3
Let f: (X, τ) → (X, σ) be rg-irresolute and semi-pre-closed. Then f maps a rps-closed set in (X, τ) into a rps-closed set in (Y, σ).

Proof: Let A be rps-closed in (X, τ). Let f(A) ⊆ U, where U is rg-open in Y. Then

A ⊆ f^{-1}(U).

Since f is rg-irresolute, $f^{-1}(U)$ is rg-open in X.

Since A is rps-closed,

$$spcl A \subseteq f^{-1}(U) \text{ that implies } f(spcl A) \subseteq U.$$

Since f is semi-pre-closed f(*spcl*A) is semi-pre-closed that implies

$$spcl(f(A)) \subseteq spcl(f(spcl A)) = f(spcl A) \subseteq U.$$

By using Definition 2.2, f(A) is rps-closed in (Y, σ).

Theorem: 4.4
Let f : X → Y and g:Y → Z be any two functions. Let h = g∘f. Then
(i) h is rps-continuous if f is rps-irresolute and g is rps-continuous.
(ii) h is rps-irresolute if both f and g are both rps-irresolute and
(iii) h is rps-continuous if g is continuous and f is rps-continuous.

Proof: Let V be closed in Z. Suppose f is rps-irresolute and g is rps-continuous. Since g is rps-continuous, $g^{-1}(V)$ is rps-closed in Y. Since f is rps-irresolute, using Definition 4.1, $f^{-1}(g^{-1}(V))$ is rps-closed in X. This proves (i). To prove (ii), let f and g be both rps-irresolute. Then $g^{-1}(V)$ is rps-closed in Y. Since f is rps-irresolute, using Definition 4.1 $f^{-1}(g^{-1}(V))$ is rps-closed in X. This proves (ii). Finally to prove (iii), let g be continuous and f be rps-continuous. Then $g^{-1}(V)$ is closed in Y. Since f is rps-continuous, using Definition 3.1, $f^{-1}(g^{-1}(V))$ is rps-closed in X. This proves (iii).

The next theorem follows easily as a direct consequence of definitions.

Theorem: 4.5
A function f: X → Y is rps-irresolute if and only if the inverse image of every rps-open set in Y is rps-open in X.

Theorem: 4.6
Let f: (X, τ) → (Y, σ) be rps-continuous and $\tau_{rps} = \tau$. Then f is rps-irresolute.

Proof: Let F be rps-closed in Y. Then by Remark 2.9, F is closed in Y. Since f is rps-continuous, using Definition 4.1, $f^{-1}(F)$ is rps-closed in X. Therefore f is rps-irresolute.

Theorem: 4.7
Suppose f: (X, τ) → (Y, σ) is rps-irresolute and $\tau_{rps} = \tau$. Then f is continuous.

Proof: Let F be closed in Y. Then F is rps-closed in Y. Since f is rps-irresolute, using Definition 4.1, $f^{-1}(F)$ is rps-closed in X. Since $\tau_{rps} = \tau$, $f^{-1}(F)$ is closed in X. Therefore f is continuous.

Definition: 4.8
A function f: (X, τ) → (Y, σ) is said to be rps-closed (resp. rps-open) if for every rps-closed (resp. rps-open) set U of X the set f(U) is rps-closed (resp. rps-open) in Y.

Theorem: 4.9
Let f: (X, τ) → (Y, σ) be a bijection. Then the following are equivalent.

(i) f is rps-open.
(ii) f is rps-closed.
(iii) f^{-1} is rps-irresolute.

Proof: Suppose f is rps-open. Let F be rps-closed in X. Then X \ F is rps-open . By Definition 4.8, f(X \ F) is rps-open. Since f is a bijection, Y \ f(F) is rps-open in Y. Therefore f is rps-closed. This proves (i) ⇒(ii).

Let g = f^{-1}. Suppose f is rps-closed. Let V be rps-open in X. Then X \ V is rps-closed in X. Since f is rps-closed , f(X \ V) is rps-closed. Since f is a bijection,
Y \ f(V) is rps-closed that implies f(V) is rps-open in Y. Since g = f^{-1} and since g and f are bijection $g^{-1}(V) = f(V)$ so that $g^{-1}(V)$ is rps-open in Y. Therefore f^{-1} is rps-irresolute. This proves (ii) ⇒ (iii).

Suppose f^{-1} is rps-irresolute. Let V be rps-open in X. Then X \ V is rps-closed in X. Since f^{-1} is rps-irresolute and $(f^{-1})^{-1}(X \setminus V) = f(X \setminus V) = Y \setminus f(V)$ is rps-closed in Y that implies f(V) is rps-open in Y. Therefore f is rps-open. This proves (iii) ⇒ (i).

Theorem 4.10
Let f : X → Y and g : X → Y be two functions. Suppose f and g are rps-closed (resp. rps-open). Then g∘f is rps-closed (resp. rps-open).

Proof: Let U be any rps-closed (resp.rps-open) set in X. Since f is rps-closed, using Definition 4.8, f(U) is rps-closed(resp. rps-open) in Y. Again since g is rps-closed (resp. rps-open), using Definition 4.8, g(f(U)) is rps-closed (resp. rps-open) in Z. This shows that g∘f is rps-closed (resp. rps-open).

CONCLUSION:
The weak and generalized forms of continuity namely rps-continuity and rps-irresoluteness are introduced and characterized with analogous recent concepts in the literature of general topology.

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