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A NEW TYPE OF DIFFERENCE SEQUENCE SPACES OF FUZZY REAL NUMBERS

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ABSTRACT

The Idea of difference sequence sets introduced by Kizmaz [1]. In this paper, we introduce certain new difference sequence spaces of fuzzy real numbers and give some topological properties and inclusion relations.

Keyword and phrases: *Difference sequence, Fuzzy real number, Solid space, Symmetric space.*

AMS Mathematical Subject Classification: *40A05, 40A25, 40A30, 40C05.*

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1. INTRODUCTION:

The concept of fuzzy was introduced by Zadeh [2]. Latter on sequences of fuzzy number have been discussed by Matloka [3]. Tripathy and Nanda [4], Nuray and Savas [5], Bilgin [6], Altin, Et, and colak [7], Kwon [8] and many others.

Let D denote the set of all closed bounded intervals A= [*A, A*] on the real line R, where *A and A* denote the end point of A. For A, B $\in$ D define A $\le$ B and iff *A $\le$* B and A* $\le$ B*, d (A,B) = *max* (|**A -*B*|, *A* -B**|). It is well known that *(D,d)* is a complete metric space and *d(A,B)* is called the distance between the intervals *A* and *B*. Also it is easy to see that $\le$ defined above is a partial order relation in D (see Matloka [3]).

A fuzzy number is a fuzzy subset of the real line *R* which is bounded, convex and normal. Let *R (I)* denote the set of all fuzzy numbers which are upper semi continuous and have compact support. For $X \in R(I)$, the α -level set X^α *for* $0 < \alpha \le 1$ is defined by, $X^\alpha = \{t \in R: X(t) \ge \alpha\}$. The 0-level i.e. X^0 is the closure of strong 0-cut, i.e. $X_0 = cl\{t \in R: X(t) > 0\}$. The absolute value of $X \in R(I)$ i.e. |X| is defined as (see Kaleva and Seikhala [9]).

$$|X|(t) = \begin{Bmatrix} \max\{X(t), X(-t)\}, \text{ for } t \ge 0 \\ 0, 0 < t \end{Bmatrix}$$

For $r \in R, \bar{r} \in R(I)$ *is defined by* $\bar{r}(t) = \begin{cases} 1 \text{ for } t = r \\ 0, \text{otherwise} \end{cases}$

For $r \in R$ *and* $X \in R(I)$ *we define* $rX(t) = \begin{cases} X(r^{-1}t) \text{ for } r \ne 0 \\ \bar{0}, \text{ for } r = 0 \end{cases}$

Define $d: R(I) \times R(I) \to R$ By

$$\bar{d}(X,Y) = \sup_{0<\alpha\le 1} d(X^\alpha, Y^\alpha), \text{ for } X, Y \in R(I)$$

The it is well known that $(R(I), \bar{d})$ is a complete metric space. A sequence $X=(X_k)$ of fuzzy numbers is said to be converge to a fuzzy number X_0 if for if for every $\varepsilon > 0$ there is a positive integer No such that $d(X_k, X_0) < \varepsilon$ for $k > N_0$. and $X=(X_k)$ of fuzzy numbers is said to be Cauchy sequence if for every $\varepsilon > 0$ there is a positive integer No such that

$$X = (X_k, X_l) < \varepsilon \text{ for } k,l > N_0.$$

A sequence space *E is said to be solid if* $(Y_n) \in E$, whenever $(X_n) \in E$ *and* $|Y_n| \le |X_n|$, for all $n \in N$. *A sequence space E is said to be monotone if E* contains the canonical pre-images of all its step space, Let $X = (X_n)$ be a sequence, the *S(X)* denotes the set of all permutations of the elements of (X_n) i.e. $S(X) = \{(X_{\pi(n)}): \pi$ is a permutation of N}. A sequence spaces *E* is said to be symmetric if $S(X) \subset E$ for all $X \in E$. A sequence space E is said to be convergence-free if $(Y_n) \in E$ wherever (X_n) *E and* $X_n = \bar{0}$ implies $Y_n = \bar{0}$.

REMARK: A sequence space *E* is solid implies that *E* is monotone. Let ℓ^0 be the set of all complex sequences and $l_{\infty,c}$ and c_0 be the sets of all bounded, convergent and null sequences $x = (x_k)$ with complex terms, respectively.

The idea of difference sequence spaces was introduced by Kizmaz [1]. In 1981, Kizmaz [1] define the sequence spaces.

$$l_\infty(\Delta) = \{x = \{x_k\} \in \ell^0 : (\Delta x_k) \in l_\infty\},$$

$$c(\Delta) = \{x = \{x_k\} \in \ell^0 : (\Delta x_k) \in c\},$$

and

$$c_0(\Delta) = \{x = \{x_k\} \in \ell^0 : (\Delta x_k) \in c_0\},$$

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where $\Delta x = (x_k - x_{k+1})$. There are Banach spaces with the norm

$$\|x\|_\Delta = |x_1| + \|\Delta x\|_\infty.$$

Then Et and Colak [10] generalized the above sequence spaces, to the sequence spaces

$$X(\Delta^r) = \{x = \{x_k\} \in \ell^0 : \Delta^r x_k \in X\},$$

for $X = l_\infty$, c and c_0, where $r \in |x|$,

$$\Delta^0 x = (x_k),\ \Delta x = (xk - x_{k+1}),\ \Delta^r x = (\Delta^r x_k - \Delta^r x_{k+1})$$

and so that

$$\Delta^r x_k = \sum_{i=0}^{r} (-1)^i \begin{bmatrix} r \\ i \end{bmatrix} x_{k+i}$$

Difference sequence spaces have been studied by Colak and Et [11], Tripathy and Esi [12], Et and Esi [13], Et, Altin, and altinok [14], Khan [15, 16] and many others.

Let c denote the space whose elements are finite sets of distinct positive integers. Given any element σ of C, we denote by $c(\sigma)$ the sequence $\{c_n(\sigma)\}$ which is such that $c_n(\sigma) = 1$ if $n \in \sigma$, $c_n(\sigma) = 0$ otherwise. Further

$$C_s = \left\{\sigma \in C : \sum_{n=1}^{\infty} c_n(\sigma) s\right\} (see[17]),$$

the set of those σ whose support has cardinality at most s, and

$$\Phi = \left\{\phi = \{\phi_k\} \in \ell^0 : \phi_1 > 0, \Delta\phi_k \geq 0 \, and \, \Delta\left(\frac{\phi_k}{k}\right) \leq 0 \, (k = 1,2...)\right\}$$

where $\Delta\phi_k = \phi_k - \phi_{k-1}$; and ℓ^0 is the set of all real sequences.

For $\phi \in \Phi$, we define the following sequence space, introduce in [18],

$$m(\phi) = \left\{x = \{x_k\} \in \ell^0 : \sup_{s\geq 1} \sup_{\sigma \in C_s} \left(\frac{1}{\phi_s} \sum_{k\in\sigma} |x_k|\right) < \infty\right\}$$

The space $m(\phi)$ was extended to $m(\phi,p)$ by Tripathy and Sen [19] as follows:

$$m(\phi,p) := \left\{x = \{x_k\} \in \ell^0 : \sup_{s\geq 1} \sup_{\sigma \in C_s} \frac{1}{\phi_s} \sum_{k\in\sigma} |x_k|^p < \infty\right\}$$

Let u be a fixed positive integer and $u = (u_k)$ be any fixed scalars sequence of non zero complex numbers (see [20,21,13]. Khan [16] generalized this sequence space and introduced the sequence space $m(\Delta_u^u, \phi, p)$: defined as follows:

$$m(\Delta_u^u, \phi, p) : \left\{x = \{x_k\} \in \ell^0 : \sup_{s\geq 1} \sup_{\sigma\in C_s} \frac{1}{\phi_s} \sum_{k\in\sigma} (|\Delta_u^u x_k|^p) < \infty, 0 p < \infty\right\}$$

where

$$\Delta_u^u x_k = (u_k x_k),$$
$$\Delta_u x_k = (u_k x_k - u_{k+1} x_{k+1}),$$
$$\Delta_u^u x_k = \left(\Delta_u^{u-1} x_k - \Delta_u^{u-1} x_{k+1}\right)$$

and so that

$$\Delta_u^u x_k = \sum_{i=0}^{u} (-1)^i \begin{bmatrix} u \\ i \end{bmatrix} u_{k+i}\, x_{k+i}$$

We introduce the sequence space $m(\Delta_u^u, \phi, p)^F$ of fuzzy real numbers as follows;

$$m(\Delta_u^u, \phi, p)^F = \left\{X = (X_k) \sup_{s\geq 1} \sup_{\sigma\in C_s} \frac{1}{\phi} \sum_{k\in\sigma} \bar{d}(\Delta_u^u X_k, \bar{0})^p > \infty, 0 < p < \infty\right\}$$

2. MAIN RESULTS:

In this section, we prove some results involving the sequence space $m(\Delta_u^u, \phi, p)^F$ with twos values of p such that $0 < p < \infty$

Theorem: 2.1. (a) The sequence space $m(\Delta_u^u, \phi, p)^F$ for $1 \leq p < \infty$ is a complete metric space by the metric,

$$\rho(X,Y) = \sum_{i=1}^{u} \bar{d}(X_i, Y_i) + \sup_{s\geq 1} \sup_{\sigma\in C_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left(\bar{d}\left(\Delta_u^u X_k, \Delta_u^u Y_k\right)^p\right)^{1/p} \tag{2.1.1}$$

for $X, Y \in m(\Delta_u^u, \phi, p)^F$.

(b) The sequence space $m(\Delta_u^u, \phi, p)^F$ for $0 < p < 1$ is a complete metric space by the metric

$$\eta(X,Y) = \sum_{i=1}^{u} \bar{d}(X_i, Y_i) + \sup_{s\geq 1} \sup_{\sigma\in C_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left(\bar{d}\left(\Delta_u^u X_k, \Delta_u^u Y_k\right)\right)^p, \tag{2.1.2}$$

for $X, Y \in m(\Delta_u^u, \phi, p)^F$.

Proof: It is clear that $m(\Delta_u^u, \phi, p)^F$ is a metric space by (2.1.1) for $1 \leq p < \infty$ and (2.1.2) for $0 < p < 1$. We need to show that $m(\Delta_u^u, \phi, p)^F$ is complete.

We give the proof only for $0 < p < 1$. Since the proof is analog for the spaces $1 \leq p < \infty$, we omit the details.

Let $(X^{(l)})$ be a Cauchy sequence in $m\left(\Delta_u^u,\phi,p\right)^F$ where $X^l = \left(X_k^l\right)_k = \left(X_k^l, X_2^1,....\right) \in m\left(\Delta_u^u,\phi,p\right)^F$ for each $l \in$ |x|. Then for given $\varepsilon > 0$ there exists $n_0 \in$ |x| such that

$$\eta\left(X^{(l)},X^{(t)}\right)<\varepsilon, \text{ for all } l,\ t > n_0.$$

Hence

$$\sum_{i=1}^{u}\bar{d}\left(X_i^{(l)},X_i^{(t)}\right)+\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k^{(l)},\Delta_u^u X_k^{(t)}\right)\right)^p<\varepsilon, \qquad (2.1.3)$$

for all $l,t > n_0$.

Now we obtain

$$\sum_{k\in\sigma}\bar{d}\left(X_i^{(l)},X_i^{(t)}\right)<\varepsilon, \text{ for all } l,t>n_0.$$

which implies that

$$\bar{d}\left(X_i^{(l)},X_i^{(t)}\right)<\varepsilon, \text{ for all } l,t>n_0 \text{ for } i=1,2,3\ldots u.$$

Hence, $\left(X_i^{(l)}\right)$ is a Cauchy sequence in $R(I)$, so it is convergent in $R(I)$, by the completeness property of $R(I)$, for $i = 1,2,3,\ldots.u$

Also,

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k^{(l)},\Delta_u^u X_k^{(t)}\right)\right)^p<\varepsilon, \text{ for all } l,\ t > n_0.$$

On taking s = 1, we have,

$$\bar{d}\left(\Delta_u^u X_k^{(l)},\Delta_u^u X_k^{(t)}\right)<\left(\varepsilon\phi_1\right)^{1/p}, \text{ for all } l,t>n_0,\ k\in|x|$$

Which implies that for each fixed k ($1 \leq k < \infty$), the sequence $\left(\Delta_u^u X_k^{(l)}\right)$ is a Cauchy sequence in $R(I)$, hence converges in $R(I)$.

Hence, we get, $\lim \Delta_u^u X_k^{(l)} = \Delta_u^u X_k$ for $k \in$ |x|

Taking limit as $t \to \infty$ in (2.1.3), we get,

$$\sum_{i=1}^{u}\bar{d}\left(X_i^{(l)},X_i\right)+\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k^{(l)},\Delta_u^u X_k\right)\right)^p<\varepsilon,$$

for all $l > n_0$. (2.1.4)

$$\Rightarrow \quad \eta\left(X^{(l)},X\right)<\varepsilon, \text{ for all } l>n_0.$$

Since $(X^{(l)}) \in m\left(\Delta_u^u,\phi,p\right)^F$ and by (2.1.4), for all $l > n_0$.

we have,

$$\eta(X^{(l)},\theta)\leq\eta(X^{(l)},\theta)+\eta(X^{(l)},\theta)<\infty.$$

Hence, $X \in m\left(\Delta_u^u,\phi,p\right)^F$. Hence, $m\left(\Delta_u^u,\phi,p\right)^F$ is a complete metric space. This completes the proof of the theorem.

Theorem 2.2: $m\left(\Delta_u^u,\phi\right)^F \subset m\left(\Delta_u^u,\phi,p\right)^F$, for all $1 \leq p < \infty$.

Proof: Let $X \in m\left(\Delta_u^u,\phi\right)^F$, then we have

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)=K\ (<\infty)$$

Hence, for each fixed s, we have

$$\sum_{k\in\sigma}\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)=K\phi_s \quad \sigma\in\phi_s \text{ for each fixed s.}$$

Hence

$$\left\{\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p\right\}^{1/p}<K\phi_s,\ \sigma\in\phi_s \text{ for each p o and } 1\leq p<\infty.$$

Thus $X \in m\left(\Delta_u^u,\phi,p\right)^F$.

Theorem 2.3: For any two sequence $\left(\phi_s\right)$ and $\left(\Psi_s\right)$ of real numbers

$$m\left(\Delta_u^u,\phi,p\right)^F \subset m\left(\Delta_u^u,\Psi,p\right)^F$$

if and only if

$$\sup_{s\geq 1}\left(\frac{\phi_s}{\Psi_s}\right)<\infty$$

Proof: Let $X \in m\left(\Delta_u^u,\phi,p\right)$. Then

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p<\infty$$

Suppose that

$$\sup_{s\geq 1}\left(\frac{\phi_s}{\Psi_s}\right)<\infty.$$

Then $\phi_s \leq K\psi_s$ and so that $\dfrac{1}{\psi_s}\leq\dfrac{K}{\phi_s}$ for some positive number K and for all s. Therefore we have

$$\frac{1}{\psi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_v^u X_k,\bar{0}\right)\right)^p\leq\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_v^u X_k,\bar{0}\right)\right)^p \text{ for each s.}$$

Now

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\psi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p\leq K\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p$$

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$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\Psi_s}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p<\infty$$

Therefore $X \in \in m\left(\Delta_u^u \Psi, p\right)^F$.

Conversely, let $m\left(\Delta_u^u \phi, p\right)^F \subseteq m\left(\Delta_u^u \Psi, p\right)^F$ and suppose that

$$\sup_{s\geq 1}\left(\frac{\phi_s}{\Psi_s}\right)=\infty .$$

Then there exists a increasing sequence (s_i) of naturals number such that $\lim \left(\frac{\phi_{s_i}}{\psi_{s_i}}\right)=\infty$. Now for every $B \in R^+$, the set of positive real numbers, there exists $i_0 \in$ |x| such that $\frac{\phi_{s_i}}{\psi_{s_i}} > B$ for all $s_i \geq i_0$. Hence $\frac{1}{\psi_{s_i}} > \frac{B}{\psi_{s_i}}$ and so that

$$\frac{1}{\psi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p \ \frac{B}{\psi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p$$

for all $s_i \geq i_0$. Now taking supremum over $s_i \geq i_0$ and $\sigma\in C_s$ we get

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\psi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p > B\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p. \tag{2.2.1}$$

Since (2.2.1) holds for all $B\in R^+$ (we may take B sufficiently large) we have

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\psi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p=\infty$$

When $X \in \in m\left(\Delta_u^u \phi, p\right)^F$ with

$$0<\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p<\infty$$

Therefore $X\notin m\left(\Delta_u^u \psi, p\right)^F$. This contradict to

$$m\left(\Delta_u^u \phi, p\right)^F \subseteq m\left(\Delta_u^u \psi, p\right)^F .$$

Hence $\sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right)<\infty.$

Form Theorem 2.3, we get the following result.

Corollary 2.4: $m\left(\Delta_u^u \phi, p\right)^F = m\left(\Delta_u^u \psi, p\right)^F$ if and only if

$$0<\inf_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right)\leq \sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right)<\infty.$$

Theorem 2.5: $l_p\left(\Delta_u^u\right)^F \subseteq m\left(\Delta_u^u, \phi, p\right)^F \subseteq l_\infty\left(\Delta_u^u\right)^F$

Proof: Since $m\left(\Delta_u^u, \phi, p\right)^F = 1_p\left(\Delta_u^u\right)^F$ for $\phi_n = 1$, for all |x|, then

$$l_p\left(\Delta_u^u\right)^F \subseteq m\left(\Delta_u^u, \phi, p\right)^F$$

Now suppose that $X\in m\left(\Delta_u^u, \phi, p\right)^F$. Then we have

For $s = 1$,

$\bar{d}\left(\Delta_u^u, X_k, \bar{0}\right) < K\phi_1$, for all $k \in$ N and for some positive integer K.

Thus $X \in l_\infty\left(\Delta_u^u\right)^F$. This completes the proof of Theorem.

The proof of the following result is obvious.

Corollary 2.6: If $0 < p < q$, then

$$m\left(\Delta_u^u, \phi, q\right)^F \subseteq m\left(\Delta_u^u, \phi, p\right)^F .$$

Theorem 2.7: The sequence space $m\left(\Delta_u^u, \phi, p\right)^F$ is not solid for $0 < p < \infty$.

Proof: The proof follows from the following example.

Take $u = 3$, $u = 1$, $p = 2$ and $\phi_s = 1$, for all $s \in$ |x|. Let $X_k = \bar{I}$ for all $k \in$N. Then we have, $\bar{d}\left(\Delta^3 X_k, \bar{0}\right) = 0$ for all $k \in$ N. Hence

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_{s_i}}\sum_{k\in\sigma}\left(\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right)^p=0 .$$

This implies that, $(X_k) \in m\left(\Delta^3\phi, 2\right)^F$. Consider the sequence (α_k) of scalars defined by

$$\alpha_k=\begin{cases}1 & , \textit{for } k \textit{ is even,}\\ 0 & , \textit{otherwise.}\end{cases}$$

So $\qquad \bar{d}\left(\Delta^3\alpha_k X_k, \bar{0}\right)=1$

$$\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{\phi_{s_i}}\sum_{k\in\sigma}\left[\bar{d}\left(\Delta_u^u X_k,\bar{0}\right)\right]^p=\sup_{s\geq 1}\sup_{\sigma\in C_s}\frac{1}{1}\sum_{k\in\sigma}[1]^p=\sup_{s\geq 1}\sup_{\sigma\in C_s} s=\infty .$$

Implies that $\left(\alpha_k x_k\right) \notin m\left(\Delta^3, \phi, 2\right)^F$. Hence $m\left(\Delta_u^u, \phi, p\right)^F$ is not solid.

Theorem 2.8 The sequence space $m\left(\Delta_u^u, \phi, p\right)^F$ is not symmetric for $0 < p < \infty$.

Proof: The proof follows from the following example.

Take u = 1 ϕ_s = 1, for all $s \in$ |x|. Let X_k = $\bar{I}$ for all $k \in$ N Then we have, $\bar{d}\left(\Delta X_k, \bar{0}\right)$ =1 for all $k \in$ N. Let $\left(Y_k\right)$ be rearrangement of $\left(X_k\right)$ such that

$$(Y_k) = (X_1, X_2, X_4, X_3, X_9, X_5, X_16, X_6, X_25\ldots.)$$

This implies that

$$\sup_{s \geq 1} \sup_{\sigma \in C_s} \frac{1}{\phi} \sum_{k \in \sigma} \left[\bar{d}\left(\Delta_u^u Y_k, \bar{0}\right)\right]^p = \infty .$$

Hence $\left(Y_k\right) \notin m\left(\Delta_u^u, \phi, p\right)^F$. Hence $m\left(\Delta_u^u, \phi, p\right)^F$ is not symmetric.

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